13
DC Choppers

A dc chopper is a dc-to-dc voltage converter. It is a static switching electrical appliance that in one electrical conversion, changes an input fixed dc voltage to an adjustable dc output voltage without inductive or capacitive intermediate energy storage. The name chopper is connected with the fact that the output voltage is a ‘chopped up’ quasi-rectangular version of the input dc voltage.

In chapters 11 and 12, thyristor devices were used in conjunction with an ac supply that forces thyristor turn-off at ac supply current reversal. This form of thyristor natural commutation, which is illustrated in figure 13.1a, is termed line or source commutation.

When a dc source is used with a thyristor circuit, energy source facilitated commutation is clearly not possible. If the load is an R-C or L-C circuit as illustrated in figure 13.1b, the load current falls to zero when the thyristor in series with the dc supply turns off. Such a natural turn-off process is termed load commutation.

If the supply is dc and the load current has no natural zero current periods, such as with the R-L load, dc chopper circuit shown in figure 13.1c, the load current can only be commutated using a self-commutating switch, such as a GTO thyristor, GCT, IGBT or MOSFET. An SCR is not suitable since once the device is latched on in this dc supply application, it remains on.

The dc chopper in figure 13.1c is the simplest of the five dc choppers to be considered in this chapter. This single-ended, grounded-load, dc chopper will be extensively analysed. See example 13.3.

13.1 DC chopper variations

There are five types of dc choppers, of which four are a subset of the fifth - the flexible but basic, four-quadrant H-bridge chopper shown in the centre of figure 13.2. Notice that the circuits in figure 13.2 are highlighted so that the derivation of each dc chopper from the fundamental H-bridge four-quadrant, dc chopper can be seen. Each chopper can be categorized depending on which output $I_o$-$V_o$ quadrant or quadrants it can operate in, as shown in figure 13.2. The five different choppers in figure 13.2 are classified according to their output $I_o$-$V_o$ capabilities as follows:

(a) First quadrant - I $+V_o$ $+I_o$
(b) Second quadrant - II $-V_o$ $-I_o$
(c) Two quadrant - I and II $+V_o$ $-I_o$
(d) Two quadrant - I and IV $+V_o$ $-I_o$
(e) Four quadrant - I, II, III, and IV $+V_o$ $-I_o$

In the five choppers in the parts a to e of figure 31.2, the subscript of the active switch or switches and diodes specify in which quadrants operation is possible. For example, the chopper in figure 13.2d uses switches $T_1$ and $T_3$, so can only operate in the first ($+I_o$-$+V_o$) and third ($+I_o$-$-V_o$) quadrants.

The first-quadrant chopper in figure 13.2a (and figure 13.1c) can only produce a positive voltage across the load since the freewheel diode $D_1$ prevents a negative output voltage. Also, the chopper can only deliver current from the dc source to the load through the unidirectional switch $T_1$. It is therefore a single quadrant chopper and only operates in the first quadrant ($+I_o$ $+V_o$).

The second-quadrant chopper, ($-I_o$-$+V_o$), in figure 13.2b is a voltage boost circuit and current flows from the load to the supply, $V_o$. The switch $T_3$ is turned on to build-up the inductive load current. Then when the switch is turned off current is forced to flow through diode $D_1$ into the dc supply. The two current paths (when the switch on and when its is off) are shown in figure 13.2b.
In both conduction cases, the average voltage across the load can be controlled by varying the on-to-off time duty cycle of the switch, $T_o$. The on-state duty cycle, $\delta$, is normally controlled by using pulse-width modulation, frequency modulation, or a combination of both. When the switch is turned off the inductive load current continues and flows through the load freewheel diode, $D_1$, shown in figure 13.2a.

The analysis to follow assumes:
- No source impedance
- Constant switch duty cycle
- Steady state conditions have been reached
- Ideal semiconductors and
- No load impedance temperature effects.

13.2.1 Continuous load current

Load waveforms for continuous load current conduction are shown in figure 13.3b.

The output voltage $v_o$, or load voltage is defined by

$$v_o(t) = \begin{cases} 
V_o & \text{for } 0 \leq t \leq T \\
0 & \text{for } t > T 
\end{cases}$$

(13.1)

The mean load voltage (hence mean load current) is
\[ I_{\text{rms}} = \frac{1}{T} \int_{0}^{T} i(t) \, dt = \frac{1}{T} \int_{0}^{T} I_{\text{rms}} \, dt \]
\[ = \frac{2}{T} \int_{0}^{T/2} I_{\text{rms}} \, dt \quad \text{whence} \quad I_{\text{rms}} = \frac{V}{\delta} \]
where the switch on-state duty cycle \( \delta = t_0/T \) is defined in figure 13.3b.

The rms load voltage is
\[ V_{\text{rms}} = \frac{1}{T} \int_{0}^{T} V_c(t) \, dt = \frac{1}{T} \int_{0}^{T} \sqrt{2} V \, dt \]
\[ = \sqrt{2} V \quad \text{thus} \quad V_{\text{rms}} = \sqrt{3} V \]

The output ac ripple voltage is
\[ V_s = \sqrt{2} V \left( \frac{1}{\delta} - \frac{1}{\sqrt{\delta}} \right) \]
\[ = \sqrt{2} \delta V \left( 1 - \frac{1}{\sqrt{\delta}} \right) \]

The maximum rms ripple voltage in the output occurs when \( \delta = \frac{1}{2} \) giving an rms ripple voltage of \( \frac{1}{2} \sqrt{2} V \).

The output voltage ripple factor is
\[ RF = \frac{V_s}{\sqrt{2} V} = \left( \frac{1}{\delta} - \frac{1}{\sqrt{\delta}} \right) \]
\[ = \frac{1}{\delta} - \frac{1}{\sqrt{\delta}} \]

Thus as the duty cycle \( \delta \to 1 \), the ripple factor tends to zero, consistent with the output being dc, that is \( V_s = 0 \).

**Steady-state time domain analysis of first-quadrant chopper**

- with load back emf and continuous output current

The time domain load current can be derived in a number of ways.
- First, from the Fourier coefficients of the output voltage, the current can be found by dividing by the load impedance at each harmonic frequency.
- Alternatively, the various circuit currents can be found from the time domain load current equations.

\[ a_n = \frac{1}{\sqrt{2}} \sin 2\pi n \delta \]
\[ b_n = \frac{1}{\sqrt{2}} (1 - \cos 2\pi n \delta) \quad \text{for} \quad n \geq 1 \]

Thus the peak magnitude and phase of the \( n \)th harmonic are given by
\[ c_n = \sqrt{a_n^2 + b_n^2} \]
\[ \phi_n = \tan^{-1} \frac{a_n}{b_n} \]

Substituting expressions from equation (13.4) yields
\[ c_n = \frac{1}{\sqrt{2}} \sin 2\pi n \delta \]
\[ \phi_n = \tan^{-1} \frac{\sin 2\pi n \delta}{1 - \cos 2\pi n \delta} = \frac{1}{2} \pi - \pi n \delta \]

where
\[ v_n = c_n \sin (n \omega t + \phi_n) \]

such that
\[ v_n(t) = I_{\text{rms}} \sum_{n=1}^{\infty} c_n \sin (n \omega t + \phi_n) \]

The load current is given by
\[ i(t) = \sum_{n=1}^{\infty} \frac{V}{R} + \frac{P_c}{R} \sum_{n=1}^{\infty} c_n \sin (n \omega t + \phi_n) \]
\[ = \frac{V}{R} + \frac{P_c}{R} \sum_{n=1}^{\infty} c_n \sin (n \omega t + \phi_n) \]
\[ \text{where} \quad c_n = \frac{1}{\sqrt{2}} \sin 2\pi n \delta \]

\[ \phi_n = \tan^{-1} \frac{\sin 2\pi n \delta}{1 - \cos 2\pi n \delta} = \frac{1}{2} \pi - \pi n \delta \]

Figure 13.4. Harmonics in the output voltage and ripple current as a function of duty cycle \( \delta = t_0/T \) and ratio of cycle period \( T \) (switching frequency, \( f_s = 1/T \)) to load time constant \( T/L \). Valid only for continuous load current conduction.

The peak-to-peak ripple current can be extracted from figure 13.4, which shows a family of curves for equation (13.15), normalised with respect to \( V_s / R \). For a given load time constant \( T = L / R \) switching frequency \( f_s = 1/T \) and switch on-state duty cycle \( \delta \), the ripple current can be extracted. This figure shows a number of important features of the ripple current.

- The ripple current \( I_{\text{pp}} \) reduces to zero as \( \delta \to 0 \) and \( \delta \to 1 \).
- Differentiation of equation (13.15) reveals that the maximum ripple current \( I_{\text{pp}} \) occurs at \( \delta = \frac{1}{2} \).
The longer the load \( R/L \) time constant, \( \tau \), the lower the output ripple current \( I_{p-p} \).

- The higher the switching frequency, \( 1/T \), the lower the output ripple.

If the switch conducts continuously (\( \delta = 1 \)), then substitution of \( t = T \) into equations (13.11) to (13.13) gives a load voltage \( V_L \) and a dc load current is

\[
I_c = I = \frac{V - V_L}{R} = \frac{V - E}{R} - \frac{\delta}{T} \left( \frac{V - E}{R} \right) \tag{13.16}
\]

The mean output current with continuous load current is found by integrating the load current over two consecutive periods, the switch conduction given by equation (13.11) and diode conduction given by equation (13.12), which yields

\[
\frac{1}{T} \int_0^T i_c(t) dt = \frac{\delta(V - E)}{R} \left( \frac{V - E}{R} \right) \tag{13.17}
\]

The input and output powers are related such that

\[
P_i = P_o
\]

\[
P_i = V_i I_i = V_L I_c = V_L \frac{\delta(V - E)}{R} \left( \frac{V - E}{R} \right) \tag{13.18}
\]

\[
P_o = V_o I_o = V_L I_o = \frac{\delta(V - E)}{R} \left( \frac{V - E}{R} \right) \tag{13.19}
\]

from which the average input current can be evaluated.

Alternatively, the average input current, which is the average switch current, \( I_{\text{avg}} \), can be derived by integrating the switch current which is given by equation (13.11), that is

\[
I_{\text{avg}} = \frac{1}{T} \int_0^T i_s(t) dt
\]

\[
= \frac{\delta(V - E)}{R} \left( \frac{V - E}{R} \right) \tag{13.20}
\]

The term \( \delta(V - E) \) is the peak-to-peak ripple current, which is given by equation (13.15). By Kirchhoff’s current law, the average diode current \( I_{\text{avg}} \) is the difference between the average output current \( I_c \) and the average input current, \( I_{\text{avg}} \), that is

\[
\frac{1}{T} \int_0^T i_d(t) dt = I_c - I_{\text{avg}}
\]

\[
= \frac{\delta(V - E)}{R} \left( \frac{V - E}{R} \right) \tag{13.21}
\]

The term \( \delta(V - E) \) represents motor back emf, then the electromotive energy conversion efficiency is given by

\[
\eta = \frac{e_{\text{in}} T_{\text{in}}}{P_o} = \frac{P_i}{P_o} \frac{T_{\text{in}}}{T_{\text{in}}}
\]

The chopper effective (dc) input impedance at the dc source is given by

\[
Z_c = \frac{V_c}{I_c}
\]

For an \( R/L \) load without a back emf, set \( E = 0 \) in the foregoing equations. The discontinuous load current analysis to follow is not valid for an \( R/L \) with \( E \neq 0 \) load, since the load current never reaches zero, but at best asymptotes towards zero during the off-period of the switch.

