AC Voltage Regulators

AC voltage regulators have a constant voltage ac supply input and incorporate semiconductor switches which vary the rms voltage impressed across the ac load. These regulators generally fall into the category of naturally commutating converters since their thyristor switches are naturally commutated by the alternating supply. This converter turn-off process is termed line commutation. The regulator output current, hence supply current, may be discontinuous or non-sinusoidal and as a consequence input power factor correction and harmonic reduction are usually necessary, particularly at low output voltage levels (relative to the input ac voltage magnitude). A feature of direction conversion of ac to ac is the absence of any intermediate energy stage, such as a capacitive dc link or energy storage inductor. Therefore ac to ac converters are potentially more efficient but usually involve a larger number of switching devices and output is lost if the input supply is temporarily lost.

There are three basic ac regulator categories, depending on the relationship between the input supply frequency $f_s$ which is usually assumed single frequency sinusoidal, possibly multi-phased, and the output frequency $f_c$. Without the use of transformers (or boost inductors), the output voltage rms magnitude $V_{rms}$ is less than or equal to the input voltage rms magnitude $V_s$, $V_{rms} \leq V_s$.  

- output frequency increased, $f_c > f_s$, for example, the matrix converter
- output frequency decreased, $f_c < f_s$, for example, the cycloconverter
- output frequency fundamental = supply frequency, $f_c = f_s$, for example, a phase controller

### 12.1 Single-phase ac regulator

Figure 12.1a shows a single-phase thyristor ac regulator supplying an L-R load. The two inverse parallel connected thyristors can be replaced by any of the bidirectional conducting and blocking switch categories of naturally commutating converters since their thyristor switches are naturally commutated by the alternating supply. The converter turn-off process is termed line commutation. The thyristor output current, hence supply current, may be discontinuous or non-sinusoidal and as a consequence input power factor correction and harmonic reduction are usually necessary, particularly at low output voltage levels (relative to the input ac voltage magnitude).

A feature of direction conversion of ac to ac is the absence of any intermediate energy stage, such as a capacitive dc link or energy storage inductor. Therefore ac to ac converters are potentially more efficient but usually involve a larger number of switching devices and output is lost if the input supply is temporarily lost.

There are three basic ac regulator categories, depending on the relationship between the input supply frequency $f_s$ which is usually assumed single frequency sinusoidal, possibly multi-phased, and the output frequency $f_c$. Without the use of transformers (or boost inductors), the output voltage rms magnitude $V_{rms}$ is less than or equal to the input voltage rms magnitude $V_s$, $V_{rms} \leq V_s$.  

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- output frequency decreased, $f_c < f_s$, for example, the cycloconverter
- output frequency fundamental = supply frequency, $f_c = f_s$, for example, a phase controller

#### 12.1.1 Single-phase ac regulator – phase control with line commutation

For control by phase angle delay, the thyristor gate trigger delay angle is $\alpha$, where $0 \leq \alpha \leq \pi$, as indicated in figure 12.1b. The fundamental of the output angular frequency is the same as the input angular frequency, $\omega = 2\pi f_s$. The thyristor current, shown in figure 12.1b, is defined by equation (11.76); that is

$$\frac{d i(t)}{dt} + R i(t) = \sqrt{V_s^2 \sin \alpha}$$  \hspace{1cm} (V) \hspace{1cm} \alpha \leq \theta \leq \beta \hspace{1cm} (rad) \hspace{1cm} (12.1)

The solution to this first order differential equation has two solutions, depending on the delay angle $\alpha$ relative to the load natural power factor angle, $\phi = \tan^{-1} \frac{V_s}{I_s}$.

**Case 1: $\alpha > \phi$**

When the delay angle exceeds the load power factor angle the load current always reaches zero before $\pi + \phi$, thus the differential equation boundary conditions are zero. The solution for $i$ is

$$i(t) = \frac{\sqrt{V_s^2 \sin \alpha}}{Z} \sin (\omega t - \phi)$$  \hspace{1cm} (A) \hspace{1cm} (12.2)

where $Z = \sqrt{R^2 + L^2} \ (\text{ohms})$ and $\tan \phi = \omega L / R = V_s / \sqrt{L^2}$

Provided $\alpha > \phi$ both ac regulator thyristors will conduct and load current flows symmetrically as shown in figure 12.1b. The thyristor conduction period is given by the angle $\theta = \alpha - \phi$. The thyristor current extinction angle $\beta$ for discontinuous load current can be determined with the aid of figure 11.9a, but with the restriction that $\beta - \alpha \leq \pi$, or figure 12.1d, or by solving equation 11.78, that is:

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{-\pi / Z \omega}$$  \hspace{1cm} (12.4)

From figure 12.1b the rms output voltage is

$$V_{rms} = \sqrt{\frac{2 \pi}{\omega}} \left[ \frac{\sqrt{V_s^2 \sin \alpha}}{Z} \right]^2$$  \hspace{1cm} (12.5)

The maximum rms output voltage is when $\alpha = \phi$ in equation (12.5), giving $V_{rms} = V_s$.

The rms load current is found by the appropriate integration of equation (12.2) squared, namely

$$I_{rms} = \frac{1}{Z} \left[ \frac{\sqrt{V_s^2 \sin \alpha}}{Z} \right]^2 \sin (\omega t - \phi) = \frac{1}{Z} \left[ \frac{\sqrt{V_s^2 \sin \alpha}}{Z} \right]^2 \sin (\omega t - \phi)$$  \hspace{1cm} (12.6)

The maximum rms output current is when $\alpha = \phi$ in equation (12.6), giving $I_{rms} = V_s / Z$.

![Figure 12.1](image-url)
From equation (12.6), the thyristor rms current is given by $I_{rms} = I_m / \sqrt{2}$ and is a maximum when $\alpha \leq \phi$. That is:

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{Z}} \quad (12.7)$$

Using the fact that the average voltage across the load inductor is zero, the rectified mean voltage (hence current) can be used to determine the thyristor mean current rating:

$$V_T = \frac{I_T}{R} = \frac{\beta}{2} \sqrt{2} V \sin \omega t \quad \text{(V)}$$

The mean thyristor current $I_T = \frac{\beta}{2} V / R$, that is:

$$I_T = \frac{\beta}{2} \sqrt{2} V \sin \omega t / R \quad (\Lambda)$$

The maximum thyristor current is for a load $\alpha = \phi$ and $\beta = \pi + \phi$. That is:

$$I_T = \frac{\beta}{2} V / \pi R = \frac{\sqrt{2}}{\pi} \sqrt{Z}$$

The thyristor forward and reverse voltage blocking ratings are both $\sqrt{2}V$.

The load current factor, using $\cos \phi = R/Z$, is:

$$F_{FF} = \frac{I_m}{I_r} = \cos \phi \left[ \sqrt{2} \frac{\beta - \alpha - \sin(\beta - \alpha)}{\cos \phi} \cos(\alpha + \phi + \beta) \right]^{1/2}$$

which is a maximum when $\alpha = \phi$, giving $F_{FF} = \pi / 2\sqrt{2}$.

The thyristor current factor is $F_{FF} = \sqrt{2} F_{FF}$, which is a maximum when $\alpha = \phi$, $F_{FF} = \pi \pi$.

The load power is:

$$P_L = \frac{V^2}{R} \cos \phi \left[ \beta - \alpha - \sin(\beta - \alpha) \right] / \cos \phi \cos(\alpha + \phi + \beta)$$

which is a maximum when $\alpha = \phi$, giving $P_L = \frac{V^2}{Z} R$.

The supply power factor is:

$$\cos \phi \left[ \frac{\beta - \alpha - \sin(\beta - \alpha)}{\cos \phi} \cos(\alpha + \phi + \beta) \right]^{1/2}$$

which is a maximum when $\alpha = \phi$, giving $\cos \phi = \cos \phi$.

For an inductive $L$-load, the fundamental load voltage components ($\cos$ and $\sin$ respectively) are:

$$a_L = \frac{\sqrt{2} V}{2\pi} \left( \cos(2\pi - \cos 2\beta) \right)$$

$$b_L = \frac{\sqrt{2} V}{2\pi} \left( 2(\beta - \alpha) - \sin 2\beta \sin 2\alpha \right)$$

$$a_n = \frac{\sqrt{2} V}{\pi} \cos(\pi + 1) \alpha - \cos(\pi + 1) \beta$$

$$b_n = \frac{\sqrt{2} V}{\pi} \sin(\pi + 1) \alpha - \sin(\pi + 1) \beta$$

for $n = 3, 5, 7, \ldots$ odd.

The Fourier component magnitudes and phases are given by:

$$c_n = \sqrt{2} V / \cos(\pi + 1) \alpha - \cos(\pi + 1) \beta$$

$$\phi_n = \tan^{-1} \left( \frac{\beta - \alpha - \sin(\beta - \alpha)}{\cos(\alpha + \phi + \beta)} \right)$$

provided $\alpha \leq \phi$.

(12.16)

If $\alpha \leq \phi$, then continuous ac load current flows, and equation (12.14) reduces to $a_L = 0$ and $b_L = \sqrt{2}V$, when $\beta = \pi + \phi$ and $a = \phi$ are substituted.

The supply apparent power can be grouped into a component at the fundamental plus components at the harmonic frequencies:

$$S^2 = V^2 P^2 / \pi R + S^2$$

Thus, giving $S^2 = V^2 P^2 / \pi R$ (12.17). $S^2 = V^2 P^2 / \pi R + S^2$ + $D^2$.

where $D$ is the supply current distortion due to the harmonic currents. The current harmonic components are found by dividing the load Fourier voltage components by the load impedance at that frequency. Equation (12.16) gives the current harmonic angles $\phi$ and magnitudes according to

$$I_n = \frac{V_n}{Z_n} = \frac{\sqrt{2} V / \sin(\pi + 1) \alpha - \cos(\pi + 1) \beta)}{\sqrt{R + (\alpha L)}}$$

(12.18)

Case 2. $\alpha \leq \phi$ (continuous gate pulses)

When $\alpha \leq \phi$, a pure sinusoidal load current flows, and substitution of $\alpha = \phi$ in equation (12.2) results in

$$i(t) = \frac{\sqrt{2} V}{\sin(\pi + 1) \alpha - \cos(\pi + 1) \beta}$$

(12.19)

If a short duration gate trigger pulse is used and $\alpha < \phi$, and directional load current may result. The device to be turned on is reverse-biased by the conducting device. Thus if the gate pulse ceases before the previous half-cycle load current has fallen to zero, only one device conducts. It is therefore usual to employ a continuous gate pulse, or stream of pulses, from a unit, then for $\alpha < \phi$ a sine wave output current results.

For both delay angle conditions, equations (12.1) to (12.14) are valid, except the simplification $\beta = \alpha + \pi$ is used when $\alpha \leq \phi$, which gives the maximum values for those equations. That is, for $\alpha \leq \phi$, substituting $\alpha = \phi$

$$i(t) = \frac{\sqrt{2} V}{\sin(\pi + 1) \alpha - \cos(\pi + 1) \beta}$$

(12.20)

If $\alpha \leq \phi$, then continuous ac load current flows, and equation (12.14) reduces to $a_L = 0$ and $b_L = \sqrt{2}V$, when $\beta = \pi + \phi$ and $a = \phi$ are substituted.

The average output voltage and current are zero. The mean half-cycle output voltage, used to determine the thyristor mean current rating, is found by integrating the supply voltage over the interval $a$ to $\pi$, $\beta = \pi$.

$$I_{rms} = \frac{\sqrt{2} V}{\pi R}$$

(12.21)

The equations (12.1) to (12.20) can be simplified if the load is purely resistive. Continuous output current only flows for $\alpha > 0$, since $\phi = \tan^{-1} (0 - 0^\circ)$. Therefore the output equations are derived from the discontinuous equations (12.2) to (12.14), with $\phi = 0$. The average output voltage and current are zero. The mean half-cycle output voltage, used to determine the thyristor mean current rating, is found by integrating the supply voltage over the interval $a$ to $\pi$, $\beta = \pi$. The average output voltage and current are zero. The mean half-cycle output voltage, used to determine the thyristor mean current rating, is found by integrating the supply voltage over the interval $a$ to $\pi$, $\beta = \pi$.
The current waveform is symmetrical about $\omega = \pm \alpha \pi$, since $v(t) = v(\omega t)$ and $a = 0$.

