CHAPTER 9  
Protecting Diodes, Transistors, and Thyristors

All power switching devices attain better switching performance if some form of switching aid circuit, called snubber, is employed. Snubber activation may be either passive or active which involves extra power switches. Only passive snubbers, which are based on passive electrical components, are considered in this chapter, while active snubbers are considered in Chapter 10. Fundamentally, the MOSFET and IGBT do not require switching aid circuits, but circuit imperfections, such as stray inductance and diode recovery, can necessitate the need for some form of switch snubber protection. Protection in the form of switching aid circuits performs three main functions:

- Divert switching losses from the switch thereby allowing a lower operating temperature, or higher electrical operating conditions for a given junction temperature.
- Prevent transient electrical stressing that may exceed I-V ratings thereby causing device failure.
- Reduce conducted and radiated electromagnetic interference

Every semiconductor switching device can benefit from switching protection circuits, but extra circuit component costs and physical constraints may dictate otherwise. The bipolar diode suffers from reverse recovery current and voltage snap which induces high but short duration circuit voltages. These voltage transients may cause interference to the associated circuit and to nearby equipment. A simple series non-polarised R-C circuit connected in parallel to the stressed or offending device is often used to help suppress the voltage oscillation at device turn-off. Such a suppression circuit can be effectively used on simple mains rectifying circuits when rectification causes any voltage oscillation by dissipating the oscillatory energy generated at turn-off when an over-voltage tends to occur.

Although the MOSFET and IGBT can usually be reliably and safely operated without external protection circuitry, stringent EMC application emission restrictions may dictate the use of snubbers. In specific applications, the IGBT is extensively current derated as its operating frequency increases. In order to attain better device current utilization, at higher frequencies, various forms of switching aid circuits can be used to divert switching losses from the stressed semiconductor switch.

Generally, all thyristor devices benefit from a polarised turn-on switching aid circuit, which is based on a series connected inductor that is active at thyristor turn-on. Such an inductive turn-on snubber is obligatory for the high-power GCT and GTO thyristor. In order to fully utilise the GTO thyristor, it is usually used in conjunction with a parallel-connected capacitive turn-off snubber, which decreases device stressing during the turn-off transient. Triacs and rectifier grade SCRs and diodes tend to use a simple R-C snubber connected in parallel to the switch to reduce interference. The design procedure of the R-C snubber for a diode is different to that for the R-C snubber design for a thyristor device, because the protection objectives and initial conditions are different. In the case of a thyristor or rectifier diode, the objective is to control both the voltage rise at turn-off and recovery overshoot effects. For the fast recovery diode or any high-speed switch, the principal objectives are to control the voltage overshoot magnitude at diode snap recovery or at turn-off respectively, which are both exacerbated because of stray circuit inductance carrying current.

9.1 The non-polarised R-C snubber

The series R-C snubber is the simplest switching aid circuit and is connected in parallel to the device being aided. It is characterized by having low series inductance and a high transient current rating. These requirements necessitate carbon type resistors for low inductance, below a few watts, and metal film resistors at higher powers. The high current and low inductance requirements are also provided by using metalised, polypropylene capacitors with high dv/dt ratings of typically hundreds of V/μs. Theoretically a purely capacitive snubber would achieve the required protection objectives, but series resistance is added to decrease the current magnitude when the capacitor is discharging and to damp any voltage oscillation by dissipating the oscillatory energy generated at turn-off when an over-voltage tends to occur.

In figure 9.1a, at switch turn-off, stray inductance \( L_s \) unclamped by the load freewheel diode, \( D_o \), produces an over voltage \( V \) on the MOSFET or IGBT. The energy associated with the inductor can be absorbed in the shown drain to source connected R-C circuit, thereby containing the voltage overshoot to a controlled safe level. Such an R-C snubber circuit is used extensively in thyristor circuits, 9.1.2, for dv/dt protection, but in such cases the initial current in the stray inductance is assumed zero. Here the initial inductor current is equal to the maximum load current magnitude, \( I_L \). The design curves in figure 9.1b allow selection of \( R \) and \( C \) values based on the maximum voltage overshoot \( V \) and an initial current factor \( \chi \)

\[
C = \frac{L_s}{(V/V_L)^2} \quad \text{(F)} \\
R = 2 \chi V_L I_L \quad \text{(Ω)}
\]

If the R-C circuit time constant, \( t = RC \), is significantly less than the MOSFET voltage rise and fall times, \( t_r \) and \( t_f \), at reset (when the capacitor is discharged through the resistor and switch at turn-on), a portion of the capacitor energy \( 1/2 CV^2 \) is dissipated in the switch, as well as in \( R \). The switch appears as a variable resistance in series with the R-C snubber. Under these conditions (\( t_r \) and \( t_f \) > RC) the resistor loss is approximately

\[
P_R = \frac{1}{2} V^2 t \quad \text{and} \quad P_R = \frac{1}{2} L_s I_f^2
\]

where

\[
P_{R_{\text{snubber}}} = \frac{t}{t + t_{\text{on}}} (P_{\text{on}} + P_{\text{off}})
\]

otherwise (\( t_r \) and \( t_f < RC \)) the resistor losses are the energy to charge and discharge the snubber capacitor, plus the energy stored in the stray inductance, that is \( 2P_{\text{on}} + P_{\text{off}} \).

The total losses are independent of snubber resistance. The snubber resistor determines the time over which the energy is dissipated, not the amount of energy dissipated. When the R-C snubber is employed across a fast recovery diode, the peak reverse recovery current is used for \( I_L \) in the design procedure.
Example 9.1: R-C snubber design for MOSFETs

A MOSFET switches a 40 A inductive load on a 200 V dc rail, at 10 kHz. The unclamped drain circuit inductance is 20 nH and the MOSFET voltage rise and fall times are both 100 ns. Design a suitable R-C snubber if the MOSFET voltage overshoot is to be restricted to 240 V (that is, 40V overshoot, viz. 20%).

Solution

From figure 9.1b, for 20 per cent voltage overshoot

\[ \zeta = 1.02, \quad \chi = 0.52 \]

Using equations (9.1) and (9.2) for evaluating \( C \) and \( R \) respectively,

\[ C = L_2 \left( \frac{V_o}{\chi I_p} \right)^2 = 20 \text{nH} \left( \frac{40 \text{A}}{0.52 \times 200 \text{V}} \right)^2 = 3 \text{nF} \]

\[ R = 2 \, \zeta \frac{V_o}{\chi I_p} \frac{1}{I_p} = 2 \times 1.02 \frac{0.52 \times 200 \text{V}}{40 \text{A}} = 5.3 \Omega \]

Use \( C = 3.3 \text{ nF} \), 450 V dc, metallised polypropylene capacitor and \( R = 5.6 \Omega \).

Since the RC time constant, 18.5 ns, is short compared with the MOSFET voltage transient times, 100 ns, the resistor power rating is given by equation (9.3).

\[ P_s = \frac{1}{2} \frac{C_s}{V_s^2} \times 200 \text{V} \times 10 \text{kHz} = 2.64 \text{W} \]

\[ P_s \approx \frac{1}{2} \frac{C_s}{V_s^2} \times 200 \text{V} \times 10 \text{kHz} = 0.16 \text{W} \]

\[ P_s = 18.5 \text{ns} \times 2.64 \text{W} + 18.5 \text{ns} \times 0.16 \text{W} = 0.85 \text{W} \]

Use a 5.6 Ω, 1 W carbon composition resistor for low self-inductance, with a working voltage of at least 250 V dc. Parallel connection of two 120 V/1 W, carbon composition resistors may be necessary since resistance values below 10 Ω are uncommon. The MOSFET switching losses are \( 2 P_s + P_s = 0.85 \text{W} = 4.95 \text{W} \), higher than those incurred by switching un-aided at 200 V and 40 A. From equations 7.9 and 7.10, the switching losses would be at least 8 W, (4W+4W).

9.1.2 Non-polarised R-C snubber circuit for a converter grade thyristor and a triac

The snubber circuit for a low switching frequency thyristor is an anode-to-cathode parallel connected R-C series circuit for off-state voltage transient suppression. Thyristor series inductance may be necessary (forming a turn-on snubber) to control anode dv/dt at turn-on. This inductive snubber is essential for the GCT and the GTO thyristor, and will be considered in section 9.3.3.

