Computer mathematical models in belaying techniques

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1. Introduction

The belaying technique, as well at the rope characteristic, plays a leading role in the determination of the rope tension during the fall of the leader in rock/ice climbing; therefore the analysis of the mechanics of belaying deserves great attention. The use of a computer model to simulate the experimental results obtained during a long set of tests with various techniques is discussed here.

Particular attention was paid to the mechanical factors that are significant for the comparison of different belaying techniques.
Considerations related to extreme occurrences, such as the brake not working correctly or the rope getting stuck in a crack, are outside the scope of this work.

The model presented here can be used to discuss the ropes properties and related belaying devices during the actual climbing practice.

2. Why a computer mathematical model in climbing activity?

It is unusual to use sophisticated mathematical tools in the evaluation of a sporting activity such as climbing: so why this "academic" approach?
The Safety Commission of the Italian Alpine Club has carried out, during the last few years, a long set of tests in order to get elements for a better understanding and for the comparison of belaying techniques.
In all tests the major parameters were recorded, in particular the loads occurring at the most crucial locations within the safety chain, such as the last runner and the belaying stance.
More than two hundred tests have been carried out for different belaying conditions and on different climbing terrain: i. e. rock and ice.
Particular situations were studied on an artificial facility, a 16 m height metallic tower located at the outskirts of Padova, where fall tests can quickly be performed and carefully repeated, using the guided fall of a steel mass.
An unforeseen problem arose when the first sets of tests were analysed: the behaviour of the various operators was often so different, in the same type of test, as to frequently lead to contradictory and misleading results. Even repeatability for the same operator was a problem. It was therefore difficult to explain the facts from the point of view of physics, thus to compare the various ways of operating and the different belaying techniques.
All our efforts could have been vanished by this problem!
In this situation, the use of a mathematical model for the comparison of the experimental results produced by various operators and belaying techniques was considered to be the most neutral and objective method.

This approach has four outstanding advantages:

- **The step by step schematisation of the problem allows the understanding of complex phenomena not perfectly deduced from experimental tests**
- **It provides a rational analysis of the most important aspects of the phenomena in which the physics play a leading role**
- **It provides a fully comparable analysis of different belaying techniques**
- **It points out the best parameters to be used for a rigorous comparison between different belaying techniques**

3. The basic assumptions for the models

It is worthwhile noting that the physics of the model is based on a long and painstaking review of the filmed actions of the operators and of the physical values (forces, masses, displacements) recorded as a function of time. **The analysis of the slowed-down motion view of the films was widely used**. The model was aimed at re-producing the experimental evidence.
A large number of runs were necessary to adjust the model parameters to the experimental records of forces as a function of time.
This approach facilitated very much the equations definition.

<u>The rope</u>

The rope is one of the key points, as well as the belayer's behaviour (see later), in the generation of the load level within the safety chain.
In the model, the rope has been considered very simply as a **spring** having a constant characteristic based on the elasticity modulus (Young modulus multiplied by the cross sectional area of the rope) and the length of the rope portion between two salient points of the safety chain (the last runner, the brake, the body of the falling mass etc.).
In parallel with the elastic behaviour of the rope, the dissipative effect of the internal friction has been applied as a **damper** in parallel with the spring.
Both the spring and the damper characteristics have been assumed by a regression analysis of the results from Dodero Tests.
The rope behaviour has strong non-linear characteristics; in order to simplify the model, constant parameters have been assumed by limiting the regression analysis up to the first tension peak in the experimental load curve. The assumption seems to be satisfactory for the aim of the model.
It is worthwhile noting that the model can also be used as a tool to analyse the different loads in the safety chain as a function of **different rope stiffness characteristics**: this approach will be more useful as long as non linear rope properties will be introduced.
The present paper can be considered as a contribution for a better understanding of the actual operating conditions of a climbing rope.
Schematically a rope span has been assumed as in **fig. 1**