From equation (12.5) the rms output voltage for a delay angle $\alpha$ is

$$V_{rms} = \frac{\sqrt{2}V}{\pi} \left(1 + \cos \alpha \right)$$

whence $\bar{I}_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{2}V}{\pi R} \left(1 + \cos \alpha \right)$, which has a maximum value of $\bar{I}_{rms} = \frac{\sqrt{2}V}{R}$ when $a = 0$.

The load current harmonics are found by dividing the voltage components by $V_{rms}$.

The load current is given by:

$$I_{n} = \frac{V_{rms}}{R} \left(1 + \frac{2\omega - \sin 2\omega}{2\pi} \right)$$

which has a maximum of $V_{rms}$ when $a = 0$.

The rms output current and supply current from $I_{rms} = V_{rms}/R$ is

$$I_{rms} = \frac{V_{rms}}{R} \left(1 + \frac{2\omega - \sin 2\omega}{2\pi} \right)$$

and

$$I_{rms} = \frac{V_{rms}}{R} \left(1 + \frac{2\omega - \sin 2\omega}{2\pi} \right)$$

The maximum rms supply current is $I_{rms} = V_{rms}/R$ at $a = 0$ when the maximum rms thyristor current is $I_{T, rms} = V_{rms}/2 \sqrt{2}$.

Therefore the output power, with $V_{rms} = R I_{rms}$ for a resistive load, is

$$P_{out} = I_{rms} V_{rms} = \frac{V_{rms}^2}{R} \left(1 + \frac{2\omega - \sin 2\omega}{2\pi} \right)$$

The input power is $P_{in} = V_{rms} I_{rms}$.

The supply power factor $\lambda$ is defined as the ratio of the real power to the apparent power, that is

$$\lambda = \frac{P_{in}}{V_{rms} I_{rms}}$$

where the apparent power is

$$S = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} \left(1 + \frac{2\omega - \sin 2\omega}{2\pi} \right)$$

and $Q = \sqrt{S^2 - P^2}$. The fundamental reactive power is

$$Q_f = V_{rms} I_{rms} \left(1 + \frac{2\omega - \sin 2\omega}{2\pi} \right)$$

The thyristor current (and voltage for a resistive load) form factor (rms to mean), shown in figure 12.2, is

$$FF_{in} = \frac{I_{rms}}{I_{in}} = \frac{1 + \cos \alpha}{\pi}$$

From equation (12.14), the thyristor current crest factor is

$$\delta = \frac{I_{rms}}{I_{in}} = \frac{2}{1 + \cos \alpha}$$

The Fourier voltage components for a resistive load (with $\beta = \pi$ in equations (12.14) and (12.15)) are

$$a_n = \frac{\sqrt{2}V}{2\pi} \left(\cos (n\pi - 1) - \cos (n\pi - 1) \omega \sin \omega \right)$$

$$b_n = \frac{\sqrt{2}V}{2\pi} \left(\cos (n\pi - 1) \omega \sin \omega \right)$$

for $n = 1, 3, 5, \ldots$ odd. Figure 12.2 show the relative harmonic rms magnitudes and dependence on $\alpha$.

The load current harmonics are found by dividing the voltage components by $R$, since $i(\omega t) = v(\omega t)/R$. For the purely inductive load case, the equations and waveforms for the half-wave controlled rectifier in section 11.3.1i, apply. Kirchhoff’s voltage law gives

$$I_{rms} = \frac{\sqrt{2}V}{\omega L} \sin \omega t$$

The load current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{\omega L} \sin \omega t$$

The current waveform is symmetrical about $\pi$. For the purely inductive load, the load power factor angle is $\phi = \frac{\pi}{2}$. Since the inductor voltage average is $V_{rms}$, current conduction will be symmetrical about $\pi$, and with short gate pulses, preventing the reverse direction thyristor from conducting, as shown in figure 12.2. Therefore, if the delay angle is less than $\frac{\pi}{2}$, but less than $\frac{\pi}{2}$, the equations and waveforms for the half-wave controller with a pure resistive load.

The thyristor current (and voltage for a resistive load) form factor (rms to mean), shown in figure 12.2, is

$$FF_{rms} = \frac{I_{rms}}{I_{in}} = \frac{1 + \cos \alpha}{\pi}$$

From equation (12.14), the thyristor current crest factor is

$$\delta = \frac{I_{rms}}{I_{in}} = \frac{2}{1 + \cos \alpha}$$

The Fourier voltage components for a resistive load (with $\beta = \pi$ in equations (12.14) and (12.15)) are

$$a_n = \frac{\sqrt{2}V}{2\pi} \left(\cos (n\pi - 1) - \cos (n\pi - 1) \omega \sin \omega \right)$$

$$b_n = \frac{\sqrt{2}V}{2\pi} \left(\cos (n\pi - 1) \omega \sin \omega \right)$$

for $n = 1, 3, 5, \ldots$ odd. Figure 12.2 show the relative harmonic rms magnitudes and dependence on $\alpha$. The load current harmonics are found by dividing the voltage components by $R$, since $i(\omega t) = v(\omega t)/R$. For the purely inductive load case, the equations and waveforms for the half-wave controlled rectifier in section 11.3.1i, apply. Kirchhoff’s voltage law gives

$$I_{rms} = \frac{\sqrt{2}V}{\omega L} \sin \omega t$$

The load current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{\omega L} \sin \omega t$$

The current waveform is symmetrical about $\pi$. For the purely inductive load, the load power factor angle is $\phi = \frac{\pi}{2}$. Since the inductor voltage average is $V_{rms}$, current conduction will be symmetrical about $\pi$, and with short gate pulses, preventing the reverse direction thyristor from conducting, as shown in figure 12.2. Therefore, if the delay angle is less than $\frac{\pi}{2}$, but less than $\frac{\pi}{2}$, the equations and waveforms for the half-wave controller with a pure resistive load.
Figure 12.3. Single-phase full-wave thyristor ac regulator with a pure inductor load:
(a) $\alpha > \frac{\pi}{2}$; (b) $\alpha < \frac{\pi}{2}$, gate pulse until $\pi$; and (c) $\alpha < \frac{\pi}{2}$, short gate pulse.

With a purely inductive load, the average output voltage is zero. If uni-directional current flows (due to the use of a narrow gate pulse), as shown in figure 12.3c, the average load current, hence average thyristor current, for the conducting thyristor, is

$$I_{\text{th ave}} = \frac{\sqrt{2} V}{\omega L} \left[ \cos \alpha - \cos \omega \alpha \right]$$

which with uni-polar pulses has a maximum of $\sqrt{2} V/\omega L$ at $\alpha = 0$.

The rectified average load voltage is

$$V_{\text{p}} = \frac{\sqrt{2} V}{\pi} \left(1 + \cos \alpha\right) \quad 0 \leq \delta \leq \frac{\pi}{2}$$

The rms load and supply (and one thyristor) current is

$$I_{\text{th rms}} = \frac{\sqrt{2}}{\omega L} \left[ \frac{\sqrt{2} V}{\pi} \left(1 + \cos \alpha - \cos \omega \alpha\right) \right]$$

The thyristor current form factor is

$$F_{\text{FF}} = \frac{\sqrt{2} V}{\sin \alpha + (\pi - \alpha) \cos \alpha}$$
which has a maximum value of $\frac{1}{2}\pi$ when $\alpha = \frac{1}{2}\pi$.

The rms load voltage is

$$V_L = \left[ \frac{1}{2} \int_{0}^{\pi} (V_b \sin \omega t)^2 \, dt \right]^{1/2}$$

(12.40)

ii. **Extended gate pulse period**

When the gate pulses are extended to $\pi$, continuous current flows, as shown in figure 12.3b, given by $I_{\text{rms}} = V_{\text{avg}} L$, lagging $V$ by $\frac{1}{2}\pi$. Each thyristor conducts an average current and rms current of

$$I_{\text{avg}} = \frac{\sqrt{2} V}{\pi \omega L}$$

$$I_{\text{rms}} = \frac{I_{\text{avg}}}{\sqrt{2}}$$

(12.41)

$$\pi \geq \alpha \geq \frac{1}{2}\pi$$

(symmetrical gate pulses)

The output voltage and current are symmetrical, as shown in figure 12.3a, hence the average output voltage and current are both zero, as is the average input current. The average thyristor current is given by

$$\bar{I_T} = \frac{1}{2} \int_{0}^{\pi} V_b \cos \omega t \, dt$$

(12.42)

which has a maximum of $V_b / 2 \sin \alpha$ at $\alpha = \frac{1}{2}\pi$.

The rectified average load voltage over half a cycle is

$$V_L = \frac{2 \sqrt{2} V}{\pi} (1 + \cos \alpha)$$

(12.43)

The rms load and supply current is

$$I_{\text{rms}} = \sqrt{2} I_{\text{rms}} = \frac{V}{\sqrt{2}} \left[ \frac{1}{2} \int_{0}^{\pi} (\cos \omega t - \cos \alpha)^2 \, dt \right]$$

(12.44)

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} \left[ \frac{1}{2} \int_{0}^{\pi} \left( \cos \omega t - \cos \alpha \right)^2 \, dt \right]$$

The rms load voltage is

$$V_L = \left[ \frac{1}{2} \int_{0}^{\pi} (V_b \sin \omega t)^2 \, dt \right]^{1/2}$$

(12.45)

The maximum rms voltage and current are $V_{\text{rms}} = V$ and $I_{\text{rms}} = V / X$ at $\alpha = \frac{1}{2}\pi$.

The rms equations for $\alpha$ greater than and less than $\frac{1}{2}\pi$ are basically the same except the maximum period over which a given thyristor conducts changes from $\pi$ to $2\pi$ (respectively), hence the rms values differ by $\sqrt{2}$. Since the output power is zero, the supply power factor is zero, for bidirectional current.

If the controller in figure 12.1a is used in the half-controlled mode (thyristor and anti-parallel diode), the resultant dc component precludes its use in ac transformer applications. The controller is limited to low power ac applications because of dc restrictions on the ac mains supply.

12.1.iii - **Load sinusoidal back emf**

When the ac controller load comprises an ac back emf $V_{\text{back}}$ of the same frequency as the ac supply $V$, as with embedded generation, then, when the thyristors conduct, the load effectively sees the vector difference between the two ac voltages, $V - V_{\text{back}}$, as shown in figure 12.5.

$$V_{\text{back}} = V - V_{\text{avg}}$$

$$= V - V_{\text{avg}} \cos \omega t = V - V_{\text{avg}} (\cos \omega t + j \sin \omega t)$$

$$= V - V_{\text{avg}} \cos \omega t - V_{\text{avg}} \sin \omega t$$

(12.46)

where

$$V_{\text{avg}} = \frac{1}{2} V \left[ \frac{1}{2} \int_{0}^{\pi} (\cos \omega t + j \sin \omega t)^2 \, dt \right]^{1/2}$$

$$= V \left[ \frac{1}{2} \int_{0}^{\pi} \cos^2 \omega t + \sin^2 \omega t \, dt \right]^{1/2}$$

$$= V \left[ \frac{1}{2} \int_{0}^{\pi} 1 \, dt \right]^{1/2}$$

$$= \frac{V}{\sqrt{2}}$$

The AC-chopper characteristics with ac back emf and purely resistive or inductive load.

(a) purely resistive and ac source load and (b) purely inductive and ac source load.

(c) Phase displacement of resultant voltage of an ac emf opposing the ac mains.
The rms output current and supply current are both given by
\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{Z_{\text{rms}}} \]

where \( I_{\text{rms}} \) is the rms output current, \( V_{\text{rms}} \) is the rms output voltage, and \( Z_{\text{rms}} \) is the rms impedance.

The passive part of the load can now be analysed as in sections 12.1.1i and ii, but the thyristor phase triggering delay angles are shifted by \( \varphi \) with respect to the original ac supply reference, as shown in the phasor diagrams in figure 12.5.

If the diode is replaced by a thyristor and the supply voltage is increased, then the load current is continuous and bidirectional, ac. The load, hence thyristor phase angle, \( \varphi \), with respect to the ac supply, is therefore given by the angle of the voltage (and the current in the case of a resistor load) across the passive part of the load.