Off-state dv/dt suppression snubber

Thyristors, other than the GCT and the GTO, normally employ a simple R-C snubber circuit as shown in figure 9.2. The purpose of the R-C snubber circuit is not primarily to reduce turn-off switching loss but rather to prevent false triggering (turn on) from applied or reapplied anode dv/dt when the switch is in a forward voltage blocking off-state. Any thyristor rate of rise of forward-voltage anode dv/dt produces a central junction charging current which may cause the thyristor to inadvertently turn on. The critical dv/dt is defined as the minimum value of dv/dt which will cause switching from the off-state to the on-state. In applications as shown in figure 9.2, an occasional false turn-on is generally not harmful to the triac or the load, since the device and the load only have to survive the surge associated with a half-cycle of the ac mains voltage supply. In other applications, such as reversible converters, a false dv/dt turn-on may prove catastrophic.

A correctly designed snubber circuit is therefore essential to control the rate of rise of anode voltage. The action of this R-C snubber circuit relies on the presence of inductance in the main current path. The inductance may be stray, from transformer leakage or a supply, or deliberately introduced. Zero inductor current is the initial condition, since the device is in the off-state when experiencing the anode positive dv/dt. Analysis is based on the response of the R-C portion of an L-C-R circuit with a step input voltage and zero initial inductor current. Figure 9.3 shows an L-C-R circuit with a step input voltage and the typical resultant voltage across the SCR or R-C components. The circuit resistor \( R \) damps (by dissipating power) any oscillation and limits the capacitor discharge current through the SCR at subsequent SCR device turn-on initiated from the gate. The snubber resistor dissipates power even if the triac is not switching, since the snubber capacitor voltage alternates, tracking the ac voltage supply.

\[ \text{Figure 9.2. Thyristor (triac) ac circuit with an R-C snubber circuit.} \]

\[ \text{Figure 9.3. Non-polarised R-C snubber equivalent circuit showing the second-order output response } e_o \text{ to a step input voltage } e_i. \]

\[ \text{Figure 9.4. Variation of snubber peak voltage, } e_p, \text{ maximum } \frac{de_o}{dt}, S; \text{ and peak current, } I_p; \text{ with } L-C-R \text{ damping factor } \xi. \]
Based on the snubber circuit analysis presented in the appendix in section 9.5 at the end of this chapter, the maximum dv/dt, \( S \), which is usually specified for a given device, seen by the SCR for a step input of magnitude \( e_0 \), is given by

\[
\hat{S} = e_R / I \tag{9.4}
\]

for a damping factor of \( \xi = 0.5 \). That is, after rearranging, the snubber resistance is given by

\[
R = L S / e_0 \quad \text{(ohms)}
\]  

(9.5)

while the snubber capacitance is given by

\[
C = 4 \mu F / \xi \hat{S} \quad \text{(F)}
\]  

(9.6)

and the peak snubber current is approximated by

\[
I = \frac{2}{\sqrt{2}} \frac{2 I}{\hat{S} R} \quad (A) \text{ for } \xi < 1.
\]  

(9.7)

Figure 9.4 shows the variation of the different normalised design factors, with damping factor \( \xi \).

Example 9.2: Non-polarised R-C snubber design for a converter grade thyristor

Design an R-C snubber for the SCRs in a circuit where the SCRs experience an induced \( \text{dv/dt} \) due to a complementary SCR turn-on, given

- peak switching voltage, \( e_0 = 200 \text{ V} \)
- operating frequency, \( f_s = 1 \text{ kHz} \)
- \( \text{dv/dt} \) limit, \( S = 200 \text{ V/\mu s} \).

Assume

- stray circuit \( L = 10 \mu \text{H} \)
- 22 per cent voltage overshoot across the SCR
- an \( L-C-R \) snubber is appropriate.

Solution

From equation (9.5) the snubber resistance is given by

\[
R = \frac{L S}{e_0} = \frac{10 \mu \text{H} \times 200 \text{V/\mu s}}{200 \text{V}} = 10 \Omega
\]

At turn-on the additional anode current from the snubber capacitor will be 200V/10Ω = 20A, which decays exponentially to zero, with a 1.8\( \mu \text{s} \) (10Ω×180nF) RC time constant.

Figure 9.4 shows the \( R-C \) snubber circuit overshoot voltage magnitude, \( e_s/e_0 \) for a range of damping factors \( \xi \). The normal range of damping factors is between \( \frac{1}{2} \) and 1. Thus from figure 9.4, allowing 22 per cent overshoot, implies \( \xi = 0.65 \). From equation (9.6)

\[
C = \frac{4 \mu F}{\xi \hat{S}} = \frac{4 \times (0.65)^2}{200 \text{V}} = 180 \text{nF} \quad \text{(preferred value)}
\]

(9.6)

which is rated at \( \geq 244 \text{V peak} \).

From equation (9.7) the peak snubber current during the applied \( \text{dv/dt} \) is

\[
I = \frac{e_0}{R} \frac{2}{\sqrt{2}} \frac{2}{\xi} = \frac{200 \text{V}}{10 \Omega} \times \frac{10}{\sqrt{2}} = 34 \text{A}
\]

(9.7)

The 10 ohm snubber resistor losses are given by

\[
P_{\text{loss}} = C I_s^2 / \xi = 180 \times 10^6 \times 244 \times 1 \times 10^3 = 11 \text{W}
\]

Resistor current flows to both charge (maximum 34A) and discharge (initially 20A) the capacitor.

The necessary 10Ω, 11W resistor must have low inductance, hence two 22 Ω, 7W, 500V dc working voltage, metal oxide film resistors can be parallel connected to achieve the necessary ratings.

Variations of the basic R-C snubber circuit are shown in figure 9.5. These circuits use extra components in an attempt to control SCR initial \( \text{dv/dt} \) arising from snubber discharge through \( R \) at thyristor turn-on. Figure 9.5a has the disadvantage that three series devices (C-R-D) provide turn-off protection. The parasitic series inductance can be decreased by using a turn-off snubber with two series components (C-D), as shown in figure 9.5b.

An R-C snubber can be used across a diode in order to control voltage overshoot at diode snap-off during reverse recovery, as a result of stray circuit inductance, as considered in 9.1.1. The R-C snubber can provide decoupling and transient overvoltage protection on both ac and dc supply rails, although other forms of R-C snubber circuit may be more applicable, specifically the soft voltage clamp.

9.2 The soft voltage clamp

A primary function of the basic R-C snubber is to suppress voltage overshoot levels. The R-C snubber commences its clamping action from zero volts even though the objective is to clamp any switch overvoltage to the supply voltage level, \( V_s \). Any clamping action below \( V_s \) involves an unnecessary transfer of energy. The soft voltage clamp reduces energy involvement since it commences clamping action once the switch voltage has reached the supply voltage \( V_s \), and the voltage overshoot commences.

The basic polarised R-C-D soft voltage clamp is shown in figure 9.6a, with resistor \( R \) and parasitic inductance, \( L_m \), and stray or deliberately introduced unclamped inductance \( L_{\text{in}} \) shown.

The voltage clamp functions at switch turn-off once the switch voltage exceeds \( V_s \). The capacitor voltage never falls below the supply rail voltage \( V_s \). Due to the stored energy in \( L \), the capacitor \( C \) charges above the rail voltage and \( R \) limits current magnitudes as the excess capacitor charge discharges through \( R \) to \( V_s \). All the energy stored in \( L \times L_{\text{in}} \) is dissipated in \( R \). The inductor current \( i_l \) and capacitor voltage \( V_c \), waveforms are shown in figure 9.6b.

At switch turn-on, the diode \( D \) blocks, preventing discharge of \( C \) which remains charged to \( V_s \). The energy drawn from the supply \( V_s \) as the capacitor overcharges, is returned to the supply as the capacitor discharges through \( R \) into the supply. The net effect is that only the energy in \( L \times L_{\text{in}} \) is dissipated in \( R \).