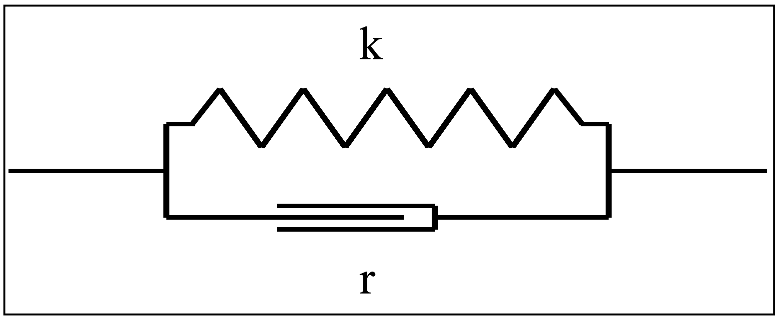

Fig. 1 : rope span schematization

The brake

The brake, whatever the type, has been modelled as a **"force multiplier"** that is a device that amplifies the force generated by the hand of the belayer. As first approximation the multiplying factor has been assumed constant for a given rope and braking device: its dependence on the tension and on the running speed of the rope through the brake is not very significant for our study.

The brake has been schematically represented in **fig. 2**

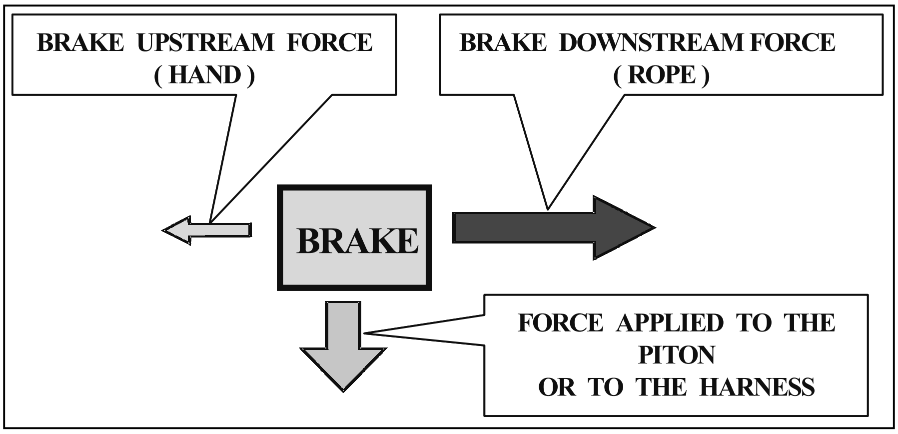

Fig. 2 : brake schematization

The belayer behaviour

This point represents the main issue of the whole model and has been identified after a careful slowed-down analysis of the filmed behaviour of the operators.

From this analysis, a first phase of the belaying action has been identified, named here **inertial phase**. In this phase the braking force generated by the belayer has an inertial characteristic, as if it were generated by the mass of an increasing part of the operator's body (the hand, the arm, the shoulder etc.) according to the muscle's rigidity.

In other words: the braking force is generated by the inertia of the increasing part of the operator's body accelerated by the rope gripped by the hand.

A second phase, named here **slipping phase**, has less defined features but nevertheless a dominant characteristic: the rope slipping throughout the brake generates a force roughly constant and independent from the body inertia of the belayer. A relevant example of this phase is represented by the impact of the belayer's hand against the brake and the subsequent rope slippage throughout the hand.

In the model the force intensity generated by the brake has been assumed linearly dependent on the rope slippage.

This assumption has been made to give flexibility to the model in order to get the best fitting of the experimental results. A more sophisticated schematization of this phase seems to be useless as far as the human behaviour appears to be, according to the slow down filmed pictures, very different from test to test and from person to person.

4. The modelling approach used

In the models definition the classical motion equations (dynamic equilibrium) have been adopted; the reason of the choice is due to the simplicity of the equilibrium equations when compared with the energetic approach.
The latter is frequently used in simplified energy balances sometimes misleading when energies evaluated in different time instants are compared; this is the case for example when the friction energy generated by the brake is correlated with the maximum tension in the rope not considering that the tension peak generally doesn't correspond in time to the end of the rope slippage as can be seen in comparing **fig.4** and **fig.5**.
Later on two models corresponding to different belaying techniques will be presented.