As seen in the waveforms in figure 12.5, the load current is dependent on the relative magnitudes and angle between the two ac sources, the type of load, and the thyristor phase delay angle. Performance features with a resistive load and inductive load are illustrated in Example 12.1d.

### Example 12.1a: Single-phase ac regulator – 1

If the load of the 50 Hz 240V ac voltage regulator shown in figure 12.1 is \( Z = 7.1 + j 7.1 \Omega \), calculate the load natural power factor angle, \( \varphi \). Then, assuming bipolar load current conduction, calculate

(a) the rms output voltage, and hence
(b) the output power and rms current, whence input power factor and supply current distortion factor, \( \mu \)

for

i. \( \alpha = \pi \)

ii. \( \alpha = \frac{\pi}{3} \radian \)

**Solution**

From equation (12.3) the load natural power factor angle is

\[ \varphi = \tan^{-1} \left( \frac{X_L}{R} \right) = \tan^{-1} \left( \frac{7.1}{7.1} \right) = \frac{\pi}{3} \radian \]

\( Z = \sqrt{R^2 + (X_L)^2} = \sqrt{71 + 71} = 10 \Omega \)

i. \( \alpha = \pi \)

(a) Since \( a = \pi \), the load current is continuous and bidirectional, ac. The rms load voltage is 240V.

(b) From equation (12.20) the power delivered to the load is

\[ P_L = P_{\text{rms}} R = \frac{V_{\text{rms}}^2}{R} \cos \varphi \]

\[ = \frac{240^2 \cos \varphi}{100} = 0.07 \text{ kW} \]

The rms output current and supply current are both given by

\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{Z_{\text{rms}}} \]

\[ I_{\text{rms}} = \frac{240}{10} = 24 \text{ A} \]

\[ I_{\text{rms}} = \frac{240}{10} = 24 \text{ A} \]

\[ \mu = \frac{P_{\text{rms}}}{P_L} = \frac{0.07}{0.07} = 1 \]

\[ \cos \varphi = \frac{P_{\text{rms}}}{P_L} = \frac{0.07}{0.07} = 1 \]

That is, the current distortion factor is \( \mu = 1 \).
Solution
As in example 12.1a, from equation (12.3) the load natural power factor angle is

\[ \phi = \tan^{-1} \frac{\omega L}{R} = \tan^{-1} \left( \frac{\pi}{4} \right) \]

The load impedance is \( Z = 10 \Omega \). The controllable delay angle range is \( \frac{\pi}{4} \leq \alpha \leq \pi \).

i. The maximum controllable output occurs when \( \alpha = \frac{\pi}{2} \).

From equation (12.2) when \( \alpha = \phi \) the output voltage is the supply voltage, \( V \), and

\[ v(t) = \frac{\sqrt{2}V}{\pi} \sin \left( \alpha - \frac{\pi}{4} \right) \quad (A) \]

The load hence supply rms maximum current, is therefore

\[ I_{\text{rms}} = \frac{240V/10\Omega}{24A} \]

ii. Power = \( \text{Power output} = \frac{\text{Apparent power output}}{\text{Power factor}} \)

\[ P = \frac{\sqrt{2}V/\pi \times 240V \times 10\Omega}{0.71} \quad (= \cos \phi) \]

iii. Each thyristor conducts for \( \alpha \) radians, between \( \alpha \) and \( \pi + \alpha \) for T1 and between \( \pi + \alpha \) and \( 2\pi + \alpha \) for T2.

The thyristor average current is

\[ I_{\text{avg}} = \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} \left( \sqrt{2}V \sin \left( \alpha - \frac{\pi}{4} \right) \right) \, d\alpha \]

\[ = \frac{\sqrt{2}V}{\pi} \times 240V - 170A \]

Maximum thyristor di/dt is derived from

\[ \frac{dv(t)}{dt} = \frac{\sqrt{2}V}{\pi} \cos \left( \alpha - \frac{\pi}{4} \right) \quad (A/s) \]

This has a maximum value when \( \alpha = \frac{\pi}{2} \), that is at \( \alpha = \alpha - \phi \), then

\[ \frac{dv(t)}{dt} = \frac{\sqrt{2}V}{\pi} \cos \left( \alpha - \frac{\pi}{4} \right) \]

\[ = 10.7A/min \]

Thyristor forward and reverse blocking voltage requirements are \( \sqrt{2}V \times \sqrt{2} \times 240 = 340Vdc \).

Example 12.1c: Single-phase ac regulator – pure inductive load

If the load of the 50 Hz 240V ac voltage regulator shown in figure 12.1 is \( Z = jX \times 10 \Omega \), and the delay angle \( \alpha \) is first \( \frac{\pi}{4} \) then second \( \frac{\pi}{4} \) calculate

i. Maximum rms output voltage and current, and hence

ii. Thyristor I-V ratings

Assume the thyristor gate pulses are of a short duration relative to the 10ms half period.

Solution
For a purely inductive load, the current extinction angle is always \( \beta = 2\pi - \alpha \), that is, symmetrical about \( \pi \) and \( \tan \phi \).
Example 12.1d: Single-phase ac regulator – with ac back emf composite load

A 230V 50Hz mains ac thyristor chopper has a load composed of 10Ω resistance in series with a 138V 50Hz ac voltage source that leads the mains by 30°. If the thyristor triggering angle is 90° with respect to the ac mains, determine:

i. The rms load current and maximum rms load current for any phase delay angle

ii. The power dissipated in the passive part of the load

iii. The thyristor average and rms current ratings and voltage ratings

iv. Power dissipated in the thyristor when modelled by $v_T = v_{mR} * i_T = 1.2 + 0.01 * i_T$

Repeat the calculations if the passive part of the load is a 20mH inductor and the ac back emf lags the 50Hz ac mains by 30°.

Solution

ac back emf with a pure resistive load

From equation (12.47), the voltage across the resistive part of the load is

$V_{ac} = V_0 - 2V_0 \cos \omega V$

$= \sqrt{230^2 + 138^2 - 2 \cdot 230 \cdot 138 \cdot \cos 30} = 130.3V$

with an angle of $\phi = 32.8°$ with respect to the ac mains, given by $\phi = 30°$ and $V_{ac} / V = 138V/230V = 0.6$ in the fourth quadrant of figure 12.5. From the phasor diagram in figure 12.5, the thyristor firing angle with respect to the load inductor voltage is $\alpha = \phi = 90° - 32.8° = 57.8°$. With a 20mH load inductor, the load rms current is given by equation (12.24), that is

$I_{rms} = I_{mR} \sqrt{2}$

$I_{rms} = 9.54\sqrt{2} = 10.1W$

The rms load current is 13A when $\alpha = 0$, that is when $\phi = 32.8°$.

ii. The 10Ω resistor losses are

$L_{res} = I_{rms}^2 \cdot 10Ω$

$L_{res} = 9.54^2 \cdot 10Ω = 910.1W$

The thyristor current ratings are

$I_{rms} = I_{rms} \sqrt{2}$

$I_{rms} = 16.83 \sqrt{2} = 11.9A$

From equation (12.22), the average thyristor current is

$I_T = \frac{\sqrt{2} V_0}{\pi R} \left(1 + \cos \omega \alpha\right)$

$I_T = \frac{\sqrt{2} \cdot 130.3V}{\pi \cdot 10Ω} \left(1 + \cos 57.8°\right) = 4.5A$

The thyristors effectively experience a forward and reverse voltage associated with a single ac source of 130.3V ac. Without phase control the maximum thyristor voltage is $V_T = 2 \cdot 130.3V = 260.6V$. If the triggering angle $\alpha$ is less than 90°-$\phi = 122.8°$ (with respect to the ac mains) then the maximum off-state voltage is less, namely

$V_T = \sqrt{2} \cdot 130.3 \cdot \sin \alpha = \sqrt{2} \cdot 130.3 \cdot \sin 32.8°$ if $\alpha < 122.3°$

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$V_T = \sqrt{2} \cdot 130.3 \cdot \sin \alpha = \sqrt{2} \cdot 130.3 \cdot \sin 32.8°$ if $\alpha < 122.3°$

The thyristor average and rms current ratings and voltage ratings

The thyristor current ratings are

$i_T = \frac{V_T}{R_T} + \frac{I_{rms}}{}$

$i_T = 1.2V + 4.5A + 0.001Ω \cdot 11.9 = 6.8W$

ac back emf with a pure reactive load

The voltage across the inductive part of the load is the same as for the resistive case, namely 130.3V. In this case the ac back emf lags the ac mains. The phase angle with respect to the ac mains is $\phi = 32.8°$, given by $\psi = 30°$ and $V_{ac} / V = 138V/230V = 0.6$ in the second quadrant of figure 12.5. Being a purely

AC voltage regulators

In thyristor heating applications, load harmonics are unimportant and integral cycle control, or burst firing, can be employed. Figure 12.6a shows the regulator when a triac is employed and figure 12.6b shows the output voltage indicating the regulator’s operating principle. Because of the low frequency sub-harmonic nature of the output voltage, this type of control is not suitable for incondensation lighting loads since flickering would occur and with ac motors, undesirable torque pulsations would result.

In many heating applications the load thermal time constant is long (relative to 20ms, that is 50Hz) and an acceptable control method involves a number of mains cycles on and then off. Because turn-on occurs at zero voltage cross-over and turn-off occurs at zero current, which is near a zero voltage cross-over, supply harmonics and radio frequency interference are low. The lowest order harmonic in the load is 1/Tr.

For a resistive load, the output voltage (and current) is defined by

$v_T = R \cdot \sin (\omega t)$

$V_{rms} = \frac{1}{2\pi} \int_{0}^{2\pi} (\sqrt{2} \cdot V_{rms}) \sin (\omega t) \, dt$

$V_{rms} = \frac{1}{2\pi} \int_{0}^{2\pi} (\sqrt{2} \cdot V_{rms}) \sin (\omega t) \, dt$

For a 20mH inductor, the impedance is $R_{ind} = 20mH \cdot \omega = 20.74Ω$, and

$V_{rms} = \frac{1}{2\pi} \int_{0}^{2\pi} (\sqrt{2} \cdot V_{rms}) \sin (\omega t) \, dt$

$V_{rms} = \frac{1}{2\pi} \int_{0}^{2\pi} (\sqrt{2} \cdot V_{rms}) \sin (\omega t) \, dt$

With an angle of $\phi = 30° - 30° = 0°$.

The power dissipated in each thyristor is

$P_T = V_T \cdot I_T + \frac{V_T^2}{R_T}$

$P_T = 7.73A \cdot 2V + \frac{(2V)^2}{20.74Ω} = 35.4W$

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$P_T = V_T \cdot I_T + \frac{V_T^2}{R_T}$

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The thyristor current ratings are

$\alpha = \phi = 30° - 30° = 0°$

$i_T = \frac{V_T}{R_T} + \frac{I_{rms}}{}$

$i_T = 1.2V + 4.5A + 0.001Ω \cdot 11.9 = 6.8W$

The thyristor average and rms current ratings and voltage ratings

The thyristor current ratings are

$\alpha = \phi = 30° - 30° = 0°$

$i_T = \frac{V_T}{R_T} + \frac{I_{rms}}{}$

$i_T = 1.2V + 4.5A + 0.001Ω \cdot 11.9 = 6.8W$
The Fourier coefficient and phase angle for each load voltage harmonic (for \( n \neq N \)) are given by

\[
c_n = \frac{2V}{\pi N} m \sin \pi n \delta \\
\phi_n = \pm (1 - n\delta) \text{ for } n < N \\
\phi_n = \pm (n\delta - 1) \text{ for } n > N
\]  

(12.52)

When \( n > N \) the harmonics are above \( 1/\delta \), while if \( n < N \) subharmonics of \( 1/\delta \) are produced.

For the case when \( n = N \), the coefficient and phase angle for the sin \( mn \) term (\( \delta_m = 0 \)) are

\[
b_{mN} = c_{mN} = \frac{2V}{\pi N} m = \sqrt{2V} \delta \quad \text{and} \quad \phi_{mN} = 0
\]  

(12.53)

Note the displacement angle between the ac supply voltage and the load voltage frequency component at the supply frequency, \( n = N \), is \( \phi_{mN} = 0 \). Therefore the fundamental power factor angle \( \cos \phi_0 = \cos 0 = 1 \).