Analysis is simplified if the resistor inductance \( L_{\text{in}} \) is assumed zero. The inductor current decreases from \( I_{l0} \) to 0 according to

\[
I_l(t) = I_{l0} e^{-\alpha t} \cos(\omega t - \varphi) \tag{A}
\]

(9.8)

where

\[
\alpha = \sqrt{\frac{R}{L}} \quad \text{(rad/s)} \quad \omega = 1 / \sqrt{L C} \quad \text{(rad/s)} \quad \varphi = \tan^{-1} \frac{\omega L_{\text{in}}}{R} \quad \text{(rad)}
\]

(9.9)
The inductor current reaches zero, termed the current reset time, \( t_r \), in time
\[
  t_r = \frac{V_0 e^{-\alpha t}}{\alpha} \quad (9.9)
\]
which must be shorter than the switch minimum off-time, \( t_{\text{off}} \). The capacitor charges from \( V_s \) according to
\[
  V_C(t) = V_s + \frac{I_C}{C_s} e^{-\alpha t} \sin \omega t \quad (9.10)
\]
The maximum capacitor voltage, hence maximum switch voltage, occurs for large \( R \)
\[
  V_C' = V_s + \frac{L_s}{C} \quad (9.11)
\]
Once the current in \( L \) has reduced to zero the capacitor discharges to \( V_s \) exponentially, with a time constant \( RC \).

The practical \( R-C \) circuit, which includes the stray inductance \( L_s \), must be over-damped, that is
\[
  R > 2 \frac{E_s}{C} \quad (9.12)
\]
The capacitor reset time \( t_r \) is the time for the capacitor to discharge to within 5 per cent of \( V_s \), as shown in figure 9.6b.

The stray inductance \( L_s \) increases the peak capacitor voltage and increases the voltage reset time. Design of the voltage clamp, including the effects of \( L_s \), is possible with the aid of figure 9.7. Design is based on specifying the maximum voltage overshoot, \( V_0 \), and minimizing the voltage reset time, \( t_r \), which limits the upper switching frequency, \( f_s \), where \( f_s \leq 1/t_r \), such that \( t_{\text{off}} \geq t_r \).

**Example 9.3:**  **Soft voltage clamp design**

A 5 \( \mu \)H turn-on snubber is used to control diode reverse recovery current and switch turn-on loss, as shown in figure 9.6a. The maximum collector current is 25 A, while the switch minimum off-time is 5 \( \mu \)s and the maximum operating frequency is 50 kHz.

i. Assuming an independent \( L-C \) resonant transfer from \( L \) to \( C \) and a subsequent \( R-C \) discharge cycle, calculate soft voltage clamp \( R \) and \( C \) requirements.

ii. Use figure 9.7 to determine the voltage clamp requirements if the discharge (reset) resistor inductance \( L_s \) is
   (a) 0
   (b) 1.0 \( \mu \)H.

In each case, the maximum switch overshoot is to be restricted to 50 V.
The reset times are given by

\[ t_r = t_c \frac{V_c}{V_g} = 4.34 \times 1.16 = 5 \mu s \quad (<20 \mu s) \]

\[ t_r = t_c \frac{V_c}{V_g} = 2.9 \times 1.16 = 3.4 \mu s \quad (<5 \mu s) \]

It is seen that smaller capacitance (0.27 \mu F versus 1.2 \mu F) can be employed if simultaneous L-C transfer and R-C discharge are accounted for. The stray inductance of the resistor discharge path has been neglected. Any inductance decreases the effectiveness of the R-C discharge. Larger C than 0.27 \mu F and \( R < 3.3 \Omega \) are needed, as is now shown.

ii. \( t_r = 1 \mu H \), that is \( L' = V_c / V_g = 0.2 \)

In figure 9.7, for a minimum voltage reset time, \( \xi = 0.7 \), \( V_c' = 0.54 \) when the \( L' = 0.2 \) curve is used. The normalised reset times are unchanged, that is \( t_r' = 2.9 \) and \( t_r = 4.34 \).

Using the same procedure as in part ii, \( m \)

\[ 0.54 = \frac{50V}{25A} \frac{1}{\sqrt{2}} \frac{5 \mu H}{C} \text{ gives } C = 0.37 \mu F \text{ (use 0.39 \mu F)} \]

\[ R = \frac{1}{2L' C} = \frac{1}{2 \times 0.7 \times 0.05 \Omega} = 2.6 \Omega \text{ (use 2.7 \Omega, 78W)} \]

Since resistor inductance has been accounted for, parallel connection of four 10GΩ, 25W wire-wound aluminium clad resistors can be used.

\[ t_r = 4.34 \times 1.4 = 6 \mu s \quad (<20 \mu s) \]

\[ t_r = 2.90 \times 1.4 = 4 \mu s \quad (<5 \mu s) \]

Note that circuit supply voltage \( V_s \) is not a necessary design parameter, other than to specify the capacitor absolute dc voltage rating. This supply independence is expected since in ac circuit analysis, as is applicable here during the transient snubber operational period, dc voltage sources are shorted.

9.3 Polarised switching-aid circuits

Optimal gate drive electrical conditions minimize collector (or drain or anode) switching times, thus minimizing switching electrical stresses and power losses. Proper gate drive techniques greatly enhance the switching robustness and reliability of a power switching device. Switching-aid circuits, commonly called snubber circuits, can be employed to further reduce device switching stresses and losses. Optimal gate drive conditions minimise the amount of snubbing needed.

During both the switch-on and the switch-off transition intervals, for an inductive load as considered in chapter 7.2, an instant exists when the switch simultaneously supports the supply voltage \( V_s \) and conducts the full load current \( I_{SL} \) as shown in figure 9.8. The gate drive conditions cannot alter this peak power loss but can vary the duration of the switching periods (\( t_{on} \) and \( t_{off} \)). From chapter 7, for an inductive load, the switching losses, \( W \), dissipated as heat in the switch, are given by

\[ W_{on} = \frac{1}{2} V_s I_{on} I_{on} \quad (J) \]

\[ W_{off} = \frac{1}{2} V_s I_{off} I_{off} \quad (J) \]

In order to reduce switching losses, two snubber circuits can be employed on a power switching device, one operational during switch turn-on, the other effective during turn-off. In the case of the turn-off snubber, energy (current) is diverted from the switch turning off into a parallel capacitor as shown in figure 9.9a thus the capacitor controls the voltage rise. The turn-on snubber utilises an inductor in series with the collector as shown in figure 9.9b in order to support part of the dc voltage supply as the collector (anode) voltage falls. The inductor therefore controls the rate of rise of collector (anode) current during the collector voltage fall time. For both snubbers, the I-V SOA trajectory is modified to be within that area shown in figure 7.8.

9.3.1 The polarised turn-off snubber circuit - assuming a linear current fall

Figure 9.10 shows a complete turn-off snubber circuit comprising a capacitor-diode plus resistor combination across the anode-to-cathode/collector-to-emitter terminals of the switching device. At switch turn-off, load current is diverted into the snubber capacitor \( C \) via the diode \( D \), while the switch principal current decreases. The anode/collar voltage is clamped to the capacitor voltage, which is initially zero. The larger the capacitor, the slower the anode/collar voltage rises for a given load current and, most importantly, turn-off occurs without a condition of simultaneous supply voltage and maximum load current (\( V_s, I_{SL} \)). Figure 9.11 shows the anode/collar turn-off waveforms for different magnitudes of snubber capacitance. The GTO/IGBT tall current has been neglected, thus the switching device is analysed without any tail current. For clarity, the terminology to henceforth used, refers to an IGBT, viz., collector, emitter, and gate. Circuit operational explanations apply to thyristors.

Figure 9.11a shows turn-off waveforms for a switch without a snubber, where it has been assumed that the collector voltage rise time is short compared with the case of the collector current fall time, which is given by \( t_c = 1 \cdot t_r \). For low capacitance values, the snubber capacitor (whence collector voltage) may charge to the rail voltage before the collector current has fallen to zero, as seen in figure 9.11b. For larger capacitance, the collector current reaches zero before the capacitor (whence collector voltage) has charged to the rail voltage level, as shown in figure 9.11c.
For analysis, the collector voltage rise time for an unaided switch is assumed zero. The device switch-off energy losses without a snubber, as shown in figure 9.11a, are given by

\[ W = \frac{1}{2} V_s I_m t_{fi} \quad (9.15) \]

With a snubber circuit, switch losses are decreased as shown in figure 9.11d, but snubber (resistor) losses are incurred. After turn-off the capacitor is charged to the rail voltage. This stored energy, \( \int C V_o^2 \), is subsequently dissipated as heat in the snubber circuit resistor at subsequent switch turn-on, when an R-C discharge current flows. If the snubber RC time constant is significantly shorter than the switch voltage fall time at turn-on, the capacitor energy dissipated in the resistor is less than \( \int C V_o^2 \) and switch losses are increased as considered in 9.1.1. A range of capacitance values exists where the total losses - snubber plus switch - are less than those losses incurred if the same device is switched unaided, when losses as given by equation (9.14) result. Two distinct snubber design cases exist, depending on capacitance magnitude, as indicated by figures 9.11b and 9.11c. The two possibilities and the associated circuit voltage and current waveforms in each case are shown in detail in figure 9.12. The waveforms are based on satisfying Kirchhoff’s voltage and current laws for each case.