5. The model of a safety chain with the wall connected brake

In order to simplifying the analysis, the case of a single runner has been presented.
A more general model, with up to five runners, is available as well.
The schematisation of this case is presented in **fig. 3**

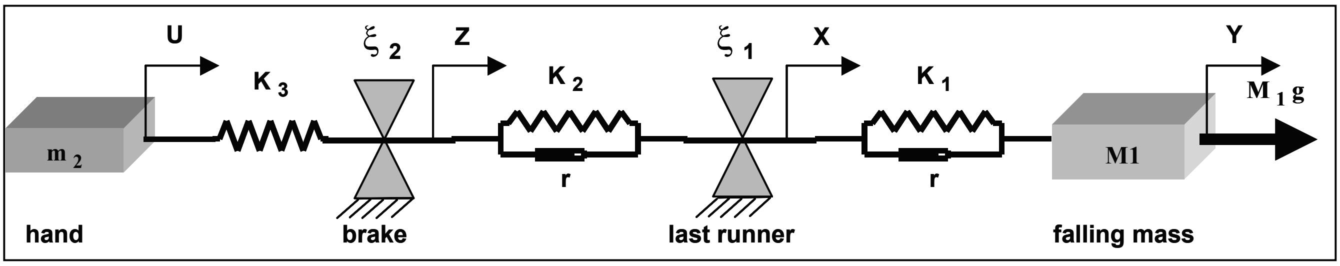

Fig.3 : wall connected brake safety chain

The model has been defined by 4 degrees of freedom (d.o.f.) represented by:

- the geometrical position of the falling mass (Y)
- the rope slipping on the last runner (X)
- the rope slipping through the brake (Z)
- the geometrical position on the belayer's hand (U)

The differential equations governing the phenomena are the following:

$$\begin{cases} M_1 \cdot Y'' + T_1 - M_1 \cdot g = 0 \\ T_1 = \xi_1 \cdot T_2 \\ T_2 = \xi_2 \cdot T_3 \\ m_2 \cdot U'' - T_3 = 0 \end{cases}$$

where

$T_1 = r * [(Y' - X')] + K_1 * (Y - X)$	tension in leader-runner rope span
$T_2 = r * (X' - Z') + K_2 * (X - Z)$	tension in runner-brake rope span
$T_3 = K_3 * (Z - U)$	tension at the belayer's hand
ξ	friction factor at the runner and at the brake

The numerical solution of the equations **(see following point 7)** gives the evolution of the main parameters as a function of time. In **fig. 4** the position of the falling mass and the slippage of the rope through the brake (both for the inertial and slipping phase) are represented for a fall defined by the following parameters:

Falling parameters (used as example)

• **falling mass**	**80**	**kg**
• **fall height**	**8**	**m**
• **rope span between the brake and the runner**	**7.15**	**m**
• **brake characteristic (force multiplier)**	**7.5**	
• **inertial braking mass**	**2.5**	**kg**

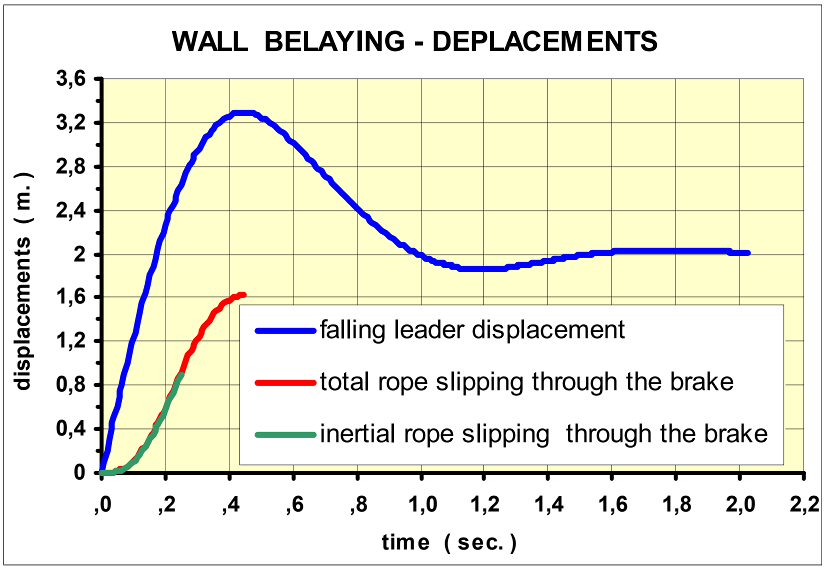

Fig.4 : wall connected brake safety chain : displacements

In **fig. 5** the forces generated by the fall are represented at the last runner, at the belaying stance and at leader harness. In the figure the correspondent experimental loads are reported as well: a good correlation can be noted between experimental and model; only a small load fluctuation, in the experimental curve, can be observed due to a non regular hand behaviour during the slippage phase.