13.2.2 Discontinuous load current

With an opposing emf \( E \) in the load, the load current can reach zero during the off-time, at a time \( t_o \) shown in figure 13.3c. The time \( t_o \) can be found by

- deriving an expression for \( \tau \) from equation (13.11), setting \( t = t_o \),

- this equation is substituted into equation (13.12) which is equated to zero, having substituted for \( t = t_o \), yielding

\[
\tau = t_o + \ln \left( \frac{1}{E/V_o} \left( \frac{E}{1 - \delta} \right) \right)
\]

(13.24)

This equation shows that \( t_o > t \). Figure 13.5 can be used to determine if a particular set of operating conditions involves discontinuous load current.

![Figure 13.5. Bounds of discontinuous load current with \( E > 0 \).](image)

The load voltage waveform for discontinuous load current conduction shown in figure 13.3c is defined by

\[
V_i(t) = \begin{cases} \frac{V}{V_o} & \text{for } 0 \leq t \leq t_o \\ 0 & \text{for } t_o < t \leq T \end{cases}
\]

(13.25)

If discontinuous load current exists for a period \( T - t_o \), from \( t_o \) until \( t \), then the mean output voltage is

\[
V_o = \frac{1}{T} \left( \int_0^{t_o} V_i dt + \int_{t_o}^T V_o dt \right) \tag{13.26}
\]

(13.26)

The rms output voltage with discontinuous load current conduction is given by

\[
V_{\text{rms}} = \frac{1}{T} \sqrt{\int_0^{t_o} V_i^2 dt + \int_{t_o}^T V_o^2 dt} \tag{13.27}
\]

(13.27)

The ac ripple voltage and ripple factor can be found by substituting equations (13.28) and (13.27) into

\[
V_{\text{rms}} = \sqrt{V_o^2 - V_{\text{rms}}^2}
\]

(13.28)
and
\[ RF = \frac{V}{P} = \left( \frac{V}{P} \right)^{-1} \]
(13.29)

**Steady-state time domain analysis of first-quadrant chopper**
- with load back emf and discontinuous output current

i. Fourier coefficients: The load current can be derived indirectly by using the output voltage Fourier series. The Fourier coefficients of the load voltage are
\[ a_n = \frac{1}{\pi} \sin(2\pi n\delta) - \frac{E}{\pi} \]
\[ b_n = \frac{1}{\pi} (1 - \cos(2\pi n\delta)) - \frac{E}{\pi} \]
(13.30)
which using
\[ c_n = \sqrt{a_n^2 + b_n^2} \]
\[ \phi_n = \tan^{-1}(-a_n/b_n) \]
give
\[ v_c(t) = \sum_{n} c_n \sin(n\omega t + \phi_n) \]
(13.31)
The appropriate division by \( Z_o = \sqrt{RF + (\text{load})} \) yields the output current.

ii. Time domain differential equations: For discontinuous load current, \( \dot{i} = 0 \). Substituting this condition into the time domain equations (13.11) to (13.14) yields equations for discontinuous load current, specifically:

- **During the switch on-period**, when \( v_o(t) = V_o \)
\[ i_c(t) = \frac{V_o - E}{R} \left( 1 - e^{-\frac{t}{R}} \right) \]
for \( 0 \leq t \leq t_s \)
(13.32)

- **During the switch off-period**, when \( v_o(t) = 0 \), after shifting the zero time reference to \( t_r \)
\[ i_c(t) = \frac{E}{R} \left( 1 - e^{-\frac{t}{R}} \right) + \frac{1}{R} e^{-\frac{t}{R}} \]
for \( 0 \leq t \leq t_s - t_r \)
(13.33)
where from equation (13.32), with \( t = t_r \)
\[ i_c(t) = \frac{V_o - E}{R} \left( 1 - e^{-\frac{t}{R}} \right) \]
(A)
(13.34)
After \( t_r \), \( v_o(t) = E \) and the load current is zero, that is
\[ i_c(t) = 0 \]
for \( t_r \leq t \leq T \)
(13.35)
The output ripple current, for discontinuous conduction, is dependent of the back emf \( E \) and is given by equation (13.34), that is
\[ I_{c_{\text{r}}}, \dot{i} = \frac{V_o - E}{R} \left( 1 - e^{-\frac{t_r}{R}} \right) \]
(13.36)
Since \( \dot{i} = 0 \), the mean output current for discontinuous conduction, is
\[ \overline{I_c} = \frac{1}{t_s} \int_{0}^{t_s} i_c(t) \, dt = \frac{1}{t_s} \int_{0}^{t_s} \frac{V_o - E}{R} \left( 1 - e^{-\frac{t}{R}} \right) \, dt + \frac{1}{t_s} \int_{t_s}^{t_s - t_r} \frac{E}{R} \left( 1 - e^{-\frac{t}{R}} \right) \, dt \]
\[ = \frac{V_o - E}{R} \left( 1 - e^{-\frac{t_s}{R}} \right) + \frac{1}{R} e^{-\frac{t_s}{R}} \]
\[ = \frac{V_o - E}{R} - \frac{E}{R} \left( 1 - e^{-\frac{t_s}{R}} \right) \]
(A)
(13.37)
The input and output powers are related such that
\[ P_o = V_o \overline{I_c} \quad P_o = I_{c_{\text{r}}} + R \quad P_o = P_c \]
(13.38)
from which the average input current can be evaluated.
i. From equations (13.2) and (13.3), assuming continuous load current, the average and rms output voltages are both independent of the back emf, namely

\[ V_{av} = \frac{1}{4\pi} \times 340V = 85V \]

\[ V_{rms} = \frac{1}{\sqrt{2}} \times 240V = 120V \]

ii. The rms ripple voltage hence ripple factor are given by equations (13.4) and (13.5), that is

\[ \delta = \frac{1}{\sqrt{2}} \times 147.2V = 1.732 \]

\[ \frac{V_{ac}}{V_{rms}} = \frac{1}{\sqrt{2}} \]

No back emf, \( E = 0 \)

iii. From equation (13.13), with \( E = 0 \), the maximum and minimum currents are

\[ I_{max} = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I_{av} \]

\[ I_{min} = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) - I_{av} \]

The peak-to-peak ripple in the output current is therefore

\[ I_{pp} = I_{max} - I_{min} \]

Alternatively the ripple can be extracted from figure 13.4 using \( T/\tau = 1 \) and \( \delta = 1/4 \).

iv. From equations (13.11) and (13.12), with \( E = 0 \), the time domain load current equations are

\[ I(t) = 34 - 28.38 e^{-\frac{t}{\tau}} + 5.62 e^{-\frac{t}{\tau}} \]

\[ I(t) = 19.90 e^{-\frac{t}{\tau}} \] for \( 0 \leq t \leq 1.25 \) ms

\[ I(t) = 34 - 28.38 e^{-\frac{t}{\tau}} + 5.62 e^{-\frac{t}{\tau}} \] for \( 0 \leq t \leq 3.75 \) ms

v. The average load current from equation (13.17), with \( E = 0 \), is

\[ I_{av} = \frac{V}{R} = \frac{85V}{10\Omega} = 8.5A \]

The average switch current, which is the average supply current, is

\[ I_{sw} = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I_{av} \]

\[ I_{sw} = 8.5A \]

\[ I_{sw} = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I_{av} \]

\[ I_{sw} = 8.5A \]

\[ I_{sw} = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I_{av} \]

\[ I_{sw} = 8.5A \]

The average diode current is the difference between the average load current and the average input current, that is

\[ I_{diode} = I_{av} - I_{av} \]

\[ I_{diode} = 8.5A - 2.22A = 6.28A \]

vi. The input power is the dc supply voltage multiplied by the average input current, that is

\[ P_{in} = V_{dc} I_{av} = 85V \times 2.22A = 185.9W \]

\[ P_{in} = 85.4W \]

From equation (13.18) the rms load current is given by

\[ V_{rms} = \frac{P_{in}}{R} \]

\[ V_{rms} = \frac{85V}{10\Omega} = 8.7A \] rms

vii. The ripple factor is the same as the \( E = 0 \) case, which is as expected since ripple current is independent of back emf with continuous output current. Alternatively the ripple can be extracted from figure 13.4 using \( T/\tau = 1 \) and \( \delta = 1/4 \).
The average switch current is the average supply current, 
\[ I_s = I_{\text{ave}} = \frac{V_s - E}{R} \]
\[ = \frac{55V - 55V}{10\Omega} = 5.5A \]
The average diode current is the difference between the average load current and the average input current, that is
\[ I_d = I_s - I_0 = 3A - 5.5A = 2.15A \]

Example 13.2: DC chopper with load back emf - verge of discontinuous conduction
A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

i. the maximum back emf before discontinuous load current conduction commences with \( \delta = 1/4 \);
ii. with 55V back emf, what is the minimum duty cycle before discontinuous load current conduction; and
iii. minimum switching frequency at \( E = 55V \) and \( t_r = 1.25ms \) before discontinuous conduction.

Solution
The main circuit and operating parameters are:
- on-state duty cycle \( \delta = 1/4 \);
- period \( T = 1/\delta = 1/0.25 = 5ms \);
- on-period of the switch \( t_r = 1.25ms \);
- load time constant \( T / R = 0.05mH/10\Omega = 5ms \).

First it is necessary to establish whether the given conditions represent continuous or discontinuous load current. The current extinction time \( t_e \) for discontinuous conduction is given by equation (13.24), and yields
\[ t_e = t_r + \tau \left( 1 - \frac{E}{E_s} \right) \]
\[ = 1.25ms + 5ms \times \left( 1 - \frac{55V}{340V} \right) \]
\[ = 5.07ms \]
Since the cycle period is 5ms, which is less than the necessary time for the current to fall to zero (5.07ms), the load current is continuous. From example 13.1 part iv, with \( E = 55V \) the load current falls from 6.4A to near zero (0.12A) at the end of the off-time, thus the chopper is operating near the verge of discontinuous conduction. A small increase in \( E \), decrease in the duty cycle \( \delta \), or increase in switching period \( T \), would be expected to result in discontinuous load current.

i. \( \delta \)

The necessary back emf can be determined graphically or analytically.

Graphically:
The bounds of continuous and discontinuous load current for a given duty cycle, switching period, and load time constant can be determined from figure 13.5. Using \( \delta = 1/4, T/\tau = 1 \) with \( r = 5ms \), and \( T = 5ms \), figure 13.5 gives \( E / V_i = 0.165 \). That is, \( E = 0.165 \times 340V = 56.2V \).
\[ t_r = \tau \left( 1 + \frac{E}{E'} \right) \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 1.25\text{ms} \]

If the switch on-state period is reduced by 0.024ms, from 1.250ms to 1.226ms ($\delta = 24.52\%$), operation is then on the verge of discontinuous conduction.