Figure 9.10. Practical capacitive turn-off snubber showing capacitor charging and discharging paths during device switching.

Figure 9.11. Switch turn-off waveforms:
(a) unaided turn-off; (b) turn off with small snubber capacitance; (c) turn-off with large snubber capacitance; (d) and switch power losses.

Figure 9.12. Switch turn-off waveforms satisfying Kirchhoff’s laws:
(a) turn-off with small snubber capacitance and (b) turn-off with large snubber capacitance.
Two factors specify the snubber discharge circuit resistance value.

- The snubber circuit RC time constant period must ensure that after turn-on the capacitor discharges before the next switch turn-off is initiated. If $t_{m}$ is the minimum switch on-time, then $I_{m}$ = $5R_{C}I_{m}$ is sufficient to ensure the correct snubber circuit initial conditions, namely, zero capacitor voltage.

- The resistor initial current at capacitor discharge is $V_{o}/R_{o}$. This component is added to the load current at switch turn-on, hence adding to the turn-on stresses. The maximum collector current rating must not be exceeded. In order to reduce the initial discharge current, a low value inductor can be added in series with the resistor, (or a wire-wound resistor used), thus producing an overdamped L-C-R discharge current oscillation at turn-on. Note that the resistor power loss in equation (9.19) is independent of resistance value. The resistance determines the period of time over which the capacitor stored energy is dissipated at switch turn-on.

![Figure 9.13. Loss components for a switch at turn-off when employing a capacitance-type snubber and assuming the collector current falls according to $i_{c} = i_{m}(3 - t/t_{o})$.](image)

![Figure 9.14. The collector I-V trajectory at turn-off with a capacitive switching-aid circuit.](image)
9.3.2 The turn-off snubber circuit - assuming a cosinusoidal current fall

As an alternative to a linear current fall at turn-off, it may be more realistic to assume that the current falls cosinusoidally according to

\[ i(t) = \frac{1}{2} I_{s} (1 + \cos(\pi t / T)) \]  

(A)

for 0 ≤ t ≤ T, as shown in figure 9.15.

As with a linear current fall, two cases exist.

(i) \( t \leq T / k \leq 1 \), that is the snubber capacitor charges to \( V_{c} \) in time \( t \), before the switch current reaches zero, at time \( T \).

(ii) \( t \geq T / k \geq 1 \), that is the snubber capacitor charges to the supply \( V_{s} \) after the switch current has fallen to zero.

These two cases are shown in figure 9.15 where \( k \) is defined as \( r / T \). Using a similar analysis as presented in the appendix (section 9.6), expressions can be derived for switch and snubber resistor losses. These and the total losses for each case are summarised in table 9.1.

Figure 9.15 shows that a minimum total loss occurs, namely

\[ W_{m} = 0.41 \times 10^{3} I_{s} f_{s} \]  

at \( k = 0.62 \)

when

\[ C_{s} = 0.16 \frac{I_{s}}{V_{s}} \]  

(F)

For \( t_{s} < 0.85T \), a cosinusoidal fall current predicts lower total losses than a linear fall current, with losses shown in figure 9.13.

Table 9.1: Normalised switching loss components at turn-off with a cosinusoidal current fall of half period \( T \)

<table>
<thead>
<tr>
<th>Component</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaided switch loss</td>
<td>( P_{sw} = \frac{V_{s} I_{s}}{100} )</td>
</tr>
<tr>
<td>Total loss</td>
<td>( P_{sw} + W_{c} + W_{r} )</td>
</tr>
<tr>
<td>Capacitor loss, ( W_{c} )</td>
<td>( \frac{k}{2} + \frac{2}{\pi^2} (\cos(k) - 1) + \frac{k}{\pi} \sin(k) - \frac{\sin^2(k)}{2\pi^2} )</td>
</tr>
<tr>
<td>Resistor loss, ( W_{r} )</td>
<td>( \frac{k - 1}{2} + \frac{1}{4} \frac{2k - 1}{\pi^2} )</td>
</tr>
</tbody>
</table>

Example 9.4: Capacitive turn-off snubber design

A 600V, 100A machine field winding is switched at 10kHz. In maintaining a constant field current, the switch operates with an on-state duty cycle ranging between 5% and 95% (5% ≤ δ ≤ 95%) and has a turn-off linear current fall time of 100ns, that is, \( T = 100 \times (1 - t / 100\text{ns}) \).

(i) Estimate the turn-off loss in the switch.

(ii) Design a capacitive turn-off snubber using the dimensionally correct identity \( i = C dv/dt \).

What is the capacitor voltage when the collector current reaches zero.

(iii) Design a capacitive turn-off snubber such that the switch voltage reaches 600V at the same time the conducting current reaches zero.

In each snubber case calculate the percentage decrease in un-aided switch turn-off power dissipation.

Solution

(i) The switch un-aided turn-off losses are given by equation (9.14). The turn-off time is greater than the current fall time (since the voltage rise time is neglected), thus the turn-off switching losses will be greater than

\[ P_{sw} = \frac{V_{s} I_{s}}{100} \times 100\text{ns} = 3\text{mJ} \]

Therefore, the turn-off loss in the switch can be calculated as

\[ P_{sw} = \frac{600V \times 100A \times 100\text{ns}}{100} = 3\text{mJ} \]

(ii) Use the equation \( i = C dv/dt \) results in a switch voltage that reaches the rail voltage after the collector current has fallen to zero. From \( k = \tanh C V / I f \) in figure 9.13, \( k = 3/2 \) satisfies the dimensionally correct capacitor charging equation. Substitution into \( i = C dv/dt \) gives the snubber capacitance

\[ 100A \times 600V = C \times 100\text{ns} \]

that is

\[ C = 16.5 \text{nF} \]

Use an 18nF, 1000V dc, metalised polypropylene, high dv/dt capacitor.

The snubber capacitor discharges at switch turn-on, and must discharge during the switch minimum on-time. That is

\[ t_{m} = 5 CR \]

5% of \( 1 / 10\text{kHz} = 5 \times R \times 18\text{nF} \)

that is \( R = 55.3 \Omega \) Use 56Ω.

The discharge resistor power rating is independent of resistance and is given by

\[ P_{df} = \frac{1}{2} C V^2 f \]

\[ = \frac{1}{2} \times 18\text{nF} \times 600^2 \times 10kHz = 32.4\text{W} \]  

Use 50W.

The resistor can be wire-wound, the internal inductance of which reduces the initial peak current when the capacitor discharges at switch turn-on. The maximum discharge current into the switch during reset, which is added to the 100A load current and any diode reverse recover current, is

\[ I_{max} = V_{c} / R = 600V / 56\Omega = 10.7\text{A} \]
which decays exponential to zero in five time constants, 5μs. The peak switch current (neglecting freewheel diode recovery) is 100A+10.7A=110.7A, at turn-on.
At switch turn-off, when the switch current reduces to zero, the snubber capacitor has charged to a voltage less than the 600V rail voltage, specifically
\[ V_C = \frac{1}{C} \int_{t_0}^{t_f} i_C dt \]
\[ = \frac{1}{16.5nF} \times 100A \times \left( \frac{t}{100ns} \right) \int_{t_0}^{t_f} dt = 300V \] (277V with 18nF)
The switch turn-off losses are reduced from 30W to
\[ P_{off} = f_s \int_0^{t_f} i_C v_C dt = f_s \int_0^{t_f} I_m \left( 1 - \frac{t}{100ns} \right)^2 \frac{t}{100ns} dt = 2.5W \] (2.3W with 18nF)
The total turn-off losses (switch plus snubber resistor) are 2.5W+32.4W=34.9W, which is more than the 30W for the unaided switch. Since the voltage rise time has been neglected in calculating the un-aided losses, 34.9W would be expected to be less than the practical un-aided switch losses. The switch losses have been reduced by 91%, (\(\frac{30}{34.9}\)) from 30W to 2.5W.

iii. As the current in the switch falls linearly to zero, the capacitor current increases linearly to 100A (\(k=1\)), such that the load current remains constant, 100A. Initially the capacitor voltage increases in a quadratic function according to
\[ V_C(t) = \frac{1}{C} \int_{t_0}^{t_f} i_C dt \]
The capacitor charges quadratically towards 600V in 100ns, as its current increases linearly from zero to 100A, that is
\[ 600V = \frac{1}{C} \int_{t_0}^{t_f} 100A \left( t \right) \frac{t}{100ns} dt \]
that is \(C = 8.5nF\)

Use a 10nF, 1000V dc, metallised polypropylene, high dv/dt capacitor.