The output power is

\[
P = \frac{V^2}{R} \frac{n}{N} \delta \left( \frac{V}{\sqrt{2R}} \right)^2 R
\]  

(12.54)

where \( n \) is the number of on cycles and \( N \) is the number of cycles in the period \( T_p \).

The average and rms thyristor currents are, respectively,

\[
I_{\text{av}} = \frac{\sqrt{2V} m}{\pi R N} \delta \quad I_{\text{rms}} = \frac{\sqrt{2V} m}{\sqrt{2} \pi R N} \delta
\]  

(12.55)

From equation (12.53), the supply displacement factor \( \cos \phi_{mN} \) is unity and supply power factor \( \lambda \) is \( \sqrt{m/N} = \sqrt{\delta} \). From \( \rho' = \lambda = \cos \phi_{mN} = \mu \), the distortion factor \( \mu \) is \( \mu = \sqrt{m/N} = \sqrt{\delta} \). The rms voltage at the supply frequency is \( V \sqrt{m/N} = 5V \) and the power transfer ratio is \( m/\delta \). For a given percentage of maximum output power, the supply power factor is the same for integral cycle control and phase angle control. The introduction of sub-harmonics tends to restrict this control technique to resistive heating type application. Temperature effects on load resistance \( R \) have been neglected, as have semiconductor on-state voltages. Finer resolution output voltage control is achievable if integral half-cycles are used rather than full cycles. The equations remain valid, but the start of multiples of half cycles are alternately displaced by \( \pi \) so as to avoid a dc component in the supply and load currents. Multiple cycles need not be consecutive within each period.

Example 12.2: Integral cycle control

The power delivered to a 120 degree resistive heating element is derived from an ideal sinusoidal supply \( \sqrt{2} \) 240 sin 2\( \pi \) 50 \( t \) and is controlled by a series connected triac as shown in figure 12.6. The triac is controlled from its gate so as to deliver integral ac cycle pulses of three (m) consecutive ac cycles from four (4N).

Calculate

i. The percentage power transferred compared to continuous ac operation

ii. The supply power factor, distortion factor, and displacement factor

iii. The supply frequency (50Hz) harmonic component voltage of the load voltage

iv. The triac maximum di/dt and dv/dt stresses

v. The phase angle \( \alpha \), to give the same load power when using phase angle control. Compare the maximum di/dt and dv/dt stresses with part iv.

vi. The output power steps when \( m \), the number of conducted cycles is varied with respect to \( N = 4 \) cycles. Calculate the necessary phase control \( \alpha \) equivalent for the same power output. Include the average and rms thyristor currents.

vii. What is the smallest power increment if half cycle control were to be used?

viii. Tabulate the harmonics and rms subharmonic component power for unit magnitudes of the load voltage for \( m = 0, 1, 2, 3, 4 \), and for harmonics \( n = 0 \) to 12. (Hint: use Excel)

Solution

The key data is: \( m = 3 \) \( N = 4 \) (\( \delta = 75\% \)) \( V = 240 \) rms ac, 50Hz

i. The power transfer, given by equation (12.54), is

\[
P = \frac{V^2}{R} \frac{3}{4} \delta \left( \frac{V}{\sqrt{2}R} \right)^2 R
\]

That is 75% of the maximum power is transferred to the load as heating losses.

ii. The displacement factor, \( \cos \phi_{mN} \), is 1.

The distortion factor is given by

\[
\mu = \frac{m}{N} = \frac{3}{4} = 0.875
\]

Thus the supply power factor, \( \lambda \), is

\[
\lambda = \mu \cos \phi_{mN} = \frac{3}{4} \cdot 1 = 0.866
\]

iii. The 50Hz rms component of the load voltage is given by

\[
V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{240}{\sqrt{2}} = 180 \text{V rms}
\]

iv. The maximum di/dt and dv/dt stresses are zero cross over, when \( t = 0 \).

\[
\frac{dV_t}{dt} \bigg|_{t=0} = \frac{d}{dt} \sqrt{2} \cdot 240 \sin 2\pi 50t \bigg|_{t=0} = \sqrt{2} \cdot 240 \cdot 2\pi \cdot 50 \cdot 0 = 0 \text{A/mus}
\]

\[
\frac{dV_t}{dt} \bigg|_{t=0} = \frac{d}{dt} \sqrt{2} \cdot 240 \sin 2\pi 50t \bigg|_{t=0} = \sqrt{2} \cdot 240 \cdot 2\pi \cdot 50 \cdot 0 = 0 \text{V/mus}
\]
v. To develop the same load power, 3600W, with phase angle control, with a purely resistive load, implies that both methods must develop the same rms current and voltage, that is, 

\[ I' = I, \quad V' = V \quad \text{when} \quad \delta = \pi \delta. \]

From equation (12.5), the extinction angle, \( \beta = \pi \delta \), since the load is resistive

\[ V' = \sqrt{R^2 + I'^2} = V \sqrt{I'^2 + \frac{P}{V^2}} = \sqrt{V^2 \left(1 + \frac{1}{R^2} \sin^2 \theta \right)} \]

that is

\[ \delta = \frac{m}{N} \left\{ \left(\pi - \alpha \right) + \frac{1}{2} \sin 2\alpha \right\} \]

Solving \( 0 = \pi - \alpha + \frac{1}{2} \sin 2\alpha \) iteratively gives \( \alpha = 63.9^\circ \).

When the triac turns on at \( \alpha = 63.9^\circ \), the voltage across it drops virtually instantaneously from \( \sqrt{2} \) 240 sin \( 63.9^\circ = 305V \) to zero. Since this is at triac turn-on, this very high dv/dt does not represent a turn-on dv/dt stress. The maximum triac dv/dt stress tending to turn it on at zero voltage cross over, which is 107 Vms, with integral cycle control. Maximum dv/dt occurs at triac turn on where the current rises from zero amperes to 305\( \sqrt{2} \) = 217A quickly. If the triac turns on in approximately 1ms, then this would represent a dv/dt of 217A. The triac initial dv/dt rating would have to be in excess of 254A/s.

vi. The output power can be varied using \( m = 0, 1, 2, 3, \) or \( 4 \) cycles of the mains. The output power in each case is calculated as in part 1 and the equivalent phase control angle, \( \alpha \), is calculated as in part v. The appropriate results are summarised in the table.

vii. Finer power step resolution can be attained if half cycle power pulses are used as in figure 12.6b. If one complete ac cycle corresponds to 1200W then by using half cycles, 600W power steps are possible. This results in nine different power levels if \( N = 4 \), from 0W to 4800W.

vii. The following table show harmonic components, rms subharmonics, etc., for \( N = 4 \), which are calculated as follows.

For \( n \neq 4 \), (that is not 50Hz) the harmonic magnitude is calculated from equation (12.52).

\[ c_n = \frac{2 \sqrt{2} V N}{\pi (N^2 - n^2)} \sin \left(\frac{x nm}{N}\right) - \frac{8 \sqrt{2} V}{\pi (16 - n^2)} \sin \left(\frac{x nm}{4}\right) \}

while equation (12.53) gives the 50Hz load component (\( n = 4 \)).

\[ c_{4,n=4} = \sqrt{2} V \frac{m}{N} = 2 \sqrt{2} V \frac{m}{4} \]

The rms output voltage is given by equation (12.51) or the square root of the sum of the squares of the harmonics, that is

\[ V' = \sqrt{ \sum_{n=1}^{N} \frac{m^2}{N^2} \left( \frac{c_n}{c_1} \right)^2 } \]

The ac subharmonic component (that is components less than 50Hz) is given by

\[ V_{ac} = \sqrt{V^2 - V'_{rms}^2} = V \sqrt{ \sum_{n=1}^{N} \frac{m^2}{N^2} \left( \frac{c_n}{c_1} \right)^2 } \]

From equations (12.51) and (12.53), the non fundamental (50Hz ac) component is given by

\[ V_n = \sqrt{V^2_{rms} - V_{ac}^2} = V \sqrt{1 - \left( \frac{c_n}{c_1} \right)^2 } \]

12.2 Single-phase transformer tap-changer – line commutated

Figure 12.7 shows a single-phase tap changer where the tapped ac voltage supply can be provided by a tapped transformer or autotransformer. Thyristor \( T_2 (T_4) \) is triggered at zero voltage cross-over (or later), subsequently under phase control \( T_1 (T_3) \) is turned on. The output voltage (and current) for a resistive load \( R \) is defined by

\[ v_o (\alpha) = i_o V, \quad V_o = \sqrt{R^2 + I_o^2} \quad \text{for} \quad 0 \leq \alpha \leq \alpha \quad \text{rad} \]

\[ v_o (\alpha) = i_o V, \quad V_o = \sqrt{R^2 + I_o^2} \quad \text{for} \quad 0 \leq \alpha \leq \pi \quad \text{rad} \]

where \( \alpha \) is the phase delay angle and \( \alpha_k < \alpha \).

If \( 0 \leq \delta \leq \frac{\alpha_k}{\sqrt{2}} \), then for a resistive load the rms output voltage is

\[ V_o = \sqrt{V^2 \left(1 - \delta^2 \right) + \left(4 \sqrt{2} \delta \right)^2} \]

\[ V_o = \sqrt{V^2 \left(1 - \delta^2 \right) + \left(4 \sqrt{2} \delta \right)^2} \]

\[ V_o = \sqrt{V^2 \left(1 - \delta^2 \right) + \left(4 \sqrt{2} \delta \right)^2} \]
The Fourier coefficients of the output voltage, which has only odd harmonics, are

\[
\begin{align*}
\alpha_1 &= V \frac{\sqrt{2}}{L} \left[ \cos \omega_0 t + 2 \sin \omega_0 t \right] \\
\beta_1 &= 0
\end{align*}
\]

The amplitude of the fundamental quadrature components, \(n = 1\), are

\[
\begin{align*}
\alpha_1 &= V \frac{1}{\sqrt{2}} (1 - \delta) \sin \alpha \\
\beta_1 &= V \frac{1}{\sqrt{2}} (1 - \delta) (\cos \alpha + \sin \alpha) (1 - \delta)
\end{align*}
\]

The thyristor voltages ratings are both \(v_1\) and \(v_2\), provided a thyristor is always conducting at any instant.

\[
\begin{align*}
I_{\text{rms}} &= \frac{V_{\text{p}}}{\sqrt{2} R} \left[ \frac{1}{\pi} \left( 2 \alpha - \sin 2\alpha \right) \right] \\
I_{\text{rms}} &= \frac{V_{\text{p}}}{\sqrt{2} R} \left[ \frac{1}{\pi} \left( \sin 2\alpha - 2\alpha \right) \right]
\end{align*}
\]

The thyristor voltages ratings are both \(v_1\) and \(v_2\), provided a thyristor is always conducting at any instant.

An extension of the basic operating principle is to use phase control on thyristors \(T_2\) and \(T_4\) as well as \(T_1\) and \(T_3\). It is also possible to use tap-changing in the primary circuit. The basic operation can also be extended from a single tap secondary to a multi-tap transformer.

The basic operating principle of any multi-output tap changer, in order to avoid short circuits, independent of the load power factor is

- Switch up in voltage when the load \(V\) and \(I\) have the same direction, delivering power
- Switch down when \(V\) and \(I\) have the opposite direction, returning power.