### Example 13.3: DC chopper with load back emf – discontinuous conduction

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and an opposing back emf of 100V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

1. the load average and rms voltages;
2. the rms ripple voltage, hence ripple factor;
3. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
4. the current in the time domain;
5. the load average current, average switch current and average diode current;
6. the input power, hence output power and rms output current;
7. effective input impedance, and electromagnetic efficiency; and
8. sketch the circuit, load, and output voltage and current waveforms.

#### Circuit Diagram

![Circuit Diagram](image)

(a) with load connected to ground and (b) load connected so that machine flash-over to ground (0V), by-passes the switch $T_1$.

\[ \tau = \frac{L}{R} = \frac{50\text{mH}}{10\Omega} = 5\text{ms} \]

\[ T = \frac{5\text{ms}}{200\text{Hz}} = 25\text{ms} \]

The main circuit and operating parameters are
- on-state duty cycle $\delta = \frac{1}{4}$
- period $T = 1\text{ms} / 200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_s = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{mH}/10\Omega = 5\text{ms}$

Confirmation of discontinuous load current can be obtained by evaluating the minimum current given by equation (13.13), that is

\[ i_r(t) = \frac{V'_r - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 1.25\text{ms} + 5\text{ms} \times \left[ 1 + \frac{340V - 100V}{100V} \left( 1 - e^{-\frac{t}{\tau}} \right) \right] = 5.07\text{ms} \]

Discontinuous conduction operation occurs if the period is increased by more than 0.07ms. In conclusion, for the given load, for continuous conduction to cease, the following operating conditions can be changed:
- increase the back emf $E$ from 55V to 56.2V
- decrease the duty cycle $\delta$ from 25% to 24.52% ($\delta$, decreased from 1.25ms to 1.226ms)
- increase the switching period $T$ by 0.07ms, from 5ms to 5.07ms (from 200Hz to 197.2Hz), with the switch on-time, $t_s$, unchanged from 1.25ms.

Appropriate simultaneous smaller changes in more than one parameter would suffice.

#### Solution

The main circuit and operating parameters are
- on-state duty cycle $\delta = \frac{1}{4}$
- period $T = 1\text{ms} / 200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_s = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{mH}/10\Omega = 5\text{ms}$

\[ i_r(t) = \frac{V'_r - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 1.25\text{ms} + 5\text{ms} \times \left[ 1 + \frac{340V - 100V}{100V} \left( 1 - e^{-\frac{t}{\tau}} \right) \right] = 5.07\text{ms} \]

The minimum practical current is zero, so clearly discontinuous current periods exist in the load current. The equations applicable to discontinuous load current need to be employed. The current extinction time is given by equation (13.24), that is

\[ i_r(t) = \frac{V'_r - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 1.25\text{ms} + 5\text{ms} \times \left[ 1 + \frac{340V - 100V}{100V} \left( 1 - e^{-\frac{t}{\tau}} \right) \right] = 5.07\text{ms} \]

i. From equations (13.26) and (13.27) the load average and rms voltages are

\[ V_r = \frac{V'_r - E}{R} = \frac{1}{4} \times \frac{340V + 100V}{5\text{ms}} = 117.4V \]

\[ V'_r = \sqrt{V_r^2 + \left( \frac{5\text{ms} \times 3.38\text{rms}}{5\text{ms}} \right)^2} = 179.3V \text{ rms} \]

ii. From equations (13.28) and (13.29) the rms ripple voltage, hence voltage ripple factor, are

\[ V'_r = \sqrt{V_r^2 + \left( \frac{5\text{ms} \times 3.38\text{rms}}{5\text{ms}} \right)^2} = 179.3V \text{ rms} \]

\[ RF = \frac{V'_r}{V_r} = \frac{179.3V}{117.4V} = 1.51 \]

iii. From equation (13.36), the maximum and minimum output current, hence the peak-to-peak output ripple in the current, are

\[ i_r(t) = \frac{V'_r - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 340V - 100V \times \left( 1 - e^{-\frac{t}{\tau}} \right) = 5.31A \]

The minimum current is zero so the peak-to-peak ripple current is $\Delta i = 5.31A$.

iv. From equations (13.32) and (13.33), the current in the time domain is

\[ i_r(t) = \frac{V'_r - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 340V - 100V \times \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ = 24 \times 1 - e^{-\frac{t}{\tau}} \]  (A) for $0 \leq t \leq 1.25\text{ms}$
From equations (13.37) to (13.40), the average load current, average switch current, and average diode current are

\[ i_{L\text{avg}} = \frac{V_s - E}{R} = \frac{117.4V - 100V}{10\Omega} = 1.74A \]

\[ i_{S\text{avg}} = \frac{E}{R} = \frac{100V}{10\Omega} = 10A \]

\[ i_{D\text{avg}} = \frac{E}{R} = \frac{100V}{10\Omega} = 10A \]

For \( 0 \leq t \leq 2.13\text{ms} \)

\[ i(t) = 0 \]

For \( 3.38\text{ms} \leq t \leq 5\text{ms} \)

\[ i(t) = 5.31A - 1.05A = 0.69A \]

v. From equations (13.37) to (13.40), the average load current, average switch current, and average diode current are

vi. From equation (13.38), the input power, hence output power and rms output current are

\[ P_{in} = V_s I_o = 340V \times 0.69A = 234.6W \]

\[ P_{out} = P_{in} - P_{R} = 234.6W - 100V \times 0.69A / 10\Omega = 1.29A \]

vii. From equations (13.42) and (13.43), the effective input impedance and electromagnetic efficiency, for \( E > 0 \) are

\[ Z_{in} = \frac{V_s}{I_o} = \frac{340V}{0.69A} = 493\Omega \]

\[ \eta = \frac{P_{out}}{P_{in}} = 74.2\% \]

viii. The circuit, load, and output voltage and current waveforms are plotted in figure example 13.3.

13.3 Second-Quadrant dc chopper

The second-quadrant dc-to-dc chopper shown in figure 13.2b transfers energy from the load, back to the dc energy source \( V_s \), a process called regeneration. Its operating principles are the same as those for the boost switch mode power supply analysed in chapter 15.4. The two energy transfer stages are shown in figure 13.6. Energy is transferred from the back emf \( E \) to the supply \( V_s \), by varying the switch \( T_2 \) on-state duty cycle. Two modes of transfer can occur, as with the first-quadrant chopper already considered. The current in the load inductor can be either continuous or discontinuous, depending on the specific circuit parameters and operating conditions. In this analysis, and all the choppers analysed, it is assumed that:

- No source impedance;
- Constant switch duty cycle;
- Steady-state conditions have been reached;
- Ideal semiconductors; and
- No load impedance temperature effects.

13.3.1 Continuous load inductor current

Load waveforms for continuous load current conduction are shown in figure 13.7a. The output voltage \( v_o \), load voltage, or switch voltage, is defined by

\[ v_o(t) = \begin{cases} V_s & \text{for } 0 \leq t \leq t_s \\ V_o & \text{for } t_s \leq t \leq T \end{cases} \]

The mean load voltage is
The output ripple current, for continuous conduction, is independent of the back emf $E$ and is given by

$$I_{r} = \frac{E}{R} \frac{1}{1 - e^{-\frac{t}{T}}}$$  \hspace{1cm} (13.51)

which in terms of the on-state duty cycle, $\delta = t_o / T$, becomes

$$I_{r,\delta} = \frac{E}{R} \left(1 - e^{-\frac{t_o}{T}}\right)$$  \hspace{1cm} (13.52)

This is the same expression derived in 13.2.1 for the first-quadrant chopper. The normalised ripple current design curves in figure 13.3 are valid for the second-quadrant chopper. The average switch current, $I_{\text{sw},\delta}$, can be derived by integrating the switch current given by equation (13.48), that is

$$I_{\text{sw},\delta} = \frac{1}{T} \int_0^T i_s(t) \, dt$$

(13.53)

The term $\bar{i} = i_o + I_{\text{sw},\delta}$ is the peak-to-peak ripple current, which is given by equation (13.51). By Kirchhoff’s current law, the average diode current $I_{\text{diode}}$ is the difference between the average output current $\bar{I}$ and the average switch current, $I_{\text{sw},\delta}$, that is

$$I_{\text{diode}} = \bar{I} - I_{\text{sw},\delta} = \frac{E}{R} \left(1 - e^{-\frac{t_o}{T}}\right)$$

(13.54)

The average diode current can also be found by integrating the diode current given in equation (13.49), as follows

$$I_{\text{diode}} = \frac{1}{T} \int_0^T i_{\text{diode}}(t) \, dt$$

(13.55)

The power produced (provide) by the back emf source $E$ is

$$P_i = E I_{\text{diode}} = \frac{E^2}{R} \left(1 - e^{-\frac{t_o}{T}}\right)$$

(13.56)

The power delivered to the dc source $V_s$ is

$$P_o = V_s I_{\text{diode}} = \frac{E}{R} \left(1 - e^{-\frac{t_o}{T}}\right)$$

(13.57)

The difference between the two powers is the power lost in the load resistor, $R$, that is

$$P_o - P_i = \frac{E}{R} \left(1 - e^{-\frac{t_o}{T}}\right)$$

(13.58)

The efficiency of energy transfer between the back emf $E$ and the dc source $V_s$ is

$$\eta = \frac{P_o}{P_i} = \frac{E}{V_s} \frac{1 - e^{-\frac{t_o}{T}}}{E}$$

(13.59)

13.3.2 Discontinuous load inductor current

With low duty cycles, $\delta$, low inductance, $L$, or a relatively high dc source voltage, $V_s$, the minimum output current may reach zero at $t_o$ before the period $T$ is complete ($t_o < T$), as shown in figure 13.7b. Equation (13.50) gives a boundary identity that must be satisfied for zero current,

$$\bar{I} = \frac{E}{R} \frac{1 - e^{-\frac{t_o}{T}}}{1 - e^{-\frac{t_o}{T}}} = 0$$

(13.60)
I

Alternatively, the time domain equations (13.48) and (13.49) can be used, such that $\dot{I} = 0$. An expression for the extinction time $t_e$, can be found by substituting $t = t_e$ into equation (13.48). The resulting expression for $I_e$ is then substituted into equation (13.49) which is set to zero. Isolating the time variable, which becomes $t_e$, yields

$$I_e = E \left( \frac{1}{R} - \frac{1}{2} \right)$$

(13.65)

The average output current can also be found by integration of the time domain output current $i_o$. By solving the appropriate time domain differential equations, the continuous load current $i_l$ shown in figure 13.7a is defined by

$$i_l(t) = \begin{cases} 0 & \text{for } 0 \leq t < t_i, \\ E & \text{for } t_i \leq t \leq T \\ \end{cases}$$

(13.66)

The average switch current, $I_s$, as the peak-to-peak ripple current, which is given by equation (13.70). By Kirchhoff’s current law, the average diode current $I_{s\delta}$ is the difference between the average output current $I_l$ and the average switch current, $I_s$, that is

$$I_{s\delta} = I_l - I_s$$

(13.71)