The necessary reset resistance to discharge the 10nF capacitor in 5μs is
\[ R = \frac{5}{5 \times 10nF} \] (1)
that is \(R = 1000\)
The power dissipated in the reset resistor is
\[ P_{reset} = \frac{1}{2} C V^2 f_s \]
\[ = \frac{1}{2} \times 10nF \times 600V^2 \times 10kHz = 18W \]

Use a 1000Ω, 25W, wire-wound, 600Vdc withstand voltage, metal clad resistor. The resistance determines the initial current magnitude and the period over which the capacitor energy is dissipated. The resistance does not determine the amount of energy dissipated. The peak switch current is therefore 100A+6A = 106A, at turn-on.

The peak switch current (neglecting freewheel diode recovery) is 100A+10.7A=110.7A, at turn-on.

The total losses (switch plus snubber resistor) with a turn-off snubber are 5W+18W =23W, which is less than the calculated 26.9W (18W+5W).

Note that the losses predicted by the equations in figure 9.13 amount to 5W+15W+20W. The discrepancy is due to the fact that the preferred value of 10nF with \(k=1\) giving 5W+18W =23W (rather than the calculated 8.5nF, \(k=1\)) has been used for the resistor loss calculation.
For low inductance the collector current reaches its maximum value $I_m$ (the load current) before the collector voltage has reached zero. As shown in figure 9.18b, from $v = L\frac{di}{dt}$, the collector current increases quadratically $i(t) = I_m\left(\frac{v}{B}\right)^2$ and the switch turn-on loss is given by

$$W_f = \frac{1}{2} V_c I_m \left(\frac{v}{B}\right)^2 \left(\frac{1}{2} \epsilon + \frac{1}{2} \epsilon^2\right)$$

(3)  

for $k \leq 1$, where $k = \frac{Ls}{\tau}$ as defined in figure 9.18.

These losses include both switch losses and stored inductor energy subsequently dissipated. For higher snubber inductance, the collector voltage reaches zero before the collector current reaches the load current level. Initially the inductor current increases quadratically $i(t) = I_m\left(\frac{v}{B}\right)^2$, then when the collector voltage has reached zero, the current increases linearly. The switch loss is given by

$$W_f = \frac{1}{2} V_c I_m \left(\frac{v}{B}\right)^2 \left(\frac{1}{6} \left(2k - 1\right)\right)$$

(9.24)

Note that these equations are similar to those for the turn-off snubber, except that the current fall time $t_o$ is replace by the voltage fall time, $t_v$. The normalised loss components for the capacitive snubber in figure 9.13 are valid for the inductive turn-on snubber.

Figure 9.18. Switch voltage and current collector waveforms at turn-on: (a) without a snubber; (b) and (c) with an inductive snubber; and (d) switch power losses.

Figure 9.19. Turn-on snubber waveforms satisfying Kirchhoff's laws: (a) turn-on with small snubber inductance and (b) turn-on with large snubber inductance.

Minimum total turn-on losses of $5/9$ those of the un-aided case, occur at $k = \frac{2}{3}$ when $L_s = \frac{2V_c I_m}{9}\left(9.25\right)$

At switch turn-off, the snubber inductance stored energy is dissipated as heat in the snubber freewheeling diode path. The maximum power loss magnitude is dependent on the operating frequency and is given by

$$P_c = \frac{1}{2} V_c I_m L_s$$

(9.26)

This power is dissipated in the inductor winding resistance, resistance $R$, and freewheeling diode $D_f$. The time constant is designed such that $t_o = S L_s/R$ where $t_o$ is the minimum device off-time, where $R$ is the effective total series resistance. The time constant can be reduced either by increasing the series resistance or by inserting a Zener diode as shown in figure 9.20.

A disadvantage of series resistance $R$ as in figure 9.20a is that the switch collector voltage at turn-off is increased from $V_r$ to $V_r + I_o R$. The resistor must also have low self-inductance in order to allow the collector current to rapidly transfer from the switch to the resistor/diode reset circuit. The advantage of using a Zener diode as in figure 9.20b is that the maximum overvoltage is fixed, independent of the load.
current magnitude. For a given maximum overvoltage, the Zener diode absorbs the inductor-stored energy quicker than would a resistor (see example 7.3 and problem 9.9). The advantages of using resistive dissipation are lower costs and more robust heat dissipation properties.

Alternatively the Zener diode can be placed across the switch as shown in figure 9.20c. The power dissipated is increased because of the energy drawn from the supply, through the inductor, during reset. At higher power, the soft voltage clamp shown in figure 9.20d, and considered in section 9.2, can be used. At switch turn-off, the energy stored in \( L_s \) along with energy from the supply, is transferred and stored in a clamp capacitor. Simultaneously energy is dissipated in \( R \) and returned to the supply as the capacitor voltage rises. The advantage of this circuit is that the capacitor affords protection directly across the switch, but with lower loss than a Zener diode as in figure 9.20c. The energy loss equation for each circuit is also shown in figure 9.20. In high-voltage applications, the combined features of the soft clamp in figure 9.20d and the low loss Zener clamp in figure 9.20b can be realised by inserting a series Zener as shown in the figure 9.20d insert. This avoids the need to series connect Zener diodes, which would be necessary if the circuit in figure 9.20c were used at voltages above a few hundred volts.

Figure 9.21 shows how a switch turn-on snubber circuit modifies the SOA trajectory during switch-on, avoiding a condition of simultaneous maximum voltage \( V_s \) and current \( I_m \).

**Example 9.5: Turn-on air-core inductor snubber design**

A 600V, 100A machine field winding is switched at 10kHz. In maintaining a constant field current, the switch operates with an on-state duty cycle between 5% and 95% (5% ≤ \( \delta \) ≤ 95%) and has a turn-on voltage fall time of 100ns, that is, \( V_s(t) = 600V(1 - t / 100ns) \).

i. Estimate the turn-on loss of the switch.

ii. Design an inductive turn-on snubber using the dimensionally correct identity \( v = Ldi/dt \). What is the current magnitude in the turn-on inductor when the switch voltage reaches zero.

iii. Design an inductive turn-on snubber such that the switch current reaches 100A at the same time the switch collector voltage reaches zero.

In each snubber case, using first a resistor and second a Zener diode for inductor reset, calculate the percentage decrease in switch power dissipation at turn-on, compared to the un-aided case.

**Solution**

i. The switch un-aided turn-on losses are given by equation (9.13). The turn-on time is greater than the voltage fall time (since the current rise time \( t_r \) has been neglected), thus the turn-on switching losses will be greater than

\[
W_{on} = \frac{1}{2} VI_{f} = \frac{1}{2} \times 600V \times 100A \times 100ns = 3mJ
\]

\[
P_{on} = W_{on} = 3mJ \times 10kHz = 30W
\]

ii. Use of the equation \( v = Ldi/dt \) results in a switch current that reaches the load current magnitude after the collector voltage has fallen to zero. From \( k = \frac{1}{2} + \frac{L_{in}}{V_s} \) in figure 9.21, \( k = 3/2 \) satisfies the dimensionally correct inductor equation. Substitution into \( v = Ldi/dt \) gives the necessary snubber inductance

\[
600V = L \times 100A
\]

The snubber inductor releases its stored energy at switch turn-off, and must discharge (demagnetise) during the switch minimum off-time, \( t_{min} \). That is

\[
t_{min} = \frac{5L}{R}
\]

5% of 1/10kHz = \( \frac{5 \times 0.6\mu H}{R} \) that is \( R = 0.6 \Omega \)

Use the preferred value 0.68Ω (nearest higher preferred value), which reduces the \( L/R \) time constant.
The discharge resistor power rating is independent of resistance and is given by
\[ P_{\text{disch}} = \frac{1}{2} \frac{L}{R} f_v \]
where \( L = 50 \text{H} \). The inductor current increases in a quadratic function according to
\[ i(t) = \frac{1}{L} \int v(t) \, dt \]

The inductive current increases quadratically to 100A in 100ns, as its voltage increases linearly from zero to 600V, that is
\[ 100A = \frac{1}{L} \int_0^{100ns} 600V \, dt \]
which is independent of resistance, is
\[ L = \frac{1}{2} \times 300\mu H = 150\mu H \]

A Zener diode, as in figure 9.20b, of \( V_z = L_1 \frac{L}{R} f_v = 0.6\mu H \times 100A/5\mu s = 12V \), will reset the inductor in the same time as 5 L/R time constants. The switch voltage is clamped to 612V during the 5ns inductor reset time at switch turn-off.