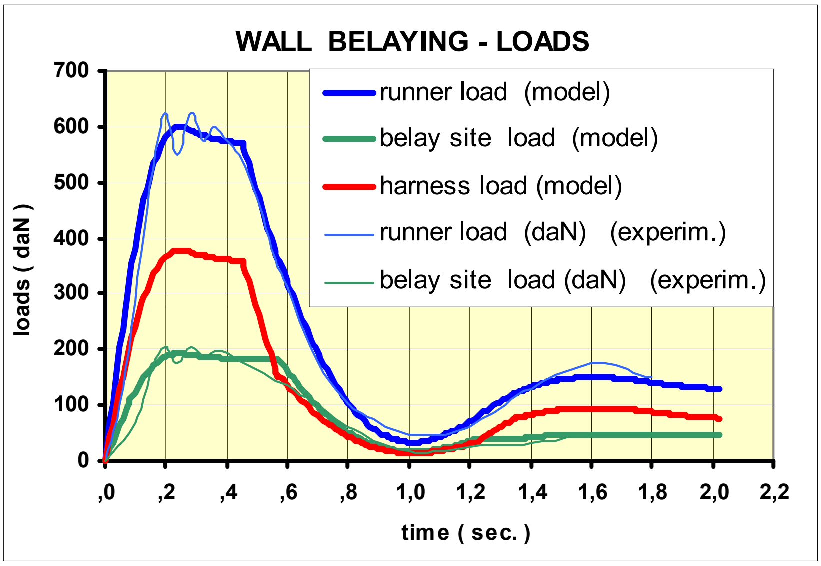

Fig.5 : wall connected brake safety chain : forces

6. The model of a safety chain with the harness connected brake

Following the same approach, a model has been set up for this kind of belaying technique. In this situation the belayer is lifted up toward the wall by the force generated by the brake : the lifting is then limited by the self-safe sling action. In order to have a better representation of the actual behaviour of the belayer, a two-dimension model has been requested representing the lifting up (vertical) and the approaching motion to the wall (horizontal).

The model for this case has been represented in **fig. 6**

In this case a 6 d.o.f. model has been requested describing:

- the geometrical position of the falling mass - **Y** (1 d.o.f)
- the rope slipping on the last runner - **X** (1 d.o.f)
- the rope slipping through the brake - **X** (1 d.o.f)
- the geometrical position of the belayer's hand – **U** (1 d.o.f.)
- the geometrical position on the belayer – **V** , **W** (2 d.o.f.)