The output ripple current, for discontinuous conduction, is dependent of the back emf $E$ and is given by equation (13.68),

$$I_{\text{rms}} = \frac{E}{R} \left( 1 - e^{-\frac{t}{R}} \right)$$

(13.72)

$$= \frac{\delta E}{R} \left( V_o - E \right)$$

(13.73)

The power produced by the back emf source $P_s$ is

$$P_s = \frac{\delta E}{R} \left( V_o - E \right)$$

(13.74)

Alternatively, the difference between the two powers is the power lost in the load resistor, $R$, that is

$$P_l = \frac{E^2}{R} - P_s$$

(13.75)

The efficiency of energy transfer between the back emf and the dc source is

$$\eta = \frac{P_s}{P_l} = \frac{1}{T} \int_0^T \frac{E}{R} \left( 1 - e^{-\frac{t}{R}} \right) dt$$

(13.76)

$$= \frac{T}{T - T_o} \left( V_o - E \right)$$

(13.77)

**Example 13.4: Second-quadrant DC chopper – continuous inductor current**

A dc-to-dc chopper capable of second-quadrant operation is used in a 200V dc battery electric vehicle. The machine armature has 1 ohm resistance in series with 1.0mH inductance.

i. The machine is used for regenerative braking. At a constant speed downhill, the back emf is 150V, which results in a 10A braking current. What is the switch on-state duty cycle if the machine is delivering continuous output current? What is the minimum chopping frequency for these conditions?

ii. At this speed, (that is, $E = 150V$), determine the minimum duty cycle for continuous inductor current, if the switching frequency is 1kHz. What is the average braking current at the critical duty cycle? What is the regenerating efficiency and the rms machine output current?

iii. If the chopping frequency is increased to 5kHz, at the same speed, (that is, $E = 150V$), what is the critical duty cycle and the corresponding average dc machine current?
Solution
The main circuit operating parameters are
- \( V_s = 200V \)
- \( E = 150V \)
- load time constant \( t = L/R = 1mH/1\Omega = 1ms \)

Figure 13.4. Circuit diagram and waveforms.

i. The relationship between the dc supply \( V_s \) and the dc machine back emf \( E \) is given by equation (13.47), that is

\[
T = \frac{E - V_s}{R} \times = \frac{E - V_s(1 - \delta)}{R}
\]

that is

\[
\delta = 0.3 = 30\% \quad \text{and} \quad P_e = 140V
\]

The expression for the average dc machine output current is based on continuous armature inductance current. Therefore the switching period must be shorter than the time \( t_s \) predicted by equation (13.62) for the current to reach zero, before the next switch on-period. That is, for \( t_s = T \) and \( \delta = 0.3 \)

\[
t_s = t + n \left( t + \frac{E}{R} \left( 1 - e^{-t/R} \right) \right)
\]

This simplifies to

\[
E = \frac{1}{3} \left( t + \frac{150V}{200V} \left( 1 - e^{-t/R} \right) \right)
\]

Iteratively solving this transcendental equation gives \( T = 0.4945ms \). That is the switching frequency must be greater than \( f_s = T = 2.022kHz \), else machine output current discontinuities occur, and equation (13.47) is invalid. The switching frequency can be reduced if the on-state duty cycle is increased as in the next part of this example.

ii. The operational boundary condition given by equation (13.61), using \( T = 1/T_s = 1/1kHz = 1ms \), yields

\[
T = \frac{E}{V_s} \left( 1 - e^{-t/R} \right)
\]

\[
150V = \frac{200V}{200V} \left( 1 - e^{-t/R} \right)
\]

Solving gives \( \delta = 0.357 \). That is, the on-state duty cycle must be at least 35.7% for continuous machine output current at a switching frequency of 1kHz.

13.4 Two-quadrant dc chopper - Q I and Q II

Figure 13.8 shows the basic two-quadrant dc chopper, which is a reproduction of the circuit in figure 13.2c. Depending on the load and operating conditions, the chopper can seamlessly change between and act in two modes:
- Devices \( T_1 \) and \( D_1 \) form the first-quadrant chopper shown in figure 13.2a, and is analysed in section 13.2. Energy is delivered from the dc source \( V_s \) to the \( R-L-E \) load.
• Devices $T_2$ and $D_2$ form the second-quadrant chopper shown in figure 13.2b, which is analysed in section 13.3. Energy is delivered from the generating load dc source $E$, to the dc source $V_s$. The two independent choppers can be readily combined as shown in figure 13.8a. The average output voltage $V_s$ and the instantaneous output voltage $v_o$ are never negative, whilst the average source current of $V_s$ can be positive (Quadrant I) or negative (Quadrant II). If the two choppers are controlled to operate independently, with the constraint that $T_1$ and $T_2$ do not conduct simultaneously, then the analysis in sections 13.2 and 13.3 are valid. Alternately, it is not uncommon to unify the operation of the two choppers, as follows.

\[
\begin{align*}
\text{if } I_\text{min} < 0 \text{ and } I_\text{max} < 0, \text{ then the chopper is active in the second-quadrant.}& \\
\text{When the minimum current (hence average output current) is greater than zero, the chopper is active in the first-quadrant.}& \\
\text{Typical output voltage and current waveforms are shown in figure 13.3a.}& \\
\text{The switch $T_2$ and diode $D_2$ do not conduct} &
\end{align*}
\]

When the output current is greater than zero, the chopper is operational in the first-quadrant. Since the load current never goes positive, switch $T_2$ and diode $D_2$ never conduct, as shown in figure 13.8c. For a highly inductive load, if the magnitude of the negative peak is greater than the positive maximum, the average is less than zero and the chopper is active in the second-quadrant. If the load is not highly inductive the boundary is determined by the average output current $\tau > 0$. The various circuit waveforms are shown in figure 13.8b.

In all cases the average output voltage is solely determined by the switch $T_1$, on-duty cycle, since when this switch is turned on the supply $V_s$ is impressed across the load, independent of the direction of the load current. When $i_o > 0$, switch $T_1$ conducts while if $i_o < 0$, the diode in parallel to switch $T_1$, namely $D_1$, conducts, clamping the load to $V_s$. The output voltage, which is independent of the load, is described by

\[
v_o(t) = 
\begin{cases} 
V_s & \text{for } 0 \leq t < T_1 \\
0 & \text{for } i_o \leq \tau \leq T_1 
\end{cases}
\]  

(13.78)

Thus

\[
\bar{V}_o = \frac{1}{T} \int_0^T V_o dt = \frac{L_o}{T} V_s - \delta V_s 
\]

(13.79)

The rms output voltage is also determined solely by the duty cycle,

\[
V_{rms} = \left[ \frac{1}{T} \int_0^T V_o^2 dt \right]^{1/2} = \sqrt{\delta} V_s
\]

(13.80)

The output ac ripple voltage, hence voltage ripple factor are given by equations (13.3) and (13.5), and are independent of the load:

\[
V_r = V_s - \bar{V}_o = V_s \delta (1 - \delta)
\]

(13.81)

and

\[
RF = \frac{V_r}{\bar{V}_o} = \frac{1}{\sqrt{1 - \delta}} = \frac{1 - \delta}{\delta}
\]

(13.82)

The Fourier series for the load voltage can be used to determine the load current at each harmonic frequency as described by equations (13.6) to (13.10). The time domain differential equations from section 13.2.1 are also valid, where there is no zero restriction on the minimum load current value.

In a positive voltage loop, when $v_o(t) = V_s$ and $v_o$ is impressed across the load, the load current condition is described by

\[
l(t) = \frac{V_s - E}{R} - \frac{1}{R} t + \frac{1}{R} v_o^2 \text{ for } 0 \leq t \leq T_1
\]

(13.83)
During the switch off-period, when \( v_o = 0 \), forming a zero voltage loop

\[
\frac{1}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I e^\frac{-t}{\tau} \text{ for } 0 \leq t \leq T - t_o
\]

(13.84)

where

\[
I = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) - \frac{E}{R} \quad (A)
\]

and

\[
I = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) - \frac{E}{R} \quad (A)
\]

The peak-to-peak ripple current is independent of \( E \),

\[
I_{pp} = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right)
\]

(13.86)

The average output current, \( T_o \), may be positive or negative and is given by

\[
T_o = \frac{1}{T} \int_0^T I(t) dt = \frac{V - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right)
\]

(A)

(13.87)

The direction of the net power flow between \( V_s \) and \( V_o \) determines the chopper operating quadrant. If \( T_o > E \) then average power flow is to the load, as shown in figure 13.8b, while if \( T_o < E \), the average power flow is back into the source \( V_s \), as shown in figure 13.8c.

\[
T_o = \frac{V - E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right)
\]

(13.88)

Thus the sign of \( T_o \) determines the direction of net power flow, hence quadrant of operation.

Calculation of individual device average currents in the time domain is complicated by the fact that the energy may flow between the dc source \( V_s \) and the load via the switch \( T_1 \) (energy to the load) or diode \( D_2 \) (energy from the load). It is therefore necessary to ascertain the zero current crossover time, when \( I \) and \( \delta \) have opposite signs, which will then specify the necessary bounds of integration.

Equations (13.83) and (13.84) are equated to zero and solved for the time at zero crossover, \( t_o \) and \( t_{co} \), respectively, shown in figure 13.8b.

\[
t_o = \frac{\tau}{2} \left( 1 - \frac{\tau}{I} \right) \text{ with respect to } t = 0
\]

(13.89)

\[
t_{co} = \frac{\tau}{2} \left( 1 + \frac{\tau}{I} \right) \text{ with respect to } t = t_o
\]

The necessary integration for each device can then be determined with the aid of the device conduction information in the parts of figure 13.8 and Table 13.1.

### Table 13.1 Device average current ratings

<table>
<thead>
<tr>
<th>Device and integration bounds, ( a ) to ( b )</th>
<th>( 0 &lt; a &lt; 0 )</th>
<th>( a &gt; 0 )</th>
<th>( 0 &lt; b &lt; 0 )</th>
<th>( b &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_o ), ( 0 \leq t \leq T - t_o ) ( e^\frac{-t}{\tau} ) ( 0 ) to ( t_o ) ( t_o ) to ( t_{co} ) ( t_{co} ) to ( 0 )</td>
<td>( t_{co} ) to ( 0 )</td>
<td>( 0 ) to ( t_o ) ( 0 ) to ( t_o )</td>
<td>( 0 ) to ( T - t_o ) ( 0 ) to ( T - t_o )</td>
<td>( 0 ) to ( T - t_o ) ( 0 ) to ( T - t_o )</td>
</tr>
<tr>
<td>( T_{co} ), ( 0 &lt; t - t_o \leq 0 ) ( e^\frac{-t}{\tau} ) ( 0 ) to ( T - t_o ) ( 0 ) to ( T - t_o )</td>
<td>( t_{co} ) to ( 0 ) ( 0 ) to ( t_o ) ( 0 ) to ( t_o )</td>
<td>( 0 ) to ( T - t_o ) ( 0 ) to ( T - t_o )</td>
<td>( 0 ) to ( T - t_o ) ( 0 ) to ( T - t_o )</td>
<td>( 0 ) to ( T - t_o ) ( 0 ) to ( T - t_o )</td>
</tr>
</tbody>
</table>

### Solution

The main circuit and operating parameters are

- on-state duty cycle \( \delta = 1/4 \)
- period \( T = 1/200 = 5 \text{ms} \)
- on-period of the switch \( t_s = 1.25 \text{ms} \)
- load time constant \( t_L = 0.05 \text{ms} \)

#### Example 13.5: Two-quadrant DC chopper with load back emf

The two-quadrant dc-to-dc chopper in figure 13.8a feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 100V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

1. the load average and rms voltages;
2. the rms ripple voltage, hence ripple factor;
3. the maximum and minimum output current, hence peak-to-peak output ripple in the current;
4. the current in the time domain;
5. the current crossover times, if applicable;
6. the load average current, average switch current and average diode current for all devices;
7. the input power, hence output power and rms output current;
8. effective input impedance and electromagnetic efficiency; and
9. sketch the circuit, load, and output voltage and current waveforms.