At turn-on when the switch voltage reduces to zero, the snubber inductor current (hence switch current) is less than the load current, 100A, specifically
\[ i_0 = \frac{1}{L} \int v_0 \, dt \]

The switch turn-on loss is reduced from 30W to
\[ P_s = i_0 \int_0^{100ns} v_0 \, dt = i_0 \int_0^{100ns} 600V \, dt = 50A \times 50A = 2.5W \]

The total turn-on losses (switch plus snubber resistor) are 2.5W + 30W = 32.5W, which is more than the 30W for the unaided switch. Since the current rise time \( t_r \) has been neglected in calculating the 30W un-aided turn-on losses, it would be expected that 32.5W would be less than the practical un-aided case. The switch loss is decreased by 92\%, \((\%)\), from 30W down to 2.5W.

### 9.3.4 The polarised turn-on snubber circuit - with saturable ferrite inductance

The purpose of a turn-on snubber circuit is to allow the switch collector voltage to fall to zero while the collector current is low. Device turn-on losses are thus reduced, particularly for inductive loads, where during switching the locus point \((V_c, I_c)\) occurs in the un-aided transition case. This turn-on loss reduction effect can be achieved with a saturable inductor in the circuit shown in figure 9.23a, rather than using a non-saturable (air core) inductor as previously considered in section 9.3.3. The saturable inductor in the snubber circuit is designed to saturate after the collector voltage has fallen to zero, at point \( y \) in figure 9.23. Before saturation the saturable inductor presents high reactance and only a low magnetising current flows. Once the collector voltage has reached zero, the inductance can saturate since the switch-on loss period is finished. From Faraday’s equation, assuming the collector voltage falls to be linear, \( V_c(1-t/t_{on}) \), the saturable inductor \( L_i \) must satisfy
\[ V_c = \frac{dV_c}{dt} = \frac{N A B_0}{L_i} \frac{dI_c}{dt} \]

Rearranging, using an inductor voltage \( V_c(t) = V_c(1-t/t_{on}) = V_c L_i / t_{on} \), and integrating gives
\[ B_0 = \frac{1}{L_i} \int_0^{t_{on}} V_c(t) \, dt = \frac{1}{L_i} \int_0^{t_{on}} V_c L_i / t_{on} \, dt \]
which yields the identity
\[ V_c = \frac{2N A B_0}{t_{on}} \]

where \( N \) is the number of turns, \( A \) is the core area, and \( B_0 \) is the core ferro-magnetic material saturation flux density.
The inductor magnetising current \( I_m \) should be much less than the load current magnitude \( I_L \), and the magnetising current at saturation is given by

\[ I_m = H_s I_m / N \]

where \( L_m \) is the core effective flux path length and \( H_s \) is the magnetic flux intensity at the onset of saturation. Before core saturation the inductance is given by

\[ L = N \varphi / I = N^2 / \mu = \mu N^2 / L_w \]

When the core saturates the inductance falls to that of an air core inductor (\( \mu = 1 \)) of the same turns and dimensions, that is, the incremental inductance is

\[ L_u = \mu N^2 / L_w \]

The energy stored in the inductor core is related to the triangular \( B-H \) area shown in figure 9.23c and magnetic volume, and is approximated by

\[ W_t = 0.5 B_H A L_w = 0.5 L t \]

The collector turn-on waveforms are shown in figure 9.23b, while the corresponding \( B-H \) curve and SOA trajectories are illustrated in figure 9.23 parts c and d. It will be seen in figure 9.23b that little device turn-on electrical stressing occurs.

**Example 9.6:**  Turn-on ferrite-core inductor snubber design

A 600V, 100A machine field winding is switched at 10kHz. In maintaining the field current constant, the switch operates with an on-state duty cycle between 5% and 95% (5% ≤ \( \delta \) ≤ 95%) and has a turn-on voltage fall time of \( t_z = 100ns \), that is, \( V_z(t) = 600V \times (1 - t / 100ns) \).

Design a saturable inductor turn-on snubber that saturates as the collector voltage reaches zero, using a ferrite core with the following parameters.

- \( A = 0.4 \text{ sq cm} \)
- \( L = 4\Omega \)
- \( B_P = 0.4T \)
- \( H_s = 100\text{At/m} \)

Calculate the switch losses at turn-on when using the saturable reactor. What is the percentage reduction in switch turn-on losses?

If an air cored inductor is used to give the same switch turn-on loss, what are the losses at reset?

**Solution**

From equation (9.31), the unaided switch turn-on loss is 30W.

\[ W_t = 0.5 B_H A L_w = 0.5 L t \]

The collector current hence magnetising current \( I_m \) at saturation, that is, when the collector voltages reaches zero, is given by equation (9.30)

\[ I_m = H_s I_m / N = 100\text{At/m} \times 0.04 / 2 = 2A \]

Since \( I_m < I_m \) (2A<<100A), this core with 2 turns produces satisfactory turn-on snubber action, resulting in greatly reduced switch losses at turn-on.

From equation (9.33) the energy stored in the core and released as heat in the reset resistor is

\[ W_e = 0.5 L I_m^2 \]

The time \( t_r \) for core reset via the resistor in five time constants, is dominated by the 16μH section (the pre-saturation section) of the \( B-H \) curve, thus

\[ t_r = 5\mu s = 5 \times 16\mu H / R \]

that is \( R = 16\Omega \)

Use a 18Ω, 1W, carbon composition resistor, for low inductance. This resistance results in a switch voltage increase above 600V of 18V.

Alternatively, the Zener diode clamps shown in figures 9.20b or c, may be suitable to dissipate the 0.32W of stored magnetic energy. The Zener voltage is determined by assuming that a fixed Zener voltage results in a linear decrease in current from 2A to zero in 5μs. That is, assuming minimal core stored energy associated with the current decrease from 100A to 2A,

\[ W_e = V_z / 2 I_m dt = 0.5 L I_m^2 \]

32μJ = \( V_z \times V_z / 2 \times 2A \times 5\mu s \)

that is \( V_z = 6.4V \)

Use a 6.8V, 1W Zener diode to clamp and dissipate the 0.32W of reset power, as in figure 9.20b. Series connected Zener diodes in parallel with the switch, as in figure 9.20c, dissipate 30W.

The energy associated with the saturation region is small and is released in an insignificant time compared to the 5μs minimum off-time. The disadvantage of the Zener diode clamping approach, as opposed to using a resistor, is that the maximum switch voltage is clamped to 606.8V, even during the short, low energy period when the inductor current falls from 100A to 2A.
ii. The switch turn-on losses with the saturable reactor are given by
\[ P_{on} = \int_{0}^{t_{on}} i V dt = \int_{0}^{t_{on}} 1.0 \times 1000 \cdot \left(1 - \frac{t}{1000}\right) \cdot 2A \cdot \left(\frac{t}{1000}\right)^2 dt \]

The switches at turn-on have been reduced from 30W to 0.1W, a 99.9% decrease in losses. The total losses (switch plus Zener diode) are 0.1W + 0.32W = 0.42W, which is significantly less than the 30W in the un-aided case.

iii. If an air core inductor of 16μH (from part i) were to replace the saturable reactor, the stored energy released would give losses
\[ W = \frac{1}{2} L \cdot I^2 = \frac{1}{2} \cdot 16 \times 10^{-6} \cdot 2^2 = 64mJ \]

Clearly the use of an air cored inductor rather than a saturable reactor, to achieve the same switch loss of 0.1W at turn-on, is impractical.