Example 12.3: Tap changing converter

The converter circuit shown in Figure 12.8 is a form of ac to dc tap changer, with a 230V ac primary. The inner voltage taps can deliver 110V ac while the outer tap develops 230V ac across the 100 resistive load. If the thyristor phase delay angle is 90°, determine

- The mean load voltage hence mean load current
- The average diode and thyristor current
- The primary rms current
- The peak thyristor and diode voltage, for any phase angle

Solution

The output voltage is similar to that shown in figure 12.7b, except rectified, and \(\alpha = 90°\).

i. The mean load voltage can be determined from equation (12.22)
\[ V_s = \frac{\sqrt{2}V_{in}}{\pi} (1 + \cos \theta) + \frac{\sqrt{2}V_{in}}{\pi} (1 + \cos \alpha) \]
\[ = \frac{\sqrt{2}}{\pi} (1 + \cos \theta) + \frac{\sqrt{2} \times 110V}{\pi} (1 + \cos \theta) + \frac{\sqrt{2} \times 230V}{\pi} (1 + \cos \theta) \]
\[ = 49.5V + 103.5V = 153V \]
whence \[ T_s = \frac{V_s}{R} = \frac{153V}{106} = 15.3A \]

ii. The diode current is associated with the 49.5V component of the average load voltage, while the thyristor component is 103.5V. Taking into account that each semiconductor has a maximum duty cycle of 50%:

The average diode current is
\[ I_{Dav} = 50\% \times 49.5V = 2.475A \]
The average thyristor current is
\[ I_{Tav} = 50\% \times 103.5V = 5.175A \]

iii. The primary rms current has two components

When the diode conducts the primary current is
\[ I_p = \frac{\sqrt{2}}{2} \times 110V \]
\[ = \frac{\sqrt{2}}{2} \times 230V \]
\[ \text{and} \]
\[ I_\alpha = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \times 110V \]
\[ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \times 230V \]
\[ \alpha \leq \alpha \leq \pi \]

The rms of each component, on the primary side, is
\[ I_{Drms} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \times 110V \]
\[ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \times 230V \]
\[ \alpha \leq \alpha \leq \pi \]

The total supply side rms current comprised the contribution of two diodes and two thyristors
\[ I_{Drms} + I_{Drms} + I_{Trms} + I_{Trms} \]
\[ = 2.63 + 2.63 + 11.5 + 11.5 = 16.68A \]

iv. The peak diode voltage is associated with the turn-on of the thyristor associated with the other half cycle of the supply and worst case is when \( \alpha < \frac{\pi}{2} \),

\[ V_{D} = \sqrt{2} \times 230V + \sqrt{2} \times 110V = 466.7V \]
and 466.7×sinα for \( \alpha > \frac{\pi}{2} \).

The thyristor peak forward and reverse voltages are experienced at \( \alpha = \frac{\pi}{2} \):
\[ V_{D}^f = \sqrt{2} \times (230V - 110V) = 169.7V \]
\[ V_{D}^r = 2 \times \sqrt{2} \times 230V = 650.5V \]

The thyristor forward voltage is controlled by its associated diode and is less than 169.7V if \( \alpha = \frac{\pi}{2} \), viz. 169.7×sinα. The peak reverse voltage of 650.5V is experienced if the complementary thyristor is turned on before \( \alpha = \frac{\pi}{2} \), otherwise the maximum is 650.5×sinα for \( \alpha > \frac{\pi}{2} \).
12.3 Single-phase ac chopper regulator – commutable switches

An ac step-down chopper is shown in figure 12.9a. The switches T1 and T3 (shown as reverse blocking IGBTs) impress the ac supply across the load while T2 and T4 provide load current freewheel paths when the main switches T1 and T3 are turned off. In order to prevent the supply being shorted, switches T1 and T3 cannot be on simultaneously when the ac supply is in a positive half cycle, while T2 and T4 can not both be on during a negative half cycle of the ac supply. Zero voltage information is necessary. If the rms supply voltage is $V$ and the on-state duty cycle of $T1$ and $T3$ is $\delta$, then rms output voltage $V_o$ is

$$V_o = \frac{V}{\sqrt{2}} \sin \omega t + \frac{V}{\sqrt{2}} \sin \theta$$

When the sinusoidal supply is modulated by a high frequency rectangular-wave carrier $\omega_c = 2(2\pi f)$, which is the switching frequency, the ac output is at the same frequency as the supply $f$, but the fundamental magnitude is proportional to the rectangular wave duty cycle $\delta$, as shown in figure 12.3b. Being based on a modulation technique, the output harmonics involve the fundamental at the supply frequency $f$, and components related to the high frequency rectangular carrier waveform $\Delta f$. The output voltage is given by

$$V_o = \sqrt{2} \sin \alpha f + \sum \sqrt{2} \sin n \alpha f \bigl[ \sin (\alpha_0 + n\omega) t - \sin \alpha_0 t \bigl]$$

(12.64)

The carrier (switching frequency) components can be filtered by using an output L-C filter, as shown in figure 12.9a, which has a cut-off frequency of $\omega_0$, complying with $\omega_0 << \alpha f$.

12.4 Three-phase ac regulator

12.4.1 Fully-controlled three-phase ac regulator with wye load and isolated neutral

The power to a three-phase star or delta-connected load may be controlled by the ac regulator shown in figure 12.10a with a star-connected load shown. The circuit is commonly used to soft start three-phase induction motors. If a neutral connection is made, load current can flow provided at least one thyristor is conducting. At high power levels, neutral connection is to be avoided, because of load triplen currents that may flow through the phase inputs and the neutral. With a balanced delta connected load, no triplen or even harmonic currents occur.

If the regulator devices in figure 12.10a, without the neutral connected, were diodes, each would conduct for $\pi$ rad in the order $T_r$ to $T_b$ at $\pi$ rad; $\alpha$ apart. As thyristors, conduction is from $a$ to $b$.

Purely resistive load

In the fully controlled ac regulator of figure 12.10a without a neutral connection, at least two devices must conduct for power to be delivered to the load. The thyristor trigger sequence is as follows. If thyristor $T_1$ is triggered at $\alpha$, then for a symmetrical three-phase load voltage, the other trigger angles are $T_2$ at $\alpha + \frac{\pi}{3}$ and $T_3$ at $\alpha + \frac{\pi}{3}$. For the antiparallel devices, $T_4$ (which is in antiparallel with $T_1$) is triggered at $\alpha + \frac{\pi}{3}$, and $T_5$ and $T_6$ at $\alpha + \frac{2\pi}{3}$. Figure 12.10b shows load neutral output voltage waveforms (which are symmetrical about zero volts) for four different phase delay angles, $\alpha$. Three distinctive conduction periods, plus a non-conduction period, exist. The waveforms in figure 12.10b are useful in determining the required bounds of integration. When three regulator thyristors conduct, the voltage (and the current) is of the form $\sqrt{2} \sin \omega t$, while when two devices conduct, the voltage (and the current) is of the form $\sqrt{2} \sin (\omega t + \phi)$.

The Fourier coefficients of the fundamental frequency are

$$a_0 = \frac{3}{4\pi} \left( \cos 2\alpha - 1 \right)$$

$$b_n = \frac{3}{4\pi} \sin 2\alpha \left( 1 - 2\alpha \right)$$

(12.66)

Using the five integration terms as in equation (12.65), not squared, gives the average half-wave load voltage, hence specifies the average thyristor current requirement with a resistive load.

$$V_o^{1/2} = 2 \sin \alpha f$$

(12.67)

The thyristor maximum average current is when $\alpha = 0$, that is $I_T = \sqrt{2V/\pi R}$.

Figure 12.10. Three-phase ac full-wave voltage controller: (a) circuit connection with a star load; (b) phase a, line-to-neutral load voltage waveforms for four firing delay angles; and (c) delta load.


**AC voltage regulators**

The transitions between 3 and 2 thyristors conducting and between the two modes involves solutions to transcendental equations, and the rms output voltage, whence currents, depend on the solution to these equations.

### Purely inductive load

For a purely inductive load the natural ac power factor angle is $\gamma/3$, where the current lags the voltage by $\gamma$. Therefore control for such a load starts from $a = \gamma/3$, and since the average inductor voltage must be zero, conduction is symmetrical about $t = 0$ and ceases at $2t - a$. The conduction period is $2(t - a)$. Two distinct conduction periods exist.

1. $\gamma/3 \leq a < \gamma/3$ (mode 3/2)—either 2 or 3 conducting thyristors

   Either two or three phases conduct and five integration terms give the load half cycle average voltage, whence average thyristor current, as
   \[
   V_{av}^{\text{rms}} = \frac{\sqrt{2}}{\pi} \left( 2 \cos \alpha - \sqrt{3} \sin \alpha + 1 + \frac{3}{2} \right) \]

   The thyristor maximum average current is $\alpha\pi/3$.

   When only two thyristors conduct, the phase current during the conduction period is given by
   \[
   I(\alpha) = \frac{\sqrt{2}}{\pi} \left( \frac{3}{2} \cos \alpha - \frac{3}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right)
   \]

   The load phase rms voltage and current are
   \[
   V_{av} = \sqrt{2} \pi \alpha \cos \alpha + \frac{3}{2} \sin 2\alpha \]

   The magnitude of the sin term fundamental $(a, \alpha = 0)$ is
   \[
   V(\alpha) = \frac{3}{2} \pi \left( \alpha - \frac{\pi}{6} \right) \left( \cos \left( \alpha - \frac{\pi}{6} \right) \right)
   \]

   while the remaining harmonics $(a, \alpha = 0)$ are given by
   \[
   V(\alpha) = \frac{3}{2} \pi \left( \sin \left( \alpha - \frac{\pi}{6} \right) \right)
   \]

2. $\gamma/3 \leq a < \gamma/3$ (mode 2/0)—either 2 or no conducting thyristors

   Discontinuous current flows in two phases, in two periods per half cycle and two integration terms (reduced to one after time shifting) give the load half cycle average voltage, whence average thyristor current, as
   \[
   V_{av}^{\text{rms}} = \frac{\sqrt{2}}{\pi} \left( 2 \cos \alpha - \sqrt{3} \sin \alpha + 1 + \frac{3}{2} \right)
   \]

   The thyristor maximum average current is $\alpha\pi/3$.

   When only two thyristors conduct, the phase current during the conduction period is given by
   \[
   I(\alpha) = \frac{\sqrt{2}}{\pi} \left( \frac{3}{2} \cos \alpha - \frac{3}{2} \cos \left( \alpha + \frac{\pi}{3} \right) \right)
   \]

   The load phase rms voltage and current are
   \[
   V_{av} = \sqrt{2} \pi \alpha \cos \alpha + \frac{3}{2} \sin 2\alpha \]

   The magnitude of the sin term fundamental $(a, \alpha = 0)$ is
   \[
   V(\alpha) = \frac{3}{2} \pi \left( \alpha - \frac{\pi}{6} \right) \left( \cos \left( \alpha - \frac{\pi}{6} \right) \right)
   \]

   while the remaining harmonics $(a, \alpha = 0)$ are given by
   \[
   V(\alpha) = \frac{3}{2} \pi \left( \sin \left( \alpha - \frac{\pi}{6} \right) \right)
   \]

### Inductive-resistive load

Once inductance is incorporated into the load, current can only flow if the phase angle is at least equal to the load phase angle, given by $\phi = \tan^{-1} \frac{v}{i}$. Due to the possibility of continuation of the load current because of the stored inductive load energy, only two thyristor operational modes occur. The initial angle at $\phi = \gamma$ operates with three then two conducting thyristors mode 3/2, then as the control angle increases, operation in a mode 2/0 occurs with either two devices conducting or all three off, until $a = \gamma/3$. The transitions between 3 and 2 thyristors conducting and between the two modes involves solutions to transcendental equations, and the rms output voltage, whence currents, depend on the solution to these equations.

### Power Electronics

The transitions between 3 and 2 thyristors conducting and between the two modes involves solutions to transcendental equations, and the rms output voltage, whence currents, depend on the solution to these equations.
The magnitude of the sin term fundamental \((a_1=0)\) is

\[
V_{f} = \frac{3}{\pi} \sqrt{\frac{2}{3}} V \left[ \frac{1}{2} \alpha - \sin(\alpha) \right] = I_{o} \alpha L
\]

while the remaining harmonics \((a_m \neq 0)\) are given by

\[
V_{n} = \frac{3}{\pi} V \left( \frac{\sin(m \pi \alpha)}{m \pi} \right) \left( \frac{\sin(m \pi \alpha / 2)}{m \pi / 2} \right) \text{ for } m \neq 0
\]

Various normalised voltage and current characteristics for resistive and inductive equations derived are shown in Figure 12.11.

---

12.4.2 Fully-controlled three-phase ac regulator with wye load and neutral connected

If the load and supply neutral is connected in the three phase thyristor controller with a wye load as shown dashed in Figure 12.10a, then possibly undesirably neutral current flow and each of the three loads can be controlled independently. Undesirably, the third harmonic and its odd multiples are algebraically summed and returned to the supply via the neutral connection. At any instant \(i_b = i_c = i_{oa}\).