Subsequently determine the necessary change in

i. duty cycle \( \delta \) to result in zero average output current and
ii. back emf \( E \) to result in zero average load current.

#### Example 13.5. Circuit diagram.
The peak-to-peak ripple current is therefore: \( \Delta i = 1.90A - (-4.38A) = 6.28A \) p-p.

iv. The current in the time domain is given by equations (13.83) and (13.84)
\[
i(t) = \frac{E}{R} \left( 1 - e^{-t/\tau} \right) + I_e
\]
where \( E = 340V \), \( R = 100\Omega \), \( I_e = 1.90A \), \( \tau = 5ms \), and \( t \) is in ms.

\[
i(t) = 340V \cdot 5ms \left( 1 - e^{-t/5ms} \right) - 4.38\times10^3 \cdot 5ms \left( 1 - e^{-t/5ms} \right)
\]
\[
= 24 \cdot 28.38 \times 10^{-3} \quad \text{for} \ 0 \leq t \leq 1.25ms
\]
\[
i(t) = \frac{100V}{5ms} \left( 1 - e^{-t/5ms} \right) + 1.90 \times 10^{-3}
\]
\[
= -10 \cdot 28.38 \times 10^{-3} \quad \text{for} \ 0 \leq t \leq 3.75ms
\]

v. Since the maximum current is greater than zero (1.9A) and the minimum is less than zero (-4.38A), the current crosses zero during the switch on-time and off-time. The time domain equations for the load current are solved for zero to give the crossover times \( t_x \) as given by equation (13.88) or solved from the time domain output current equations as follows.
During the switch on-time
\[
i_o(t) = 24 \cdot 28.38 \times 10^{-3}
\]
where \( 0 \leq t = t_o \leq 1.25ms \)

\[
t_o = 5ms \times (1/28.38) = 0.838ms
\]

During the switch off-time
\[
i_o(t) = -10 \cdot 28.38 \times 10^{-3}
\]
where \( 0 \leq t = t_o \leq 3.75ms \)

\[
t_o = 5ms \times (11.90/28.38) = 0.870ms
\]

(1.250ms + 0.870ms = 2.12ms, with respect to switch \( T_1 \) turn-on)

vi. The load average current, average switch current, and average diode current for all devices;
\[
T_{AV} = \frac{(V_o - E)}{R} + (6V - E)/R
\]
\[
= (85V - 100V)/100\Omega = -1.5A
\]

When the output current crosses zero current, the conducting device changes. Table 13.1 gives the necessary current equations and integration bounds for the condition \( i_o > 0 \). Table 13.1 shows that all four semiconductors are involved in the output current cycle.

\[
T_{av} = \frac{1}{5ms} \int_{t_o}^{t_o+5ms} 24 \cdot 28.38 \times 10^{-3} dt = 0.081A
\]
viii. Since the average output current is negative, energy is being transferred from the back emf $E$ to the dc voltage source $V_s$, the electromagnetic efficiency of conversion is given by

$$\eta = \frac{\delta V_s - E}{\delta V_s} \text{ for } \delta < 0$$

$$= \frac{95.2}{150} = 63.5\%$$

The effective input impedance is

$$Z_{\text{in}} = \frac{V_s}{I_s + I_m} = \frac{340}{0.080 + 0.357} = 85\Omega$$

ix. The circuit, load, and output voltage and current waveforms are sketched in the figure for example 13.5.

x. Duty cycle $\delta$ to result in zero average output current can be determined from the expression for the average output current, equation (13.87), that is

$$I_o = \frac{\delta V_s - E}{R} = 0$$

that is

$$\delta = \frac{E}{V_s} = \frac{1000}{3400} = 29.4\%$$

xi. As in part x, the average load current equation can be rearranged to give the back emf $E$ that results in zero average load current

$$I_o = \frac{\delta V_s - E}{R} = 0$$

that is

$$E = \delta V_s = \frac{1}{2} \times 3400 = 85V$$

13.5 Two-quadrant dc chopper - Q1 and Q IV

The unidirectional current, two-quadrant dc chopper, or asymmetrical half H-bridge shown in figure 13.9a incorporates two switches $T_s$ and $T_o$ and two diodes $D_s$ and $D_o$. In using switches $T_s$ and $T_o$ the chopper operates in the first and fourth quadrants, that is, bi-directional voltage output $v_o$ but unidirectional current, $i_o$. The chopper can operate in two quadrants (I and IV), depending on the load and switching sequence. Net power can be delivered to the load, or received from the load provided the polarity of the back emf $E$ is reversed. Because of this need to reverse the back emf for regeneration, this chopper is not commonly used in dc machine control. On the other hand, the chopper circuit configuration is commonly used to meet the converter requirements of the switched reluctance machine, which only requires unidirectional current, two-quadrant dc chopper, or asymmetrical half H-bridge shown in figure 13.9b. The circuit, load, and output voltage and current waveforms are sketched in the figure for example 13.5.

State #1

When both switches $T_s$ and $T_o$ conduct, the supply $V_s$ is impressed across the load, as shown in figure 13.10a. Energy is drawn from the dc source $V_s$.

$\text{State #2}$

If only one switch is conducting, and therefore also one diode, the output voltage is zero, as shown in figure 13.10b. Either switch (but only one on at any time) can be the on-switch, hence providing redundancy, that is

- $T_s$ and $D_s$ conducting: $v_o = 0$
- $T_o$ and $D_o$ conducting: $v_o = 0$

State #3

When both switches are off, the diodes $D_s$ and $D_o$ conduct load energy back into the dc source $V_s$, as in figure 13.10c. The output voltage is $-V_s$ that is

- $T_s$ and $T_o$ are not conducting: $v_o = -V_s$

Figure 13.9. Two-quadrant (I and IV) dc chopper

(a) circuit where $i_o > 0$; (b) operation in quadrant IV, regeneration into $V_o$; and (c) operation in quadrant I.

Figure 13.10. Two-quadrant (I and IV) dc chopper operational current paths: (a) $T_s$ and $T_o$ forming a $+V_s$ path; (b) $T_s$ and $D_s$ (or $T_o$ and $D_o$) forming a zero voltage loop; and (c) $D_s$ and $D_o$ creating a $-V_s$ path.
The two zero output voltage states can most effectively be used if alternated during any switching sequence. In this way, the load switching frequency (load ripple current frequency) is twice the switching frequency of the switches. This reduces the output current ripple for a given switch operating frequency (which minimises the load inductance necessary for continuous load current conduction). Also, by alternating the zero voltage loop, the semiconductor losses are evenly distributed. Specifically, a typical sequence to achieve these features would be

\begin{align*}
T_1 \text{ and } T_2 & : V_s \\
T_1 \text{ and } D_2 & : 0 \\
T_2 \text{ and } T_1 & : V_s \\
T_1 \text{ and } D_1 & : 0 \text{ (not } T_2 \text{ and } D_2 \text{ again)} \\
T_2 \text{ and } D_1 & : 0 \text{ (not } T_1 \text{ and } D_2 \text{ again)} \\
T_1 \text{ and } D_2 & : 0, \text{ etc.}
\end{align*}

The sequence can also be interleaved in the regeneration mode, when only one switch is on at any instant, as follows

\begin{align*}
D_1 \text{ and } D_2 & : -V_s \quad (\text{that is } T_1 \text{ and } T_2 \text{ off}) \\
T_1 \text{ and } D_2 & : 0 \\
D_1 \text{ and } D_2 & : -V_s \\
T_2 \text{ and } D_1 & : 0 \text{ (not } T_1 \text{ and } D_2 \text{ again)} \\
D_1 \text{ and } D_2 & : -V_s \\
T_1 \text{ and } D_2 & : 0, \text{ etc.}
\end{align*}

In switched reluctance motor drive application there may be no alternative to using only \( \pm V_s \) control loops without the intermediate zero voltage state.

There are two basic modes of chopper switching operation.

- **Multilevel switching** is when both switches are controlled independently to give all three output voltage states (three levels), namely \( \pm V_s \), 0V.
- **Bipolar switching** (or two-level switching) is when both switches operate in unison, where they turn on together and off together. Only two output voltage states (hence the term bipolar), are possible, \( \pm V_s \) and 0V.

### 13.5.1 dc chopper: – Q I and Q IV – multilevel output voltage switching (three level)

The interleaved zero voltage states are readily introduced if the control carrier waveforms for the two switches are displaced by 180°, as shown in figure 13.9b and c, for continuous load current. This requirement can be realised if two up-down counters are displaced by 180°, when generating the necessary triangular carriers. As shown in figures 13.9b and c, the switching frequency 1/\( T_{CS} \) is determined by the triangular wave frequency 1/2\( T \), whilst advantageously the load experiences twice that frequency, 1/\( T \), hence the output current has reduced ripple, for a given switch operating frequency.

1. \( 0 \leq \delta \leq 1/2 \)

It can be seen in figure 13.9b that when \( \delta \leq 1/2 \) both switches never conduct simultaneously hence the output voltage is either 0 or \(-V_s\). Operation is in the fourth quadrant. The average output voltage is load independent and for \( 0 \leq \delta \leq 1/2 \), using the waveforms in figure 13.9b, is given by

\[
\bar{V}_o = \frac{1}{\bar{T}} \int_{t_1}^{t_2} V_s dt = \bar{V}_s \left( 1 - \frac{\delta}{2} \right) = -\bar{V}_s \left( 1 - \frac{\delta}{2} \right)
\]

(13.91)

Examination of figure 13.9b reveals that the relationship between \( t_1 \) and \( \delta \) must produce

\[
\delta = \frac{t_1}{T}
\]

(13.92)

(13.92) gives

\[
\bar{V}_o = -\bar{V}_s \left( 1 - \frac{\delta}{2} \right) = -V_s \left( 1 - \frac{\delta}{2} \right) \quad \text{for} \quad 0 \leq \delta \leq 1/2
\]

The rms output voltage is independent of the duty cycle and is \( V_s \).

At the output ac ripple voltage is

\[
V_c = \sqrt{\bar{V}_o^2 - \bar{V}_s^2} = V_s \left( 1 - \frac{\delta}{2} \right)
\]

(13.97)

which is a maximum at \( \delta = 1/2 \) and a minima for \( \delta = 0 \) and \( \delta = 1 \).