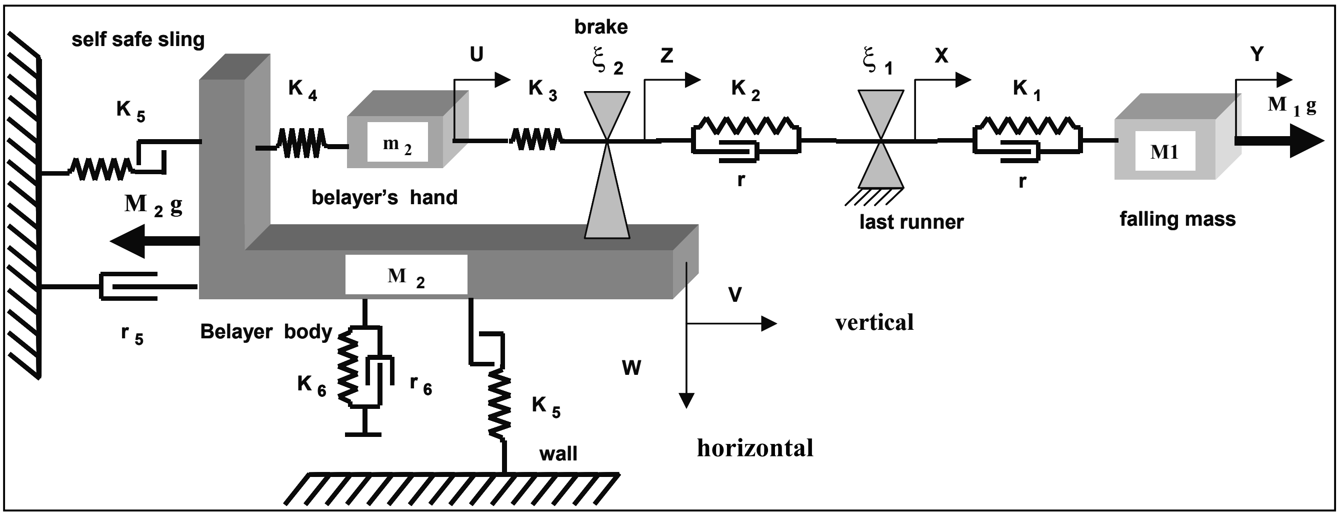

Fig.6 : harness connected brake safety chain

In the model the friction of the belayer's body against the wall is represented (dumpers r_5 and r_6) as well as the impact absorption (spring k_6); the belayer motion limitation is represented by the self-safe sling (spring k_5). The spring k_4 represents the connection between the belayer's hand and his body.

The differential equations governing the phenomena are the following:

$$\begin{cases} M_1 * (Y_{rel}'' + V'') + T_1 - M_1 * g = 0 \\ T_1 = \xi_1 * T_2 \\ T_2 = \xi_2 * T_3 \\ m_2 * (U_{rel}'' + \lambda * V'') - T_3 + T_4 + \lambda * m_2 * g = 0 \\ M_2 * W'' + r_{6\,horiz} * W' - (T_2 - T_3) * \sin(\alpha) - K_5 * W + K_6 * W = 0 \\ M_2 * V'' + r_{5\,vert} * V' - T_4 + M_2 * g - (T_2 - T_3) * \cos(\alpha) + K_5 * V = 0 \end{cases}$$

Where

$T_1 = r_1 * [(Y_{rel}' + V') - X'] + K_1 * [(Y_{rel} + V) - X]$ tension in leader-runner rope span
$T_2 = r_1 * [X' - (Z_{rel}' + V')] + K_2 * [X - (Z_{rel} + V)]$ tension in runner-brake rope span
$T_3 = K_3 * (Z_{rel} - U_{rel})$ tension at the belayer's hand
$T_4 = K_4 * U_{rel}$ elastic force generated by the hand-shoulder connection
α angle between T_2 and vertical due to the belayer's position with respect to the wall
λ parameter =1 if hand weight is considered (hand vertical motion); otherwise (horizontal motion) =0

Again the displacements and the loads generated are represented in **figs. 7** , **8** respectively starting from the same parameters used for the wall connected brake; the belayer mass has been assumed 85 kg.