9.3.5 The unified turn-on and turn-off snubber circuit

Figure 9.25 shows a switching circuit which incorporates both an inductor turn-on and a capacitor turn-off snubber. Both \( C_\text{on} \) and \( L_\text{on} \) are dimensioned by the analysis outlined in sections 9.3.1 and 9.3.3, respectively. The power rating of the dissipating resistor \( R \) incorporates a contribution from both the turn-on inductor \( L_\text{on} \) and turn-off capacitor \( C_\text{on} \), according to
\[ P_{\text{diss}} = \frac{1}{2} L \cdot I^2 + \frac{1}{2} C \cdot V^2 \]  

(9.34)

Calculated resistance values to satisfy both minimum off and on time reset according to \( I_{\text{on}} = 5R_\text{on}C_\text{on} \) and \( t_{\text{off}} = 5L_\text{on}/R_\text{on} \), may result in irreconcilable resistance and/or switch voltage/current requirements. The snubber capacitor discharges at turn-on via an L-C-R circuit rather than the usual R-C circuit, hence reducing the turn-on current stressing of the switch.

In example 9.4 the resistor requirement for the 16μF capacitive turn-off snubber is \( R < 560 \), while the 0.6μF inductive turn-on snubber in example 9.5 requires \( R > 68 \). Thus 0.68Ω < \( R < 560 \) satisfies snubber \( L_\text{on} \) and \( C_\text{on} \) reset requirements. The maximum reset current and voltage are related to \( V = \sqrt{2L/R_\text{on}} \).

In combining the two snubber functions, the single resistor may reduce the maximum switch over-voltage at turn-off, \( I_{\text{on}} = Z \), and the maximum switch snubber current at turn-on \( V_z/Z \). If \( R \) is too small a high switch snubber current \( V_z/Z \) flows at turn-on, while if \( R \) is too large, a large switch over-voltage, \( I_{\text{on}} \times Z \), occurs at switch turn-off.

An important by-product from using a turn-on snubber circuit is that the inductor controls the reverse recovery process of the load freewheeling bipolar Si diode at switch turn-on.
Figure 9.26. Bridge leg configurations: (a) Undeland indirect leg snubber circuit; (b) leg with turn-on snubber and turn-off snubber C_s discharge path shown; (c) L-C oscillation at switch-on; (d) blocking circuit to prevent snubber capacitor discharge when D_f conducts; and (e) dual soft voltage clamps for high current single switch modules.

In each circuit in figure 9.27, at switch turn-off, $t_2$, the energy $\frac{1}{2}mL_i^2$ stored in the turn-on snubber inductor is dissipated in the resistor of the discharge circuit. The energy $\frac{1}{2}rmL_i^2$, due to diode recovery, is dissipated in the resistor at time $t_1$ in circuits (a), (b) and (c). In figure 9.27d the energy in excess of that associated with the load, $\frac{1}{2}L_i^2$, due to diode recovery, is dissipated in the switch and its parallel connected diode. At time $t_1$

$$W = \frac{1}{2}mL_i^2 + \frac{V_{ce}}{V_{os}}I_mL_i + V_{os}I_m$$

(9.35)

is dissipated in the two semiconductor components. Since the energy is released into a low voltage $v_{os}$, the reset time $t_2$ - $t_1$ is large.

Magnetic coupling of the inductors in figures 9.27c and d does not result in any net energy savings.

Figure 9.27. Turn-on snubbers for bridge legs: (a) single inductor in dc link; (b) unified L-R-D snubber; (c) soft voltage clamp; and (d) soft voltage clamp with load clamped.
9.5 Appendix: Non-polarised turn-off R-C snubber circuit analysis

When a step input voltage is applied to the L-C-R circuit in figure 9.3, a ramped voltage appears across the R-C part of the circuit. If this dv/dt is too large, a thyristor in the off-state will turn on as a result of the induced central junction displacement current, which causes carrier injection from the outer junctions.

The differential equations describing circuit current operation are

\[
(D^2 + 2\xi D + \omega^2)I = 0
\]

and

\[
(cD + 1) = CDe_i
\]

where

\[ D = \text{differential operator} = \frac{d}{dt}\]

and

\[ \xi = \text{damping ratio} = \frac{\alpha R}{2\xi \omega} \]

\[ \omega = \text{oscillation frequency} = \frac{1}{\sqrt{LC}} \]

\[ \omega_s = \text{natural frequency} = \frac{1}{\sqrt{LC}} \]

Solution of equations (9.36) and (9.37), for initial current \( I_0 = 0 \), leads to

(a) The snubber current

\[
I(t) = \frac{e_i}{R} \left( \frac{2\xi}{\sqrt{1 - \xi^2}} \right) e^{-\xi \omega_s t} \sin \omega_s t
\]

(A)

(b) The rate of change of snubber current

\[
\frac{dI}{dt} = \frac{e_i}{L} \left( \cos \omega_s t - \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_s t \right)
\]

(A/s)

(c) Snubber R-C voltage

\[
e_s = e_i \left( 1 - \frac{e_i}{\sqrt{1 - \xi^2}} \cos \omega_s t - \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_s t \right)
\]

(V)

(d) The rate of change of R-C voltage

\[
\frac{d\epsilon_s}{dt} = \frac{e_i}{L} \left( 2\xi \cos \omega_s t + \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_s t \right)
\]

(V/s)

The maximum value expressions for each equation can be found by differentiation

(a) Maximum snubber current

\[
I_{max} = \frac{e_i}{R} \left( \frac{2\xi}{\sqrt{1 - \xi^2}} \right)
\]

when \( \cos \omega_s t = \xi \)

(b) The maximum snubber dv/dt is given by

\[
\frac{dI_{max}}{dt} = \frac{e_i}{L} \left( \cos \omega_s t - \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_s t \right)
\]

(A/s)

(c) Maximum R-C voltage

\[
e_{max} = e_i \left( 1 + \frac{e_i}{\sqrt{1 - \xi^2}} \right)
\]

(V)

(d) Maximum slew rate, \( \frac{d\epsilon_s}{dt} \), for \( \xi < \frac{1}{2} \)

\[
\frac{d\epsilon_s}{dt} = \frac{e_i}{L} \left( \cos \omega_s t - \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_s t \right)
\]

(V/s)

when \( \cos \omega_s t = \frac{2\xi}{\sqrt{1 - \xi^2}} \). The minimum value of the maximum slew rate is 0.81 pu at \( \xi = 0.265 \).

at \( \xi = \frac{1}{2} \), \( \frac{d\epsilon_s}{dt} = n_0 e_i \),

for \( \xi > \frac{1}{2} \)

\[
\frac{d\epsilon_s}{dt} = \frac{e_i}{L} \left( \cos \omega_s t - \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_s t \right)
\]

(V/s)

when \( t = 0 \)

9.6 Appendix: Polarised turn-off R-C-D switching aid circuit analysis

Switch turn-off loss for an unaided switch, assuming the collector voltage rise time is negligible compared with the collector current fall time, is

\[
W = \frac{1}{2} W_{t_{so}}
\]

If \( r \) is the time in figure 9.11 for the snubber capacitor \( C \), to charge to the supply \( V_s \), and \( t_o \) is the switch collector current fall time, assumed linear such that \( I(t) = I_m (1 - t / t_o) \), then two capacitor charging conditions can exist

\[
\begin{align*}
\tau & \leq t_o \\
\tau & \geq t_o
\end{align*}
\]

Let \( k = \frac{\tau}{t_o} \) and electrical energy

\[
W = \int_{t_0}^{\tau} V_i \, dt
\]

Case 1: \( \tau \leq t_o, k \leq 1 \)

Figure 9.11b shows ideal collector voltage and current waveforms during aided turn-off for the condition \( \tau \leq t_o \). If, assuming constant maximum load current, \( I_m \), during the switching interval, the collector current falls linearly, then the load deficit, \( I(t) \), charges the capacitor \( C \). From the collector voltage, the capacitor voltage, and collector voltage, therefore increase quadratically. The collector voltage \( v_c \) and current \( i_c \) are given by

\[
\begin{align*}
i_c(t) & = I_m \left( 1 - \frac{t}{t_o} \right) \quad 0 \leq t \leq \tau \\
v_c(t) & = V_s \left( 1 - \frac{t}{t_o} \right) \quad \tau \leq t \leq t_o
\end{align*}
\]

The final capacitor charge is given by

\[
Q = CV_s \left[ \int_{t_0}^{t} (I_m - i_c(t)) \, dt + \int_{t}^{\tau} V_c(t) \, dt \right] (C)
\]