The output voltage ripple factor is

\[
RF = \frac{V_c}{\bar{V}_o} = \frac{V_s \left( 1 - \frac{\delta}{2} \right)}{\bar{V}_s} = \frac{2V_s}{\sqrt{3}} \frac{1}{\sqrt{1 - \delta}}
\]

(13.98)

Although the average output voltage may reverse, the load current is always positive but can be discontinuous or continuous. Equations describing bipolar output are presented within the next section, 13.5.3, which considers multilevel (two and three level) output voltage switching states.
which gives

\[ i(t) = \frac{V - E}{R} \left(1 - e^{-\frac{t}{\tau}}\right) + I e^{-\frac{t}{\tau}} \quad \text{for} \quad 0 \leq t \leq r \]

(13.99)

During the first switching cycle the current starts from zero, so \( j = 0 \). Otherwise \( j \) is the lower reference, \( j' \), from the end of the previous cycle.

The current at the end of the positive voltage loop period is the reference level \( j' \), whilst the time to rise to \( j' \) is derived by equating equation (13.99) to \( j' \) and solving for time \( t' \) at the end of the period. Solving \( i(t') = j' \) for \( t' \), gives

\[ t' = \tau \ln \left( \frac{V - E - j' R}{V - E - j'R} \right) \]

(13.100)

13.5.3 Multilevel output voltage states, dc chopper

In switched reluctance machine drives it is not uncommon to operate the asymmetrical half H-bridge shown in figure 13.9 such that:

- both switches operate in the on-state together to form +V voltage loops;
- switches operate independently to give zero voltage loops; and
- both switches are simultaneously off, forming –V voltage output loops.

The control objective is to generate a current output pulse that tracks a reference shape which starts from zero, rises to maintain a fixed current level, with hysteresis, then the current falls back to zero. The waveform shown in figure 13.12 fulfils this specification.

The switching strategy to produce the current waveform in figure 13.12 aims at:

- For rising current- use +V loops (and zero volt loops only if necessary)
- For near constant current- use zero voltage loops (and ±V loops only if necessary to increase or decrease the current)
- For falling current- use –V loops (and zero volts loops only if necessary to reduce the fall rate)

Operation is further characterised by continuous load current during the pulse.

Energy is supplied to the load from the source during +V loops, and returned to the supply during –V loop periods.

The chopper output current during each period is described by equations previously derived in this chapter, but reproduced as follows.

In a positive voltage loop, \( (T_1 \text{ and } T_4 \text{ are both on}) \), when \( v_s(t) = V_s \) and \( V_s \) is impressed across the load, the load circuit condition is described by

\[ L \frac{di}{dt} + Ri + E = V_s \]

which yields

\[ i(t) = \frac{V - E}{R} \left(1 - e^{-\frac{t}{\tau}}\right) + I e^{-\frac{t}{\tau}} \quad \text{for} \quad 0 \leq t \leq r \]

(13.101)

where \( j \) equals the reference current level, \( j' \), from the previous switching period.

The current at the end of the period is the reference level \( j' \), whilst the time to fall to \( j' \) is given by equating equation (13.101) to \( j' \) and solving for time \( t' \) at the end of the period.

\[ t' = \tau \ln \left( \frac{E + j'R}{E + j'R} \right) \]

(13.102)

In a negative voltage loop, when both switches \( T_1 \) and \( T_4 \) are off, the current falls rapidly and the circuit equation, when \( v_s(t) = -V_s \) is

\[ L \frac{di}{dt} + Ri + E = -V_s \]

which gives

\[ i(t) = \frac{V - E}{R} \left(1 - e^{-\frac{t}{\tau}}\right) + I e^{-\frac{t}{\tau}} \quad \text{for} \quad 0 \leq t \leq r \]

(13.103)

where \( j \) equals the lower reference, \( j' \), from the end of the previous cycle.

The current at the end of the negative voltage loop period is the lower reference, \( j' \), whilst the time to fall to \( j' \) is derived by equating equation (13.103) to \( j' \) and solving for time \( t' \) at the end of the period.

\[ t' = \tau \ln \left( \frac{E - j'R}{E - j'R} \right) \]

(13.104)
The following table. For longer current chopping, 𝑡3 dominate the switching frequency.

The current pulse period is given by

\[ \tau_{\text{pulse}} = \frac{E_V + E_i + E_L}{E_V + E_i + E_L} \]

for \( 0 \leq t \leq \tau \).

The same equation is used to determine the time for the final current period when the current decays to zero, whence \( \tau = 0 \).

The characteristics and features of the three output voltage states are illustrated in the following example, 13.6.

**Example 13.6: Asymmetrical, half H-bridge, dc chopper**

The asymmetrical half H-bridge, dc-to-dc chopper in figure 13.9 feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc voltage source. The chopper output current is controlled in a hysteresis mode within a current band between limits 5A and 10A.

 Determine the period of the current shape shown in the figure example 13.6:

\( t_1 \): when only \( V_L \) loops are used

\( t_2 \): when a zero volt loop is used to maintain tracking within the 5A band.

In each case calculate the switching frequency if the current were to be maintained within the hysteresis band for a prolonged period. How do the on-state losses compare between the two control approaches?

**Solution**

The main circuit and operating parameters are:

- \( E = 340V \) and \( V_L = 340V \)
- load time constant \( \tau = L/R = 0.05mH/10\Omega = 5ms \)
- \( I = 5A \) and \( \dot{I} = 10A \)

Examination of the figure shows that only one period of the cycle differs, namely the second period, \( t_2 \), where the current is required to fall to the lower hysteresis band level, -5A. The period of the other three regions \( (t_1, t_3, \text{and} t_4) \) are common and independent of the period of the second region, \( t_2 \).

\( t_1 \): The first period, the initial rise time, \( r_1 = t_1 \) is given by equation (13.100), where \( \dot{I} = 10A \) and \( \dot{I} = 0A \).

\[ r_1 = r \left( \frac{E_l + E_i + E_L}{E_l + E_i + E_L} \right) \]

that is \( t_1 = 5ms \times \frac{340V + 55V - 5A \times 10\Omega}{340V + 55V + 10A \times 10\Omega} = 2.16ms \)

\( t_2 \): In the third period, the current rises from the lower hysteresis band limit of 5A to the upper band limit 10A. The duration of the current increase is given by equation (13.100) again, but with \( \dot{I} = \dot{I} = 5A \).

\[ r_2 = r \left( \frac{E_l + E_i + E_L}{E_l + E_i + E_L} \right) \]

that is \( t_2 = 5ms \times \frac{340V + 55V + 5A \times 10\Omega}{340V + 55V + 10A \times 10\Omega} = 1.20ms \)

\( t_3 \): The fourth and final period is a negative voltage loop where the current falls from the upper band limit of 10A to the \( -I \) which equals zero. From equation (13.104) with \( \dot{I} = \dot{I} = 10A \) and \( \dot{I} = 0A \).

\[ r_3 = r \left( \frac{E_l + E_i + E_L}{E_l + E_i + E_L} \right) \]

that is \( t_3 = 5ms \times \frac{340V + 55V + 10A \times 10\Omega}{340V + 55V + 10A \times 10\Omega} = 1.13ms \)

The current pulse period is given by

\[ \tau_{\text{pulse}} = \frac{E_V + E_i + E_L}{E_V + E_i + E_L} \]

for \( 0 \leq t \leq \tau \).

When a zero volt loop is used to maintain the current within the hysteresis band, the current decays slowly, and the period time \( t_2 \) is given by equation (13.102), with \( I = 5A \) and \( I = 10A \).

\[ t_2 = 5ms \times \frac{340V + 55V + 10A \times 10\Omega}{340V + 55V + 5A \times 10\Omega} = 0.53ms \]

The total period, \( T_1 \) of the chopper current pulse when a 0V loop is not used, is

\[ T_1 = t_1 + t_2 + t_3 = 2.16ms + 1.20ms + 1.13ms = 4.49ms + t_1 \]

When a zero volt loop is used to maintain the current within the hysteresis band, the current decays slowly, and the period time \( t_2 \) is given by equation (13.102), with \( I = 5A \) and \( I = 10A \).

\[ t_2 = 5ms \times \frac{340V + 55V + 10A \times 10\Omega}{340V + 55V + 5A \times 10\Omega} = 1.95ms \]

The total period, \( T_2 \) of the chopper current pulse when a 0V loop is used, is

\[ T_2 = t_1 + t_2 + t_3 = 2.16ms + 1.95ms + 1.20ms + 1.13ms = 6.44ms \]

The current falls significantly faster within the hysteresis band if negative voltage loops are employed rather than zero voltage loops, 0.53ms versus 1.95ms.

The switching frequency within the current bounds has a period \( t_2 + t_3 \) and each case is summarized in the following table. For longer current chopping, \( t_3 \) and \( t_4 \) dominate the switching frequency.
Using zero voltage current loops reduces the switching frequency of the H-bridge switches by a factor of over three, for a given peak-to-peak ripple current. If the on-state voltage drop of the switches and the diodes are similar for the same current level, then the on-state losses are similar, and evenly distributed for both control methods. The on-state losses are similar because each of the three states always involves the same current variation flowing through two semiconductors. The principal difference is in the significant increase in switching losses when only ±V loops are used (13.42).

### Table 13.6. Switching losses.

<table>
<thead>
<tr>
<th>Voltage loops</th>
<th>( t_2 + t_3 )</th>
<th>Current ripple frequency</th>
<th>Switch frequency</th>
<th>Switch loss ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>±V</td>
<td>0.53ms+1.20ms = 1.73ms</td>
<td>578Hz</td>
<td>578Hz</td>
<td>( 3.42 )</td>
</tr>
<tr>
<td>+V and zero</td>
<td>1.95ms+1.20ms = 3.15ms</td>
<td>317Hz</td>
<td>169Hz</td>
<td>1</td>
</tr>
</tbody>
</table>

#### 13.6 Four-quadrant dc chopper

The four-quadrant H-bridge dc chopper is shown in figure 13.13 where the load current and voltage are referenced with respect to \( T_1 \), so that the quadrant of operation with respect to the switch number is preserved.

The H-bridge is a flexible basic configuration where its use to produce single-phase ac is considered in chapter 14.1.1, while its use in acms applications is considered in chapter 15.6.2. It can also be used as a dc chopper for the four-quadrant control of a dc machine. With the flexibility of four switches, a number of different control methods can be used to produce four-quadrant output voltage and current (bidirectional voltage and current). All practical methods should employ complementary device switching in each leg (either \( T_1 \) or \( T_3 \) on but not both and either \( T_2 \) or \( T_4 \) on, but not both) so as to minimise distortion by ensuring current continuity around zero current output.

One control method involves controlling the H-bridge as two virtually independent two-quadrant choppers, with the over-riding restriction that no two switches in the same leg conduct simultaneously. One chopper is formed with \( T_1 \) and \( T_4 \) grouped with \( D_1 \) and \( D_4 \) which gives positive current \( i_1 \) but bidirectional voltage \( ±V \) (Q1 and Q4 operation). The second chopper is formed by grouping \( T_2 \) and \( T_3 \) with \( D_2 \) and \( D_3 \), which gives negative output current \( -i_1 \) but bi-directional voltage \( ±V \) (Q1 and Q2 operation).