For a resistive balanced load there are three modes of thyristor conduction.

When 3 thyristors conduct \(i_b = i_c = I_{oa} = 0\), two thyristor conduct \(I_a = \sqrt{2} V / R \sin(\alpha t - \frac{1}{3} \pi)\), and for one thyristor \(I_a = I_{oa} = \sqrt{2} V / R \sin(\alpha t)\).

Mode [3/2] \(0 \leq \alpha \leq 5\pi / 6\)

Periods of zero neutral current occur when three thyristors conduct and the rms of the discontinuous neutral current is given by

\[
I_{na}^2 = \frac{3}{\pi} \left( \frac{\sqrt{2} V}{R} \sin(\alpha t - \frac{1}{3} \pi) \right)^2 \int dt
\]

\[
I_{na} = \frac{V}{R} \left[ \frac{3}{\pi \alpha} \sin(\pi \alpha) \right]^{1/2}
\]

The average neutral current is

\[
\bar{I}_n = 3 \frac{V}{R} \left( 1 - \cos \alpha \right) (12.87)
\]

At \( \alpha = 0^\circ\), no neutral current flows since the load is seen as a balance load supplied by the three-phase ac supply, without an interposing controller.

Mode [2/1] \(\pi / 3 \leq \alpha \leq 5\pi / 6\)

From \( \alpha \) to \( \pi / 3 \) two phase conduct and after \( \pi / 3 \) the neutral current is due to one thyristor conducting.

The rms neutral current is given by

\[
I_{na}^2 = \frac{3}{\pi} \left( \frac{\sqrt{2} V}{R} \sin(\alpha t - \frac{1}{3} \pi) \right)^2 \int dt + \frac{3}{\pi} \left( \frac{\sqrt{2} V}{R} \sin(\alpha t) \right)^2 \int dt
\]

\[
I_{na} = \frac{V}{R} \left[ 1 - \frac{3}{\pi} \cos \pi \alpha \right]^{1/2}
\]

Maximum ms neutral current occurs at \( \alpha = 5\pi / 6 \), when \( I_{oa} = V / R \).

The average neutral current is

\[
\bar{I}_n = \frac{3 \sqrt{3} V}{2 R} \left( \sin \alpha - 1 \right)
\]

The maximum average neutral current, at \( \alpha = 5\pi / 6 \), is

\[
\bar{I}_n = \frac{3 \sqrt{3} V}{2 R} \left( \frac{\sqrt{3} \sin \alpha - 1}{1} \right) = 0.9886 \frac{V}{R}
\]

Figure 12.11. Three-phase ac full-wave voltage controller characteristics for purely resistive and inductive loads: (a) normalised rms output voltages; (b) normalised half-cycle average voltages; (c) normalised output current for a purely inductive load; and (d) fundamental ac output voltage.

Figure 12.12. Three-phase ac full-wave voltage neutral-connected controller with resistive load, normalised rms neutral current and normalised average neutral current.
Mode [1/0]  \(0 \leq a \leq \frac{\pi}{3}\)

The neutral current is due to only one thyristor conducting. The rms neutral current is given by

\[
I_{n} = \frac{\sqrt{3}}{2} \left( \frac{2V_{R} \sin \alpha}{R} \right) I_{Lr} \sin \left( \frac{\alpha}{2} \right)
\]

The average neutral current is

\[
\overline{I}_{n} = \frac{3\sqrt{2}V}{\pi R} \left( 1 + \cos \alpha \right)
\]

The neutral current is greater than the line current until the phase delay angle \(\alpha > 67^\circ\). The neutral current reduces to zero when \(\alpha = \pi\), since no thyristors conduct.

The normalised neutral current characteristics are shown plotted in figure 12.12.

### 12.4.3 Fully-controlled three-phase ac regulator with delta load

The load in figure 12.10a can be replaced with the start delta in figure 12.10c. Star and delta load equivalence applies in terms of the same line voltage, line current, and thyristor voltages, provided the load is linear. A delta connected load can be considered to be three independent single phase ac regulators, where the total power (for a balanced load) is three times that of one regulator, that is

\[
\text{Power} = 3 \times V_{L} \cos \phi = \sqrt{3} V_{L} \cos \phi
\]

For delta-connected loads where each phase end is accessible, the regulator shown in figure 12.13 can be employed in order to reduce thyristor current ratings. Each phase forms a separate single-phase ac controller as considered in section 12.1 but the phase voltage is the line-to-line voltage, \(\sqrt{3} V\). For a resistive load, the phase rms voltage, hence current, given by equations (12.23) and (12.24) are increased by \(\sqrt{3}\), viz.:

\[
V_{r} = \sqrt{3} V_{L} = \frac{\alpha}{\pi} \left( \frac{3 \sin 2\alpha}{2\pi} \right) = \sqrt{3} \left( I_{r} \frac{R}{V} \right) 0 \leq \alpha \leq \pi
\]

The line current is related to the sum of two phase currents, each phase shifted by \(120^\circ\). For a resistive delta load, three modes of phase angle dependent modes of operation can occur.

Figure 12.13. A delta connected three-phase ac regulator: (a) circuit configuration and (b) normalised line rms current for controlled and semi-controlled resistive loads.

Mode [3/2]  \(0 \leq a \leq \frac{\pi}{3}\)

The line current is given by

\[
I_{L} = \frac{\sqrt{3}}{R} \left( \frac{3}{\pi} \cos \alpha + \frac{1}{2} \sin 2\alpha \right)
\]

### 12.4.4 Half-controlled three-phase ac regulator

The half-controlled three-phase regulator shown in figure 12.14a requires only a single trigger pulse per thyristor and the return path is via a diode. Compared with the fully controlled regulator, the half-controlled regulator is simpler and does not give rise to dc components but does produce more line harmonics.

Figure 12.14b shows resistive symmetrical load, line-to-neutral voltage waveforms for four different phase delay angles, \(a\).

**Resistive load**

Three distinctive conduction periods exist.

i. \(0 \leq a \leq \frac{\pi}{3}\)  – [Mode 3/2]

Before turn-on, one diode and one thyristor conduct in the other two phases. After turn-on two thyristors and one diode conduct, and the three-phase ac supply is impressed across the load. The output phase voltage is asymmetrical about zero volts, but with an average voltage of zero. Examination of the \(a = \frac{\pi}{3}\) waveform in figure 12.14b shows the voltage waveform is made from three segments. The rms load voltage per phase (line to neutral) is

\[
V_{rms} = I_{rms} \cos \alpha = \frac{3}{\pi} \cos 2\alpha - \frac{1}{2} \sin 2\alpha \quad 0 \leq \alpha < \pi
\]

The Fourier co-efficients for the fundamental voltage, for a resistive load are

\[
a_{0} = \frac{3}{8\pi} V_{L} \cos 2\alpha - 1 \quad a_{1} = \frac{3}{8\pi} V_{L} \cos \alpha
\]

Using three integration terms, the average half-wave (half-cycle) load voltage, for both halves, specifies the average thyristor and diode current requirement with a resistive load. That is

\[
V_{rms} = 2 \times \overline{I}_{r} R = 2 \times \overline{I}_{d} \cos \alpha = \frac{3}{2\pi} \left( \cos 2\alpha - \frac{1}{2} \sin 2\alpha \right) \quad 0 < \alpha < \pi
\]

The diode and thyristor maximum average current is when \(\alpha = 0\), that is

\[
\overline{I}_{r} = \overline{I}_{d} = \frac{\sqrt{2}V}{\pi R}
\]
After $\alpha = \frac{\pi}{2}$, only one thyristor conducts at one instant and the return current is a diode. Examination of the $\alpha = \pi$ and $\alpha = \frac{3}{2}\pi$ waveforms in figure 12.14b show the voltage waveform is made from three segments, although different segments of the supply around $\omega t = \pi$. Using three integration terms, the average half-wave (half-cycle) load voltage, for both halves, specifies the average thyristor and diode current requirement with a resistive load. That is

$$V_{\text{rms}} = 2T_i R = 2T_{\text{ave}} = \frac{V^2}{2\pi} \left[ 1 + \cos \alpha + \frac{3}{4 \pi} \right]$$

(12.103)

iii. $\pi \leq \alpha \leq \frac{7}{6} \pi$ – [mode2/0]

Current flows in only one thyristor and one diode and at $7\pi/6$ zero power is delivered to the load. The output is symmetrical about zero. The output voltage waveform shown for $\alpha = \pi$ in figure 12.14b has one component.

$$V_{\text{rms}} = I_{\text{ave}} R = 2T_{\text{ave}} R = \frac{V}{2\pi} \sin \alpha$$

(12.107)

with a fundamental given by

$$a_1 = -\frac{3}{4 \pi} \cos ^2 \left( \alpha - \frac{\pi}{6} \right) \quad b_1 = \frac{3}{4 \pi} \sin ^2 \left( \alpha - \frac{\pi}{6} \right)$$

(12.108)

Using one integration term, the average half-wave (half-cycle) load voltage, for both halves, specifies the average thyristor and diode current requirement with a resistive load. That is

$$V_{\text{rms}} = 2T_i R = 2T_{\text{ave}} = \frac{V^2}{2\pi} \left[ 1 + \cos \frac{\alpha}{2} \right]$$

(12.109)

Purely inductive load

Two distinctive conduction periods exist.

i. $\pi \leq \alpha \leq \frac{3}{2} \pi$ – [mode3/2]

For a purely inductive load (cycle starts at $\alpha = \frac{3}{2} \pi$)

$$V_{\text{rms}} = I_{\text{ave}} L \omega = \frac{V}{\pi} \sin \alpha + \frac{3}{4 \pi} \sin \frac{2 \alpha}{3} \pi$$

(12.110)

while for a purely inductive load the fundamental voltage is ($a_1 = 0$)

$$b_1 = V \left( \frac{3}{2} \pi - \sin 2 \alpha \right) = I_{\text{ave}} L \omega$$

(12.111)

ii. $\frac{3}{2} \pi \leq \alpha \leq \pi$ – [mode2/0]

For a purely inductive load, no mode 3/2 exist and rms load voltage for mode 2/0 is
Three output voltage modes can be shown to occur, depending on the delay control angle. Repeat the calculations if each phase load is a 20mH.

\[ V_{cm} = I_{cm} \omega L - V \left( \frac{1}{2} + \frac{\alpha}{\pi} + \frac{1}{\pi} \sin(2\alpha - \frac{\pi}{2}) \right) \]  
(12.112)

with a fundamental given by \( \alpha = 0 \)

\[ I_t = \frac{3}{\pi} V \left[ \sin(2\alpha - \sin(2\alpha - \frac{\pi}{2}) \right] = I_{.ol} \]  
(12.113)

When \( \alpha > \pi \), the load current is dominated by harmonic currents. Normalised semi-controlled inductive and resistive load characteristics are shown in figure 12.15.

### 12.4.5 Other thyristor three-phase ac regulators

#### i. Delta connected fully controlled regulator

For star-connected loads where access exists to a neutral that can be opened, the regulator in figure 12.16a can be used. This circuit produces identical load waveforms to those for the regulator in figure 12.10 regardless of the type of load, except that mean device current ratings are halved (but the line currents are the same). Only one thyristor needs to be conducting for load current, compared with the circuit of figure 12.10 where two devices must be triggered. The triggering control is simplified but the maximum thyristor blocking voltage is increased by 2/\( \sqrt{3} \), from 3\( \sqrt{X/2} \) to \( 6V \).

Three output voltage modes can be shown to occur, depending on the delay control angle.

- Mode [1/0]: \( 0 \leq \alpha \leq \frac{\pi}{3} \)
- Mode [1]: \( \frac{\pi}{3} \leq \alpha \leq \frac{\pi}{2} \)
- Mode [1/0]: \( \frac{\pi}{2} \leq \alpha \leq \pi \)

In figure 12.16a, at \( \alpha = 0 \), each thyristor conducts for \( \frac{\pi}{3} \), which for a resistive line load, results in a maximum thyristor average current rating of

\[ I_T = \frac{3}{\pi} V \sqrt{3}R - \frac{3}{\pi} \sqrt{3}V \]  
(12.114)

A half-controlled version is not viable.

