Outline Solutions to odd-numbered exercises from the book: 
*An Introduction to Financial Option Valuation: Mathematics, Stochastics and Computation*,
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2.1 We have
\[ e^{rt}D_0 = \left(1 + \frac{r_c t}{m}\right)^m D_0. \]
This rearranges to
\[ r_c = m \left(e^{rt/m} - 1\right)/t. \]
Then, using the given approximation,
\[ r_c \approx m (1 + rt/m - 1)/t = r. \]

2.3 Suppose the time-zero value of \( \hat{\pi}_B \) is greater than the time-zero value of \( \pi_A \), that is, \( S > C + Ee^{-rT} \). Then we could sell \( \hat{\pi}_B \) and buy \( \pi_A \) at time zero. This gives us a profit at time zero. But we know that the portfolio \( \hat{\pi}_B \) has a payoff that is never greater than that of \( \pi_A \). Hence, we can be certain that at expiry we will not have to pay out more than we gain. So we have locked into a guaranteed profit, at no cost. This violates the no-arbitrage assumption. Hence, by contradiction, we must have \( S \leq C + Ee^{-rT} \).

2.5 Consider
- holding a European put option with exercise price \( E_1 \), and
- holding a European put option with exercise price \( E_3 \), and
- writing two European put options with exercise price \( E_2 = \frac{1}{2}(E_1 + E_3) \).

The value at expiry is
\[ \max(E_1 - S, 0) + \max(E_3 - S, 0) - 2 \max(\frac{1}{2}(E_1 + E_3) - S, 0) \]
To show that this matches the payoff in Exercise 1.3, we note that it is piecewise linear with corners at \( S = E_1, S = E_3 \) and \( S = \frac{1}{2}(E_1 + E_3) \). At \( S = E_1 \) the payoff is
\[ 0 + E_3 - E_1 - 2(\frac{1}{2}(E_1 + E_3) - E_1) = 0. \]
At \( S = \frac{1}{2}(E_1 + E_3) \) the payoff is
\[
0 + E_3 - \frac{1}{2}(E_1 + E_3) + 0 = \frac{1}{2}(E_3 - E_1).
\]

At \( S = E_3 \) the payoff is
\[
0 + 0 + 0 = 0.
\]

Hence, we have matched the payoff from Exercise 1.3.

Let \( C(E_1, 0) \) denote the time-zero value of the call with exercise price \( E_1 \). With this notation, the difference between the time zero values of the two portfolios is
\[
[C(E_1, 0) - P(E_1, 0)] + [C(E_3, 0) - P(E_3, 0)] - 2[C(E_2, 0) - P(E_2, 0)] = 0.
\]

Now put-call parity (2.2) says (with \( t = 0 \)):
\[
\begin{align*}
C(E_1, 0) - P(E_1, 0) &= S_0 - E_1 e^{-rT}, \\
C(E_2, 0) - P(E_2, 0) &= S_0 - E_2 e^{-rT}, \\
C(E_3, 0) - P(E_3, 0) &= S_0 - E_3 e^{-rT}.
\end{align*}
\]

In (1) we get
\[
S_0 - E_1 e^{-rT} + S_0 - E_3 e^{-rT} - 2(S_0 - E_2 e^{-rT}) = e^{-rT} (2E_2 - (E_1 + E_3)) = 0,
\]
as required.