![Four-quadrant dc chopper circuit](image)

**Figure 13.13.** Four-quadrant dc chopper circuit, showing first quadrant \( i_1 \) and \( v_o \) references.

The second control method is to unify the operation of all four switches within a generalised control algorithm. With both control methods, the chopper output voltage can be either multilevel or bipolar, depending on whether zero output voltage loops are employed or not. Bipolar output states increase the ripple current magnitude, but do facilitate faster current reversal, without crossover distortion. Operation is independent of the direction of the output current \( i \).

Since the output voltage is reversible for each control method, a triangular based modulation control method, as used with the asymmetrical H-bridge dc chopper in figure 13.9, is applicable in each case. Two generalised unified H-bridge control approaches are considered – bipolar and three-level output.

### 13.6.1 Unified four-quadrant dc chopper - bipolar voltage output switching

The simpler output to generate is bipolar output voltages, which use one reference carrier triangle as shown in figure 13.14 parts (c) and (d). The output voltage switches between \( +V \) and \( -V \), and the relative duration of each state depends on the magnitude of the modulation index \( \delta \).

If \( \delta = 0 \) then \( T_1 \) and \( T_4 \) never turn-on since \( T_2 \) and \( T_3 \) conduct continuously which impresses \( -V \) across the load.

At the other extreme, if \( \delta = 1 \) then \( T_1 \) and \( T_4 \) are on continuously and \( V \) is impressed across the load. If \( \delta \neq 0 \) then \( T_1 \) and \( T_4 \) are turned on for half of the period \( T \), while \( T_2 \) and \( T_3 \) are on for the remaining half of the period. The output voltage is \( V \) for half of the time and \( -V \) for the remaining half of any period. The average output voltage is therefore zero, but disadvantageously, the output current needlessly ripples about zero (with an average value of zero).

The chopper output voltage is defined in terms of the triangle voltage reference level \( v_o \) by

- \( v_o > \delta, \quad v_o = -V \)
- \( v_o < \delta, \quad v_o = +V \)

From figure 13.14c and d, the average output voltage varies linearly with \( \delta \) such that

\[
T = \frac{1}{T} \left[ \int_{v_o}^{v_o + V} \delta \, dt + \int_{v_o - V}^{v_o} \delta \, dt \right]
\]

(13.105)

Examination of figures 13.14c and d reveals that the relationship between \( t_o \) and \( \delta \) must produce

- \( \delta = 0: \quad t_o = 0 \quad \text{and} \quad v_o = -V \)
- \( \delta = \frac{1}{2}: \quad t_o = \frac{1}{2} T \quad \text{and} \quad v_o = 0 \)
- \( \delta = 1: \quad t_o = T \quad \text{and} \quad v_o = +V \)

that is

\[
\delta = \frac{t_o}{T}
\]

which on substituting for \( t_o/T \) in equation (13.105) gives

\[
T = \frac{2(1-\delta)}{T} V_o
\]

(13.106)

The average output voltage can be positive or negative, depending solely on \( \delta \). No current discontinuity occurs since the output voltage is never actually zero. Even when the average voltage is zero, ripple current flows through the load, with an average value of zero amps. The rms output voltage is independent of the duty cycle and is \( V_o \).

The output ac ripple voltage is

\[
V_o = \sqrt{V_o^2 - V_o^2}
\]

(13.107)

The ac ripple voltage is zero at \( \delta = 0 \) and \( \delta = 1 \), when the output voltage is pure dc, namely \( V_o \) or \( -V_o \) respectively. The maximum ripple voltage occurs at \( \delta = \frac{1}{2} \), when \( V_o = V_o \).

The output ripple factor is

\[
RF = \frac{V_o}{V_o} \sqrt{\frac{1}{4} (1-\delta)}
\]

(13.108)

\[
RF = \frac{2V_o}{2V_o} \sqrt{\frac{1}{4} (1-\delta)}
\]

Circuit operation is characterized by two time domain equations:

During the on-period for \( T_1 \) and \( T_4 \), when \( v_o(t) = V_o \)

\[
L \frac{dv}{dt} + R_i + E = V_o
\]

which yields

\[
t = \frac{V_o - E}{R} \left[ 1 - e^{-\frac{R}{L}} \right] + t_e \]

for \( 0 \leq t \leq T \).
During the on-period for T2 and T3, when \( v_0(t) = -V_e \),
\[
\frac{d}{dt}i_1(t) + R_i i_1 + E = -V_e
\]
which, after shifting the zero time reference to \( t_s \), gives
\[
i_1(t) = \frac{V_e}{R} \left[ 1 - e^{-t_L} \right] - \frac{E}{R} \quad \text{for } 0 \leq t \leq t_s
\]
(13.110)

The initial conditions \( i_1 \) and \( i_2 \) are determined by using the steady-state boundary conditions:
\[
\begin{align*}
& i_1 = \frac{V_e}{R} \left( 1 - e^{-t_L} \right) - \frac{E}{R} \\
& i_2 = \frac{V_e}{R} \left( 1 - e^{-t_L} \right) - \frac{E}{R}
\end{align*}
\]
(13.111)

The peak-to-peak ripple current is independent of load emf, \( E \), and twice that given by equation (13.15). The mean output current is given by
\[
\bar{i} = \frac{V_e}{R} \left( 1 - e^{-t_L} \right) - \frac{E}{R}
\]
which can be positive or negative, as seen in figure 13.14c and d.

Figures 13.14c and d show chopper output voltage and current waveforms for conditions of positive average voltage and current in part (c) and negative average voltage and current in part (d). Each part is shown with the current having a positive maximum value and a negative minimum value. Such a load current condition involves activation of all possible chopper conducting paths (sequences) as shown at the top of each part in figure 13.14 and transposed to table 13.3a. The table shows how the conducting device possibilities (states) decrease if the minimum value is positive or the maximum value is negative.

<table>
<thead>
<tr>
<th>Conducting devices sequences</th>
<th>( P &gt; 0 )</th>
<th>( P &lt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i &gt; 0 )</td>
<td>( i &gt; 0 )</td>
<td>( i &gt; 0 )</td>
</tr>
<tr>
<td>( i &gt; 0 )</td>
<td>( i &gt; 0 )</td>
<td>( i &gt; 0 )</td>
</tr>
<tr>
<td>( i &lt; 0 )</td>
<td>( i &lt; 0 )</td>
<td>( i &lt; 0 )</td>
</tr>
<tr>
<td>( i &lt; 0 )</td>
<td>( i &lt; 0 )</td>
<td>( i &lt; 0 )</td>
</tr>
</tbody>
</table>

If the minimum output current is positive, that is, \( i \) is positive, then only components for a first and fourth quadrant chopper conduct. Specifically \( T_3, T_2, D_2, \) and \( D_3 \) do not conduct. Examination of figure 12.14c shows that the output current conduction states are as shown in table 13.3a for \( i > 0 \).

If the output current never goes positive, that is \( i \) is negative, then \( T_1, T_4, D_1, \) and \( D_4 \) do not conduct. The conducting sequence becomes as shown in table 13.3a for \( i < 0 \). Because the output is bipolar (\( \pm V_e \)), the average chopper output voltage, \( \bar{v} \), does not affect the three possible steady-state sequences. Table 13.3a shows that the conducting devices are independent of the average output voltage polarity. That is, the switching states are the same on the left and right sides of table 13.3a. The transition between these three possible sequences, due to a current level polarity change, is seamless. The only restriction is that both switches in any leg do not conduct simultaneously. This is ensured by inserting a brief dead-time between a switch turning off and its leg complement being turned on. That is, dead-time between the switching of the complementary pair (\( T_1 - T_2 \)), and in the other leg the complementary pair is (\( T_3 - T_4 \)).

13.6.2 Unified four-quadrant dc chopper - multilevel voltage output switching

In order to generate three output states, specifically \( \pm V_e \) and \( 0 \), two triangular references are used which are displaced by 180° from one another as shown in figure 13.14a and b. One carrier triangle is
used to specify the state of the leg formed by \( T_1 \) and \( T_2 \) (the complement of \( T_1 \)), while the other carrier triangle specifies the state of the leg formed by switches \( T_3 \) and \( T_4 \). The output voltage levels switches between \( +V_s \), \( 0 \), and \( -V_s \) depending on the modulation index \( \delta \), such that \( 0 \leq \delta \leq 1 \). A characteristic of the output voltage is that, depending on \( \delta \), only a maximum of two of the three states appear in the output, in steady-state. An alternative method to generate the same switching waveforms, is to us one triangular carrier and two references, \( \delta \) and 1-\( \delta \).

If \( \delta = 0 \) then \( T_1 \) and \( T_4 \) never turn-on since \( T_1 \) and \( T_4 \) conduct continuously which impresses \( -V_s \) across the load. As \( \delta \) increases, the \( 0 \) state appears as well as the \( +V_s \) state, the later of which decreases in duration as \( \delta \) increases.

At \( \delta = \frac{1}{2} \) the output is zero since \( T_2 \) and \( T_3 \) (or \( T_1 \) and \( T_4 \)) are never on simultaneously to provide a path involving the dc source. The output voltage is formed by alternating \( 0 \) loops (\( T_1 \) and \( T_3 \) on, alternating to \( T_2 \) and \( T_4 \) on, etc.). The average output voltage is therefore zero. The ripple current for zero voltage output is therefore minimised and independent of any load emf.

At the extreme \( \delta = 1 \), \( T_1 \) and \( T_4 \) are on continuously and \( V_s \) is impressed across the load. As \( \delta \) is reduced from one, the \( 0 \) state is introduced, progressively lengthening to all of the period as \( \delta \) falls to \( \frac{1}{2} \).