#### ii. Three-thyristor delta connected regulator

The number of devices and control requirements for the regulator of figure 12.16a can be simplified by employing the regulator in figure 12.16b. In figure 12.16b, because of the half-wave configuration, at \( \alpha = \frac{\pi}{3} \), each thyristor conducts for \( \frac{\pi}{3} \), which for a resistive line load, results in a maximum thyristor average current rating of

\[ I_T = \frac{3}{\pi} V \sqrt{3}R - \frac{3}{\pi} \sqrt{3}V \]  
(12.115)

Two thyristors conduct at any time as shown by the six sequential conduction possibilities that complete one mains ac cycle in figure 12.17.

Three output voltage modes can be shown to occur, depending on the delay control angle.

### Table 12.1. Thyristor electrical ratings for four ac controllers

<table>
<thead>
<tr>
<th>Circuit figure</th>
<th>Thyristor voltage ( \sqrt{2}V )</th>
<th>Thyristor rms current pu</th>
<th>Control delay angle range</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.10</td>
<td>( \frac{3}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( 0 \leq \alpha \leq \frac{\pi}{4} )</td>
</tr>
<tr>
<td>12.13</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{\pi}{3} \leq \alpha \leq \pi )</td>
</tr>
<tr>
<td>12.14</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( 0 \leq \alpha \leq \frac{\pi}{4} )</td>
</tr>
<tr>
<td>12.16a</td>
<td>( 1/\sqrt{2} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( 0 \leq \alpha \leq \frac{\pi}{3} )</td>
</tr>
<tr>
<td>12.16b</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\pi}{3} \leq \alpha \leq \frac{2\pi}{3} )</td>
</tr>
</tbody>
</table>

### Example 12.4: Star-load three-phase ac regulator – untapped neutral

A 230V (line to neutral) 50Hz three-phase mains ac thyristor chopper has a symmetrical star load composed of 100 resistances. If the thyristor triggering delay angle is \( \alpha = 90^\circ \), determine:

- The rms load current and voltage, and maximum rms load current for any phase delay angle
- The power dissipated in the load
- The thyristor average and rms current ratings and voltage ratings
- Power dissipated in the thyristors when modelled by \( v_i = v_o + c_i = 1.2 + 0.01 \times i_t \)

Repeat the calculations if each phase load is a 20mH.
### Solution

**Power Electronics**

(a) **10Ω Resistive load - α = 90°**

1. **Rms voltage from equation (12.68)**
   
   \[ V_{rms} = I_{rms} R = V \left( \frac{3}{2} + \frac{1}{2} \cos(2\alpha + \phi) \right) \]
   
   \[ = 230V \left( \frac{3}{2} + \frac{1}{2} \cos(2 \times 90° + 30°) \right) \]
   
   \[ = 230V \times 0.377 = 86.6V \]

   Whence the rms current
   
   \[ I_{rms} \frac{V_{rms}}{R} = \frac{86.6V}{10\Omega} = 8.66A \]

2. **The load power is**
   
   \[ P_{Load} = I_{rms}^2 R = 8.66^2 \times 10\Omega = 750.7W \]

3. **Thyristor average current from equation (12.71)**
   
   \[ I_T = \frac{3V}{2\pi} \left( \sin \alpha - \frac{1}{2} \right) \]
   
   \[ = \sqrt{3/2} \times 230V \left( \sin \frac{\pi}{2} - \frac{1}{2} \right) \]
   
   \[ = 4.48A \]

   Thyristor rms current
   
   \[ I_{rms} = \frac{I_T}{\sqrt{2}} \]
   
   \[ = \frac{4.48A}{\sqrt{2}} = 6.12A \]

4. **Thyristor loss**
   
   \[ P_{Thyristor} = V_T I_T + 0.01 I_T^2 \]
   
   \[ = 1.2 \times 4.48A + 0.01 \times 6.12^2 = 5.75W \]

(b) **20mH Inductive load - α = 90°**

1. **Rms voltage and current from equation (12.77)**
   
   \[ V_{rms} = \frac{V}{\sqrt{2}} \left( \frac{3}{2} - \frac{1}{2} \sin(2\omega) \right) \]
   
   \[ = 230V \left( \frac{3}{2} - \frac{1}{2} \sin(2 \times 90°) \right) \]
   
   \[ = 230V \]

   \[ I_{rms} = \frac{V}{\sqrt{2}} \left( \frac{3}{2} - \frac{1}{2} \cos^2 \alpha + \frac{1}{2} \sin(2\omega) \right) \]
   
   \[ = 230V \left( \frac{3}{2} - \frac{1}{2} \cos^2 \alpha + \frac{1}{2} \sin(2 \times 90°) \right) \]
   
   \[ = 36.6A \]

2. **The load power is zero.**

3. **Since the delay angle is 90°, the natural power factor angle, continuous sinusoidal current flows and the thyristor average current is**
   
   \[ I_T = \frac{3}{2\pi} \frac{V_{rms}}{I_{rms}} \]
   
   \[ = \frac{3}{2\pi} \frac{230V}{36.6A} = 23.3A \]

   Thyristor rms current
   
   \[ I_{rms} = \frac{I_T}{\sqrt{2}} \]
   
   \[ = \frac{23.3A}{\sqrt{2}} = 35.88A \]

4. **Thyristor loss**
   
   \[ P_{Thyristor} = V_T I_T + 0.01 I_T^2 \]
   
   \[ = 1.2 \times 23.3A + 0.01 \times 35.88^2 = 44.45W \]

### 12.5 Cycloconverter

The simplest cycloconverter is a single-phase, two-pulse, ac input to single-phase ac output circuit as shown in figure 12.18a. It synthesises a low-frequency ac output from selected portions of a higher-frequency ac voltage source and consists of two converters connected back-to-back. Thyristors T1 and T2 form the positive converter group P, while T3 and T4 form the negative converter group N.

Figure 12.18b shows how an output frequency of one-fifth of the input supply frequency is generated. The P group conducts for five half-cycles (with T1 and T2 alternately conducting), then the N group conducts for five half-cycles (with T3 and T4 alternately conducting). The result is an output voltage waveform with a fundamental of one-fifth the supply with continuous load and supply current. The harmonics in the load waveform can be reduced and rms voltage controlled by using phase control as shown in figure 12.18c. The phase control delay angle is greater towards the group changeover portions of the output waveform. The supply current is now distorted and contains a subharmonic at the cycloconverter output frequency, which for figure 12.18c is at one-fifth the supply frequency.

With inductive loads, one blocking group cannot be turned on until the load current through the other group has fallen to zero, otherwise the supply will be short-circuited. An intergroup reactor, L, as shown in figure 12.18a can be used to limit any inter-group circulating current, and to maintain a continuous load current.

A single-phase ac load fed from a three-phase ac supply, and three-phase ac load cycloconverters can also be realised as shown in figures 12.19a and both of 12.19b and c, respectively. A transformer is needed in figure 12.19a, if neutral current is to be avoided. The three-pulse per ac cycle cycloconverter in figure 12.19b uses 18 thyristors, while the 6-pulse cycloconverter in figure 12.19c uses 36 thyristors (inter-group reactors are not shown), where the load (motor) neutral connection is optional. The output frequency, with considerable harmonic content, is limited to about 40% of the input frequency, and motor reversal and regeneration are achievable.
If a common neutral is used, no transformer is necessary. Most cycloconverters are 6-pulse, and the neutral connection in figure 12.19c removes the zero sequence component.

The positive features of the cycloconverter are

- Natural commutation
- No intermediate energy storage stage
- Inherently reversible current and voltage

The negative features of the cycloconverter are

- High harmonics on the input and output
- Requires at least 18 thyristors usually 36
- High reactive power

### 12.6 The matrix converter

Commutation of the cycloconverter switches is restricted to natural commutation instances dictated by the supply voltages. This usually results in the output frequency being significantly less than the supply frequency if a reasonable low harmonic output is required. In the matrix converter in figure 12.20c, the thyristors in figure 12.19b are replaced with fully controlled, bidirectional switches, like those shown in figures 12.20a and b. Rather than eighteen switches and eighteen diodes, nine switches and thirty-six diodes can be used if a unidirectional voltage and current switch in a full-bridge configuration is used as shown in figure 6.11. These switch configurations allow converter current commutation as and when desired, provide certain conditions are fulfilled. These switches allow any one input supply ac voltage and current to be directed to any one or more of the output lines. At any instant, only one of the three input voltages can be connected to a given output. This flexibility implies a higher quality output voltage can be attained, with enough degrees of freedom to ensure the input currents are sinusoidal and with unity (or adjustable) power factor. The input L-C filter prevents matrix modulation frequency components from being injected into the input three-phase ac supply system. The relationship between the output voltages \( v_1, v_2, v_3 \) and the input voltages \( v_a, v_b, v_c \) is determined by the states of the nine bidirectional switches \( S_{ij} \), according to

\[
\begin{align*}
v_1 &= S_{11} S_{22} S_{33} v_a + S_{12} S_{23} S_{31} v_b + S_{13} S_{21} S_{32} v_c \\
v_2 &= S_{11} S_{22} S_{33} v_a + S_{12} S_{23} S_{31} v_b + S_{13} S_{21} S_{32} v_c \\
v_3 &= S_{11} S_{22} S_{33} v_a + S_{12} S_{23} S_{31} v_b + S_{13} S_{21} S_{32} v_c
\end{align*}
\]

From Kirchhoff's voltage law, the number of switches on in each row must be either one or none, otherwise at least one input supply is shorted, that is

\[
\sum_{i=1}^{3} S_{ij} \leq 1 \quad \text{for any } j
\]

With the balanced star load shown in figure 12.20c, the load neutral voltage \( v_c \) is given by

\[
v_c = \frac{2}{3} (v_a + v_b + v_c)
\]

The line-to-neutral and line-to-line voltages are the same as those applicable to SVM (space voltage modulation, Chapter 14.3.iv), namely

\[
\begin{align*}
v_{ac} &= \begin{bmatrix} 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \\
v_{bc} &= \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}
\end{align*}
\]

from which

\[
\begin{align*}
v_{ac} &= \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \\
v_{bc} &= \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}
\end{align*}
\]

Similarly the relationship between the input line currents \( i_a, i_b, i_c \) and the output currents \( I_a, I_b, I_c \) is determined by the states of the nine bidirectional switches \( S_{ij} \), according to

\[
\begin{align*}
i_a &= S_{11} S_{22} S_{33} i_a + S_{12} S_{23} S_{31} i_b + S_{13} S_{21} S_{32} i_c \\
i_b &= S_{11} S_{22} S_{33} i_a + S_{12} S_{23} S_{31} i_b + S_{13} S_{21} S_{32} i_c \\
i_c &= S_{11} S_{22} S_{33} i_a + S_{12} S_{23} S_{31} i_b + S_{13} S_{21} S_{32} i_c
\end{align*}
\]
The key feature of the matrix converter is that the capacitor voltage filter, capacitor size \( L-C \), and cost requirements are similar. The advantage of the matrix converter over a dc link approach to ac to ac conversion lies not in the fact that the capacitor voltage is ½ \((v)\cos \phi\) and ½ \((v)\sin \phi\) for all values of \( n \). The maximum voltage gain, the ratio of the peak fundamental ac output voltage to the peak ac input voltage, can be used, and only 27 states of the switch matrix can be utilised.

The maximum voltage gain is given by

\[
\eta = \frac{\sqrt{2} V_{\text{out}}}{V_{\text{in}}} \quad (12.123)
\]

and \( V_{\text{in}} = 0 \) elsewhere.

Fourier analysis of \( v_{\text{in}} \) yields the load voltage Fourier coefficients \( v_{\text{in}} \) and \( v_{\text{out}} \), such that

\[
v_{\text{in}}(\omega) = \sum_{n=1}^{\infty} V_{\text{n}}(\omega) \sin \omega t
\]

for all values of \( n \). The load current can be evaluated by solving

\[
R_i + L_i \frac{dI_i}{dt} = \sqrt{2} V_{\text{in}} \sin \omega t
\]

and is defined by

\[
l_i(\omega) = \sum_{n=1}^{\infty} l_{i,n} \sin(\omega t - \phi_{i,n}) \quad (12.124)
\]

where \( \phi = \tan^{-1}(v_o / V_{\text{out}}) \).