The energy stored by the capacitor, \( W_c \), and energy dissipated in the switch, \( W_s \), are given by

\[
W_c = \frac{1}{2} CV_s^2 \quad (\text{ storing energy })
\]

\[
W_s = \frac{1}{2} CV_s^2 \quad (\text{ energy dissipated }) (J)
\]

The total circuit losses \( W_{total} \) are

\[
W_{total} = W_c + W_s = \frac{1}{2} CV_s^2 \times (1 - \frac{k^2}{k^2 + 1}) \quad k \leq 1 (J)
\]

Case 2: \( \tau \geq t_o, k \geq 1 \)

Figure 9.11c shows the ideal collector voltage and current switch-off waveforms for the case when \( k \geq 1 \). When the collector current falls to zero the snubber capacitor has charged to a voltage, \( V_s \), where

\[
v_s = \frac{1}{C} \int_{t_0}^{t} V_s \, dt = \frac{1}{C} \times \frac{1}{2} \int_{t_0}^{t} V_s \, dt
\]

(V)

The collector voltage \( v_c \) and current \( i_c \) are given by

\[
\begin{align*}
i_c(t) & = I_m \left( 1 - \frac{t}{t_o} \right) \\
v_c(t) & = \frac{V_s}{k} \left( 1 - \frac{t}{t_o} \right) \quad 0 \leq t \leq t_o
\end{align*}
\]

\[
\begin{align*}
i_c(t) & = 0 \\
v_c(t) & = \frac{V_s}{k} \quad t_o \leq t \leq \tau
\end{align*}
\]
The final capacitor charge is given by

\[ Q = C V = \int_0^V \left( I_c - I_{\text{f}}(t) \right) dt + \int_{I_c}^{I_{\text{f}}(t)} I_c dt \]

\[ = I_c f_{c}(k - 1) \]  \hspace{1cm} (9.55)

The energy stored by the capacitor \( W_c \) and energy dissipated in the switch \( W_s \) are given by

\[ W_c = \frac{1}{2} C V^2 \left( I_{\text{f}}(t) \right) \]  \hspace{1cm} (9.56)

\[ W_s = \int_0^V V I(t) dt \]

Using equations (9.53) and (9.55) to eliminate \( V \) yields

\[ W_s = \frac{1}{2} C V^2 \left( k - k^\frac{1}{2} \right) \]  \hspace{1cm} (9.57)

The total circuit losses \( W_{\text{total}} \) are

\[ W_{\text{total}} = W_c + W_s = \frac{1}{2} C V^2 \left( k - k^\frac{1}{2} \right) \]  \hspace{1cm} (9.58)

The equations (9.50) to (9.52), and (9.56) to (9.58) have been plotted, normalised with respect to unaided losses \( \frac{1}{2} C V^2 f_{c} f_{c} \), in figure 9.13.

Reading list


Problems

9.1. The figure 9.28 shows GTO thyristor turn-off anode \( I-V \) characteristics. Calculate

i. turn-off power loss at 1 kHz. What percentage of the total loss does the tail current account for?

ii. losses when a capacitive turn-off snubber is used and the anode voltage rises quadratically to 600V in 0.5\( \mu \)s. What percentage of the total losses does the tail current account for? What is the necessary capacitance?

iii. losses when a capacitive turn-off snubber is used and the anode voltage rises quadratically to 600V in 2\( \mu \)s. What percentage of the total losses does the tail current account for? What is the necessary capacitance?

[10.5 W]

9.2. Prove that the minimum total losses (switch plus snubber resistor), associated with a switch which utilises a capacitive turn-off switching-aid circuit, occur if the snubber capacitor is fully charged when the collector current has fallen to \( \frac{1}{3} \) its original value. Derive an expression for this optimal snubber capacitance.

\[ C_{\text{o}} = \frac{1}{2} \frac{I_{\text{f}}(t)}{V_{\text{f}}} \]  \hspace{1cm} (F)

9.3. Derive an expression for the optimal turn-on switching-aid circuit inductance, assuming the collector current rise time in the unaided circuit is very short compared with the collector voltage fall time.

\[ L_{\text{o}} = \frac{1}{2} \frac{I_{\text{f}}}{V_{\text{f}}} \]  \hspace{1cm} (H)

9.4. A ferrite toroid has \( B-H \) characteristics as shown in figure 9.29 and a cross-sectional area, \( A, \) of 10 mm\(^2\) and effective length, \( L_{\text{eff}} \) of 50 mm.

A number of such toroid cores are to be stacked to form a core for a saturable inductor turn-on snubber in a switching circuit. The circuit supply voltage is \( V \) and the switch voltage fall time at turn-on is \( t_{\text{f}}. \) Assume \( t_{\text{f}} \) is independent of supply voltage and falls linearly from \( V \) to 0 V.

i. Using Faraday’s Law, show that if the ferrite inductor is to saturate just as the switch collector voltage falls to zero at turn-on, then the number of turns \( N \) for \( n \) cores is given by

\[ N = \frac{V_{\text{f}}}{2B_{\text{max}}A} \]

ii. Derive an expression for the inductance before saturation.

iii. It is required that the maximum magnetising current before saturation does not exceed 1 A. If only 10 turns can be accommodated through the core window, what is the minimum number of cores required if \( V = 200 \) V and \( t_{\text{f}} = 1 \mu s? \)

iv. How many cores are required if the supply voltage \( V \) is increased to the peak voltage of the three-phase rectified 415 V ac mains, and the load power requirements are the same as in part iii?

v. Calculate the percentage change in the non-saturated inductance between parts iii and iv.

vi. What are the advantages of saturable inductance over linear non-saturable inductance in turn-on snubber applications? What happens to the inductance and stored energy after saturation?

[\( t = N^2 R, n = 5, n = 4, 1:9 \)]

9.5. Prove, for an inductive turn-on snubber, where the voltage fall is assumed linear with time, that

\[ k = \frac{I_{\text{f}}}{V_{\text{f}}} \]  \hspace{1cm} \text{for } k \geq 1

\[ k = \frac{I_{\text{f}}}{V_{\text{f}}} + \frac{1}{2} \]  \hspace{1cm} \text{for } k \leq 1

where \( k = I_{\text{f}}/R \) (see figures 9.18 and 9.21).

9.6. Derive the expressions in table 9.1 for a turn-off snubber assuming a cosinusoidal current fall. Prove equation (9.21), the optimal capacitance value.
9.7 Show that when designing a capacitive turn-off snubber using the dimensionally correct equation \( i = C \frac{dv}{dt} \), as in example 9.4b, the capacitor charges to \( \frac{1}{2} V_s \) when the switch current reaches zero.

9.8 Show that when designing an inductive turn-on snubber using the dimensionally correct equation \( v = L \frac{di}{dt} \), as in example 9.5b, the inductor current reaches \( \frac{1}{2} I_m \) when the switch voltage reaches zero.

9.9 Reset of inductive turn-on snubber energy \( \frac{1}{2} L I^2 \) can be affected through a resistor, \( R \), as in figure 9.20a or through a Zener diode, \( D_z \), as in figure 9.20b. Show that for the same reset voltage, namely \( V_z = I_m R \), in each case, Zener diode reset is \( n \) times faster the resistor reset when \( n R C_s < t_o \).

9.10 In figure 9.12a show that for \( \tau < t_f \), the collector current is given by

\[
I_c = I_{so} - \frac{2 I_{so} V_s}{t_f}
\]

when the collector voltage reaches the supply voltage rail \( V_s \).

9.11 In figure 9.19a show that for \( \tau < t_v \), the collector voltage is given by

\[
v_c = V_c - \frac{2 I_{so} V_c}{t_v}
\]

when the collector current reaches the load current \( I_m \).

9.12 An RCD turn-off snubber is used across a switch in a 600V dc, 10A, 20kHz chopper application. The switch current fall time at 10A is 100ns.

i. What is the capacitor voltage when the switch current reaches zero at turn-off, if the switch turn-off loss is to be 1W?
ii. What snubber capacitance is necessary?

For the same un-aided switching conditions, the total losses (switch plus snubber resistor) are to be
(a) 10.3W
(b) 9W

What is the capacitor requirement and what is the capacitor voltage when the switch current reaches zero?

[600V, 0.83nF]

9.13 For a cosinusoidal current fall at turn-off as shown in figure 9.15, derive expressions for the switch current \( i_o \) when \( k < 1 \) and collector voltage \( v_o \) when \( k > 1 \).