The voltage output in terms of the triangular level \( V_s \) reference is defined by:

\[
\begin{align*}
\text{For } 0 \leq \delta < \frac{1}{2} & : \\
& v_a = \delta \cdot V_s, v_c = -V_s, \\
& v_b < \delta, v_d = 0
\end{align*}
\]

\[
\begin{align*}
\text{For } \delta = \frac{1}{2} & : \\
& v_a = \delta, v_b, v_c = 0 \\
& v_d < \delta, v_e = V_s
\end{align*}
\]

\[
\begin{align*}
\text{For } \frac{1}{2} < \delta \leq 1 & : \\
& v_a > \delta, v_b = -V_s \\
& v_c < \delta, v_d = 0
\end{align*}
\]

# From figure 13.14b for \( \delta < \frac{1}{2} \), the average output voltage varies linearly with \( \delta \) such that

\[
\bar{V}_r = \frac{1}{T} \left[ \int_0^T (0.5(t \cdot \delta) + (-V_s)) \, dt \right]
\]

\[
= \frac{1}{T} \left( \frac{t}{T} - V_s \right) = \frac{1}{2} \left( \frac{t}{T} - 1 \right) V_s
\]

Examination of figure 13.14b reveals that the relationship between \( t \) and \( \delta \) must produce

when \( \delta = 0 \): \( t_r = 0 \) and \( v_r = -V_s \),

when \( \delta = \frac{1}{2} \): \( t_r = T \) and \( v_r = 0 \)

that is

\[
\delta = \frac{1}{2} \frac{t_r}{T}
\]

which on substituting for \( t_r/T \) in equation (13.113) gives

\[
\bar{V}_r = \frac{1}{T} \left( \frac{t}{T} - 1 \right) V_s = \frac{1}{2} (2\delta - 1) V_s
\]

# From figure 13.14a for \( \delta > \frac{1}{2} \), the average output voltage varies linearly with \( \delta \) such that

\[
\bar{V}_r = \frac{1}{T} \left[ \int_0^T V_s \, dt + \int_0^{0.5T} 0 \, dt \right]
\]

\[
= \frac{V_s}{2}
\]

Examination of figure 13.14a reveals that the relationship between \( t \) and \( \delta \) must produce

when \( \delta = \frac{1}{2} \): \( t_r = 0 \) and \( v_r = 0 \),

when \( \delta = 1 \): \( t_r = T \) and \( v_r = V_s \),

that is

\[
\delta = \frac{1}{2} \frac{t_r}{T}
\]

which on substituting for \( t_r/T \) in equation (13.115) gives

\[
\bar{V}_r = \frac{V_s}{2} = \frac{1}{2} (2\delta - 1) V_s
\]
During the on-period for T1 and T4, when \( v_o(t) = V_o \)
\[
\frac{d}{dt} \left( \frac{L}{R} \right) + R I + E = V_o 
\]
which yields
\[
i_o(t) = \frac{E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I e^{\frac{t}{\tau}} \quad \text{for} \quad 0 \leq t \leq \tau, \quad \text{and} \quad \delta \geq \frac{1}{2} \tag{13.124}
\]
During the on-period for T2 and T3, when \( v_o(t) = -V_o \)
\[
\frac{d}{dt} \left( \frac{L}{R} \right) + R I + E = -V_o
\]
which, after shifting the zero time reference to \( \tau \), gives
\[
i_o(t) = \frac{E}{R} \left( 1 - e^{\frac{t}{\tau}} \right) + I e^{\frac{t}{\tau}} \quad \text{for} \quad 0 \leq t \leq \tau - \tau, \quad \text{and} \quad \delta \leq \frac{1}{2} \tag{13.125}
\]
The third equation is for a zero voltage loop.
During the switch off-period, when \( v_o(t) = 0 \)
\[
\frac{dL}{dt} + Ri + E = 0
\]
which, after shifting the zero time reference, in figure 13.14a or b, gives
\[
i_o(t) = \frac{E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I e^{-\frac{t}{\tau}} \quad 0 \leq t \leq \tau, \quad \text{and} \quad \delta \geq \frac{1}{2} \tag{13.126}
\]
The initial conditions \( i_o \) and \( i \) are determined by using the usual steady-state boundary condition method and are dependent on the transition states. For example, for continuous steady-state transitions between +V, loops and 0V loops, the boundary conditions are given by
\[
I = \frac{V}{R} \left( 1 - e^{-\frac{\tau}{\tau}} \right) - \frac{E}{R} \quad (A)
\]
and
\[
i = \frac{V}{R} e^{\frac{\tau}{\tau}} - \frac{E}{R} \quad (A)
\]
Figures 13.14a and b show output voltage and current waveforms for conditions of positive average voltage and current in part (a) and negative average voltage and current in part (b). Each part is shown with the current having a positive maximum value and a negative minimum value. Such a load current condition involves the activation of all possible chopper conducting paths, which are shown at the top of each part in figure 13.14 and transposed to table 13.3B. The conducting device possibilities decrease if the minimum value is positive or the maximum value is negative.

### Table 13.3B. A Four-quadrant chopper multilevel (three-level) output voltage states

<table>
<thead>
<tr>
<th>Conducting devices sequences</th>
<th>( P &gt; 0 )</th>
<th>( P &lt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1, T2, D1, D2, T3, T4</td>
<td>( P &gt; 0 )</td>
<td>( P &gt; 0 )</td>
</tr>
<tr>
<td>T1, T2, T3, T4</td>
<td>( P &gt; 0 )</td>
<td>( P &lt; 0 )</td>
</tr>
<tr>
<td>T1, T2, D1, D2, T3, T4</td>
<td>( P &lt; 0 )</td>
<td>( P &lt; 0 )</td>
</tr>
<tr>
<td>D1, D2, T1, T2, D3, D4</td>
<td>( P &gt; 0 )</td>
<td>( P &gt; 0 )</td>
</tr>
<tr>
<td>T1, D1, T2, T2, D3, D4</td>
<td>( P &lt; 0 )</td>
<td>( P &lt; 0 )</td>
</tr>
<tr>
<td>D1, D2, T1, T2, D3, D4</td>
<td>( P &lt; 0 )</td>
<td>( P &lt; 0 )</td>
</tr>
</tbody>
</table>

If the minimum output current is positive, that is, \( i_o \) is positive, then only components for a first and fourth quadrant chopper conduct. Specifically \( T_2, T_3, D_2, \) and \( D_3 \) do not conduct, thus do not appear in the output sequence. Examination of figure 12.14c shows that the output current conduction states are as shown in table 13.3B for \( i_o > 0 \).

If the output current never goes positive, that is \( i_o \) is negative, then \( T_1, T_4, D_1, \) and \( D_4 \) do not conduct, thus do not appear in the output device sequence. The conducting sequence is as shown in table 13.3B for \( i_o < 0 \).

Unlike the bipolar control method, the output sequence is affected by the average output voltage level, as well as the polarity of the output current swing. The transition between the six possible sequences due to load voltage and current polarity changes, is seamless. The only restriction is that both switching devices in any leg do not conduct simultaneously. This is ensured by inserting a brief dead-time between a switch turning off and its leg complement being turned on.

**Example 13.7: Four-quadrant dc chopper**

The H-bridge, dc-to-dc chopper in figure 13.13 feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 56V dc, from a 340V dc source. If the chopper is operated with a 200Hz multilevel carrier as in figure 13.14 a and b, with a modulation depth of \( \delta = \frac{1}{4} \), determine:
- \( i_o \), the average output current and switch \( T_1 \) on-time
- \( v_o \), the rms output voltage and ac ripple voltage, hence voltage ripple factor
- \( i_o \), the average output current, hence quadrant of operation
- \( v_o \), the electromagnetic power being extracted from the back emf \( E \).

If the mean load current is to be halved, which is
- \( v_o \), the modulation depth, \( \delta \), requirement
- \( v_o \), the average output voltage and the corresponding switch \( T_1 \) on-time
- \( v_o \), the electromagnetic power being extracted from the back emf \( E \).

**Solution**

The main circuit and operating parameters are:
- modulation depth \( \delta = \frac{1}{4} \)
- period \( T_{carrier} = \frac{1}{f_{carrier}} = \frac{1}{200} = 5 \text{ms} \)
- \( E = 55V \) and \( V_o = 340V \) dc
- load time constant \( \tau = L/R = 0.05 \text{ms} \)

\( i_o \) is determined by equation (13.114), and for \( \delta < \frac{1}{2} \)
\[
\bar{P}_o = \left( \frac{V}{R} \right)^2 - \left( \frac{2\delta - 1}{2} \right)^2 V_o E = 340^2 \left( \frac{2\delta - 1}{2} \right)^2 V_o E = -170V
\]
where
\[
\tau = \frac{2\delta T}{2\delta - 1} = 0.25 \text{ms}
\]

Figure 13.14 reveals that the carrier frequency is half the switching frequency, thus the 5ms in the above equation has been halved. The switches \( T_1, T_4 \) turn on for 1.25ms, while \( T_2, T_3 \) are subsequently turned on for 3.75ms.

\( v_o \) is determined, from equation (13.118), as
\[
V_o^2 = \frac{1}{2} \bar{P}_o = \frac{\bar{P}_o}{2} V_o E = 340^2 \left( \frac{2\delta - 1}{2} \right) = 240 \text{Vrms}
\]
From equation (13.119), the output ac ripple voltage, hence ripple voltage factor, are
\[
V_o = \sqrt{2} V_o \sqrt{\frac{1}{2 - 2\delta}} = 170 \text{V ac}
\]
\[
RF = \frac{V_o}{V_o} = \frac{170}{170} = 1
\]

\( i_o \) is determined, from equation (13.117)
\[
\tau = \frac{\bar{P}_o - E}{R} = \frac{\bar{P}_o - E}{R} = \frac{340 \left( \frac{2\delta - 1}{2} \right)}{55} = 22.5A
\]

Since both the average output current and voltage are negative (-170V and -22.5A) the chopper with a modulation depth of \( \delta = \frac{1}{4} \) is operating in the third quadrant.
iv. The electromagnetic power developed by the back emf $E$ is given by
\[ P_e = E I = 55V \times (-22.5A) = -1237.5W \]

v. The average output current is given by
\[ I_o = \frac{(2\delta - 1)V - E}{R} \]
when the mean current is -11.25A, $\delta = 0.415$, as derived in part vi.

vi. Then, if the average current is halved to -11.25A
\[ I_o = \frac{E + 7R}{R} \]
\[ = -55V - 11.25A \times 10\Omega = -57.5V \]
The average output voltage rearranged in terms of the modulation depth $\delta$ gives
\[ \delta = \frac{1}{2} \left( 1 + \frac{57.5V}{340V} \right) = 0.415 \]
The switch on-time when $\delta < 0.5$ is given by
\[ t_o = 2\delta = 2 \times 0.415 + 5ms = 2.07ms \]
From figure 13.14b both $T_1$ and $T_4$ are turned on for 2.07ms, although, from table 13.3B, for negative load current, $I_o = -11.25A$, the parallel connected freewheel diodes $D_2$ and $D_3$ conduct alternately, rather than the switches (assuming $j_o < 0$). The switches $T_1$ and $T_4$ are turned on for 1.25ms, while $T_2$ and $T_3$ are subsequently turned on for 2.93ms.

vii. The electromagnetic power developed by the back emf $E$ is halved and is given by
\[ P_e = E I_o = 55V \times (-11.25A) = -618.75W \]

Reading list

Dewan, S. B. and Straughen, A., Power Semiconductor Circuits,

Dubey, G.K., Power Semiconductor Controlled Drives,

Mohan, N., Undeland, T. M., & Robbins, W.P., Power Electronics: Converters, Applications & Design,

Problems

13.1. The dc GTO thyristor chopper shown in figure 13.1c operates at 1 kHz and supplies a series 5 $\Omega$ and 10 mH load from an 84 V dc battery source. Derive general expressions for the mean load voltage and current, and the load rms voltage at an on-time duty cycle of $\delta$. Evaluate these parameters for $\delta = 0.25$.

[21 V, 4.2 A; 42 V]

13.2. The dc chopper in figure 13.1c controls a load of $R = 10$ $\Omega$, $L = 10$ mH and 40 V battery. The supply is 340 V dc and the chopping frequency is 5 kHz. Calculate (a) the peak-to-peak load ripple current, (b) the average load current, (c) the rms load current, (d) the effective input resistance, and (e) the rms switch current.