The load current \( l_i \) is given by

\[
l_i(\omega) = \sum_{n=1}^{\infty} l_{i,n} \sin(\omega t - \phi_{i,n}) \quad (12.125)
\]

The load efficiency, \( \eta \), which is related to the power dissipated in the resistive component \( R \) of the load, is defined by

\[
\eta = \frac{\text{fundamental active power}}{\text{total active power}}
\]

where

\[
\eta = \frac{\eta_1 \alpha + \eta_2 \beta}{\eta_1 \alpha + \eta_2 \beta + \eta_3 \gamma}
\]

In general, the total load power is

\[
\sum_{n=1}^{\infty} V_{\text{n}}(\omega) \sin \omega t \cos \phi_{i,n}
\]

where

\[
\sum_{n=1}^{\infty} V_{\text{n}}(\omega) \sin \omega t \cos \phi_{i,n}
\]

Figure 12.20. Three-phase input to three-phase output matrix converter circuit: bidirectional switches (a) reverse blocking igbt conventional igbt; and (b) switching matrix; and (c) three-phase ac supply to three-phase ac load.

where the switches \( S_i \) are constrained such that no two or three switches short between the input lines or cause discontinuous output current. Discontinuous output current must not occur since no natural default current freewheel paths exist. The input short circuit constraint is satisfied by ensuring that only one switch in each row of the 3×3 matrix in equation (12.116) (hence row in equation (12.121)) is on at any time, viz., equation (12.117), while continuous load current in equation (12.121) (hence column in equation (12.116)) is ensured by Kirchhoff’s current law, that is

\[
\sum_{i=1}^{3} S_{ij}i_j = \eta_{\text{active}} \frac{\text{fundamental active power}}{\sum_{i=1}^{3} \sum_{j=1}^{3} (S_{ij})^2}
\]

More than one switch on in a column implies that a load phase is parallel feeding more than one output phase, which is allowable.

Thus given Kirchhoff’s voltage and current law constraints, not all the 512 (2^9) states for nine switches can be used, and only 27 states of the switch matrix can be utilised.

The maximum voltage gain, the ratio of the peak fundamental ac output voltage to the peak ac input voltage is \(1/\sqrt{3} = 0.577\). Above this level, called over-modulation, distortion of the input current occurs. Since the switches are bidirectional and fully controlled, power flow can be bidirectional. Control involves the use of a modulation index that varies sinusoidally.

Since no intermediate energy storage stage is involved, such as a dc link, this so called total silicon solution to ac to ac conversion provides no ride-through, thus is not well suited to ups application. The advantage of the matrix converter over a dc link approach to ac to ac conversion lies in the fact that a dc link capacitor is not required. Given the matrix converter requires an input \( L-C \) filter, capacitor size and cost requirements are similar. The key feature of the matrix converter is that the capacitor voltage requirement is ac. For a given temperature, ripple current, etc., the lifetime of an ac capacitor is significantly longer than a dc voltage electrolytic capacitor, as is required for a dc link. The use of oil impregnated paper bipolar capacitors to improve dc-link inverter reliability, significantly increases capacitor volume and cost for a given capacitance and voltage.

### 12.7 Power Quality: load efficiency and supply current power factor

One characteristic of ac regulators is non-sinusoidal load current, hence supply current as illustrated in figure 12.1b. Difficulty therefore exists in defining the supply current power factor and the harmonics in the load current may detract from the load efficiency. For example, with a single-phase motor, current components other than the fundamental detract from the fundamental torque and increase motor heating, noise, and vibration. To illustrate the procedure for determining load efficiency and supply power factor, consider the circuit and waveforms in figure 12.1.

#### 12.7.1 Load waveforms

The load voltage waveform is constituted from the sinusoidal supply voltage \( v \) and is defined by

\[
v_{\text{i}}(\omega) = \sqrt{2} V_{\text{in}} \sin \omega t
\]

and is defined by

\[
l_i(\omega) = \sum_{n=1}^{\infty} l_{i,n} \sin(\omega t - \phi_{i,n}) \quad (12.126)
\]
12.7.2 Supply waveforms

Linear load:

For sinusoidal single and three phase ac supply voltages feeding a linear load, the load power and apparent power are given by

\[ P = V_I \cos \phi \quad S = V_I \]

and the supply power factor is

\[ \cos \phi = \frac{P}{S} \]  \hspace{1cm} (12.129)

The rms supply voltage is 230V, at 50Hz. The supply current is a 10ms, 10A current block occurring every 20ms. The rms supply current is therefore 10/\sqrt{2} = 7.07A.

Non-linear loads (e.g. rectification):

i. The supply distortion factor \( \mu \), displacement factor \( \cos \psi \), and power factor \( \lambda \) give an indication of the adverse effects that a non-sinusoidal load current has on the supply as a result of SCR phase control.

In the circuit of figure 12.1a, the load and supply currents are the same and given by equation (12.2). The supply current Fourier coefficients \( I_m \) and \( I_m \) are the same as for the load current Fourier coefficients \( I_m \) and \( I_m \) respectively, as previously defined. The total supply power factor \( \lambda \) can be defined as

\[ \lambda = \frac{\text{real power}}{\text{total mean input power}} = \frac{\text{total mean input power}}{\text{rms input VA}} \]  \hspace{1cm} (12.131)

The supply voltage is sinusoidal hence supply power is not associated with the harmonic non-fundamental currents.

\[ \lambda = \frac{V_m}{I_m} \cos \psi \]  \hspace{1cm} (12.132)

where \( \cos \psi \), termed the displacement power factor, is the fundamental power factor defined as

\[ \cos \psi = \cos \left( -\frac{1}{\tan \phi} \frac{1}{\mu} \right) \]  \hspace{1cm} (12.133)

Equating with equation (12.132), the total supply power factor is defined as

\[ \lambda = \mu \cos \psi, \quad 0 \leq \lambda \leq 1 \]  \hspace{1cm} (12.134)

The supply current distortion factor \( \mu \) is the ratio of fundamental rms current to total rms current \( I_{rms} \), that is

\[ \mu = \frac{I_1}{I_{rms}} \]  \hspace{1cm} (12.135)

ii. The supply fundamental harmonic factor \( \rho_h \) is defined as

\[ \rho_h = \frac{\text{total harmonic (non-fundamental) rms current (or voltage)}}{\text{fundamental rms current (or voltage)}} \]  \hspace{1cm} (12.136)

\[ \rho_h = \frac{I_h}{I} \sqrt{\sum I_h^2} \]  \hspace{1cm} (12.137)

The general relationships between the various current forms can be summarised as

\[ I_m = \sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots} \quad I_m = \sqrt{I_1^2 + I_2^2} \quad I_{rms} = \sqrt{I_1^2 + I_2^2} \quad \text{etc.} \]  \hspace{1cm} (12.138)

Example 12.5: Power quality - load efficiency

If a purely resistive load \( R \) is fed with a voltage

\[ v(t) = \sin \omega t + \sin 3\omega t \]

what is the fundamental load efficiency?

Solution

The load current is given by

\[ i(t) = \frac{\sqrt{3} V}{R} \sin \omega t + \frac{1}{3\sqrt{3} R} \sin 3\omega t \]

The load efficiency is given by equation (12.128), that is

\[ \eta = \frac{\left( \frac{\sqrt{3} V}{R} \right)^2}{\left( \frac{\sqrt{3} V}{R} \right)^2 + \frac{1}{3\sqrt{3} R}} \]

The introduced third harmonic component decreases the load efficiency by 10%.

Example 12.6: Power quality - sinusoidal source and constant current load

A half-wave rectifier with a load freewheel diode as shown in figure 11.3 has a 10A constant current load, \( I_0 \). If rectifier circuit is supplied from the ac mains with voltage \( v(t) = 230\sin 2\pi t \) determine:

i. the supply apparent power and average load power

ii. the total supply power factor, \( \lambda \), hence distortion, \( \mu \), and displacement factors

iii. the average and rms current rating of each diode and diode reverse voltage requirements

Solution

The rms supply voltage is 230V, at 50Hz. The supply current is a 10ms, 10A current block occurring every 20ms. The rms supply current is therefore 10/\sqrt{2} = 7.07A.
i. The supply apparent power is
\[ S = V_m I_m = 230V \times 7.07A = 1626.1\text{VAR} \]
The average load voltage is that for half wave rectification, viz.,
\[ V_L = \frac{\sqrt{2}}{\sqrt{2}} = 103.5V \]
The average load power, which must be equal to the input power from the 50Hz source, is
\[ P_L = P_S = V_L I_L = 103.5V \times 10A = 1035W \]
The fundamental of a square wave, with a dc offset of half the magnitude is
\[ i(t) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \text{A} = 4.50A \]
which is in phase with the ac supply, that is \( \cos \Phi = 1 \).
Alternately, the load power, hence input power, which is at the supply voltage frequency of 50Hz, can be confirmed by
\[ P_L = V_L I_L \cos \Phi = 1035W \]
Example 12.7: Power quality - sinusoidal source and non-linear load
An unbalanced single-phase rectifier circuit is supplied from the ac mains with voltage \( v(\omega t) = \sqrt{2} \times 230\sin 2\pi 50t \). The dominant resultant harmonics in the supply current are
\[ i(t) = 10 + 15 \cos(\pi + \frac{\pi}{6}) + 3 \sin(2 \pi + \frac{\pi}{6}) + 2 \sin(4 \pi + \frac{\pi}{6}) \]
Determine
i. the fundamental power factor hence power delivered from the supply
ii. the total supply power factor, hence distortion factor
iii. the harmonic current and the ac current
iv. the total harmonic distortion with respect to the fundamental current and the total rms current
v. the current crest factor.
Solution
i. The power from the supply delivered to the load is only at the supply frequency
Reading list


Problems

12.1. Determine the rms load current for the ac regulator in figure 12.14, with a resistive load \( R \).

Consider the delay angle intervals \( 0 \) to \( \frac{\pi}{3} \), \( \frac{\pi}{3} \) to \( \frac{2\pi}{3} \), and \( \frac{2\pi}{3} \) to \( \frac{7\pi}{6} \).

12.2. The ac regulator in figure 12.14, with a resistive load \( R \) has one thyristor replaced by a diode. Show that the rms output voltage is

\[
V'_{\text{rms}} = \frac{1}{2\pi} \left( 2\pi - \alpha + \frac{\pi}{2} \sin 2\alpha \right)
\]

while the average output voltage is

\[
\bar{V} = \frac{\sqrt{2\pi}}{2\pi} \left( \cos \alpha - 1 \right)
\]

12.3. Plot the load power for a resistive load for the fully controlled and half-controlled three-phase ac regulator, for varying phase delay angle, \( \alpha \). Normalize power with respect to \( \sqrt{\bar{V}^2/R} \).

12.4. For the tap changer in figure 12.7, with a resistive load, calculate the rms output voltage for a phase delay angle \( \alpha \). If \( v_2 = 200 \) V ac and \( v_1 = 240 \) V ac, calculate the power delivered to a 10 ohm resistive load at delay angles of \( \frac{\pi}{6} \), \( \frac{\pi}{3} \), and \( \frac{\pi}{2} \). What is the maximum power that can be delivered to the load?

12.5. A, 0.01 H inductance is added in series with the load in problem 12.4. Determine the load voltage and current waveforms at a firing delay angle of \( \frac{\pi}{6} \). Assuming a 50 Hz supply, what is the minimum delay angle?

12.6. The thyristor \( T_2 \) in the single-phase controller in figure 12.1a is replaced by a diode. The supply is 240 V ac, 50 Hz and the load is 10 \( \Omega \) resistive. Determine the

i. rms output voltage
ii. supply power factor
iii. mean output voltage
iv. mean input current.

[207.84 V; 0.866 lagging; 54 V; 5.4 A]

12.7. The single-phase ac controller in figure 12.6 operating on the 240 V, 50 Hz mains is used to control a 10 \( \Omega \) resistive heating load. If the load is supplied repeatedly for 75 cycles and disconnected for 25 cycles, determine the

i. rms load voltage,
ii. input power factor, \( \lambda \), and
iii. the rms thyristor current.