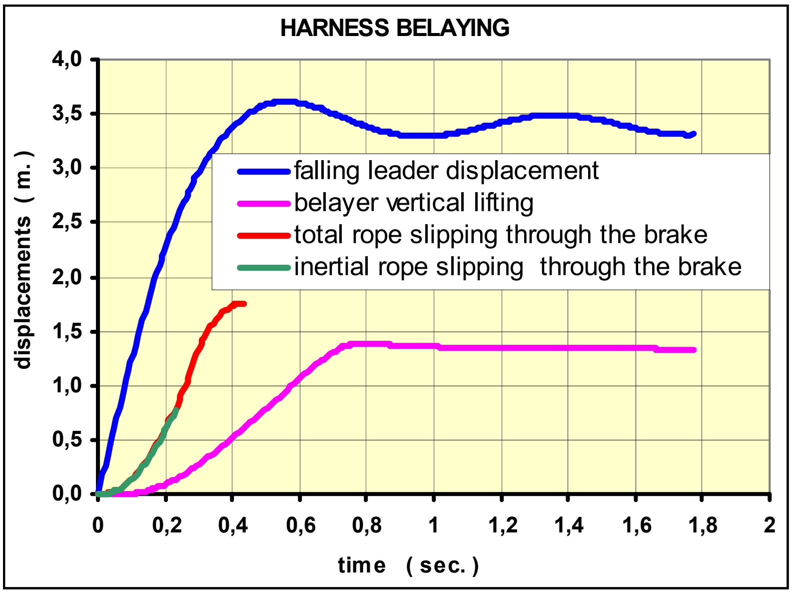

Fig.7 : harness connected brake safety chain : displacements

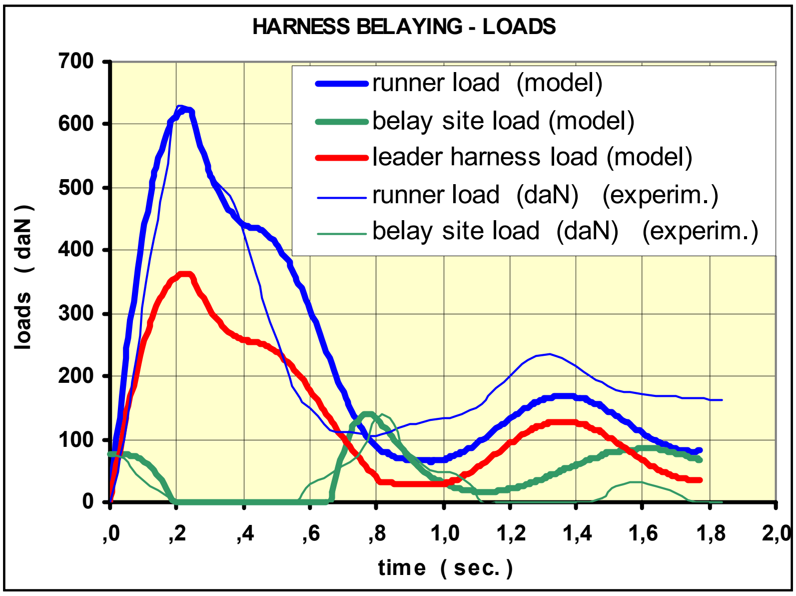

Fig.8 : harness connected brake safety chain : forces

It can be noted that the belayer's lift at the maximum safety load instant (roughly 0.2 sec) is very little: this is somewhere in contrast with the current believing of the climbing world.
In fact it is a common belief that the belayer's lifting reduces the safety chain loads.
Actually it is the low braking force generated by the belayer the true origin of the low load of the harness belaying technique also according to the tests: the low inertial force is due to the small mass involved in the braking action typical of this type of braking.
In fig. 8 the experimentally registered loads are reported as well: it can be noted a good correlation between the model and the experimental data for the load peak; the correlation is not as good as well for the second part of the phenomena when the friction and the bumps of

the belayer's body against the wall play a fundamental role that is not easily outlined by equations because of the casual behaviour of the belayer.

7. The solution of the differential equations

The differential equations have been solved by Taylor forward finite differences transforming the system into a linear equations system.
In order to have a friendly tool, mainly for graphic representation, in a situation in which a trial and error approach has been widely applied, the EXCELL has been used; surely a more "scientific" language such as "matlab" or "FORTRAN" could be used taking advantage from the better fitting between the tool and the problem to be solved but, on the other hand, having less flexibility in treating the problem. As second approach and only after a correct and well-defined "specification" of the problem, more suitable simulating languages can be applied.
The equations shown previously represent only a part of the whole analysis: suited controls allow switching from a set of equations to other sets valid for different time spans. In fact these equations are valid for the inertial phase only, being a little bit different for the slipping phase.
The handling of the aforementioned controls has been one of the most difficult part on the analysis carried out due to the multiplicity of situations to be considered.

8. Conclusion

The simulation, through a computer model, of different belaying techniques has been carried out.
A comparison with the experimental data has been performed with a good correlation between experimental and simulated data giving soundness to the model results.
The computer use, in the analysis of falls during the climbing activity, has been proven to be very useful mainly when physical phenomena have to be studied.
Furthermore the analysis by models allow the reduction of the problem to its main physical aspects not always clear in experimental tests: in fact spurious effects frequently shadow the basic aspects of the phenomena as we unfortunately experimented, making the understanding more difficult.
The mathematical models can be efficiently used for comparing the different belaying techniques and for analysing parameters influence such as :

- the falling leader and the belayer's masses
- the fall height
- different rope stiffness
- the brake efficiency
- the belayer position
- the belaying stance organization
- the intermediate runners influence
- others

All these analysis give relevant suggestions for better belaying techniques and for a better human behaviour to be adopted during this critical operation.

For the people having, may be, a critical approach to this method considered too far from the practical situation faced during an actual fall, it can be underlined that

> **the model suggests solutions to specific problems by highlighting their physical aspects: only a well finalized experimental test can confirm the soundness the model suggestions avoiding a very expensive, complex and time consuming test campaign.**

This approach has been successfully experimented during the activities carried out by our Commission.