Hand in solutions to problems 1 and 2 by 2PM on Thursday 8 March, in dept. office

>>>>>>> **SOLUTIONS MUST BE MACHINE WRITTEN** <<<<<<<

1. (a) Give an unambiguous and complete description of Kruskal's algorithm (in your own words; do *not* copy from lecture notes or elsewhere). 2pts

   (b) Use this to find a minimum spanning tree for the graph drawn below, where the edge between vertices i and j has weight given by the non-negative difference between the i-th and the j-th digit in your nine digit *Registration number*. For example, if your number is 201261834, then the weight of the edge between vertices 1 and 2 is $|2-0|=2$, and for the edge between vertices 3 and 7 it is $|1-8|=7$. 8pts

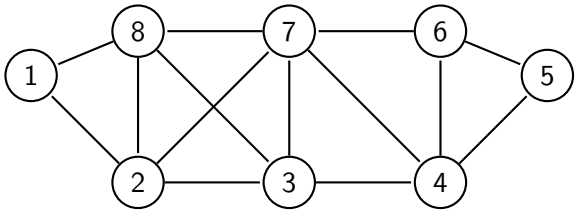
   Your Registration number begins with the year you started at Strathclyde

   Draw the spanning tree and write down your Registration number, the total weight of the spanning tree and a *list of the edges in the tree, in the order in which they were picked*. Write this as in the following example,with the weight of each edge below it:

   Follow these instructions

   $$\begin{array}{lll} 201261834 & \text{weight} = 9, & 23\text{–}46\text{–}34\text{–}\cdots \\ & & 1\ \ 1\ \ 1\ \ \cdots \end{array}$$

   (this is just an example of the form). Write each edge in the list with its digits in increasing order. For example, write 27, not 72.



2. (a) Give an unambiguous and complete description of Prim's algorithm. 2pts

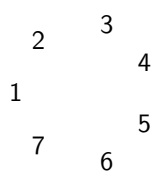
   (b) Given the *complete graph* on 7 vertices, labeled $1, 2, \ldots, 7$, let the weight of the edge between vertices i and j be the non-negative difference between the i-th and j-th digits in the *reverse* of your Registration number. For example, if your Registration number is 201456749 then the reverse is 947654102, so the edge between vertices 1 and 4 has weight $|9-6|=3$ and for vertices 2 and 4 the weight is $|4-6|=2$. 6pts

   A complete graph has an edge between each pair of vertices

   Find a minimum weight spanning tree for this graph, *using Prim's algorithm, starting from vertex 1*. In your solution, write the reverse of your Registration number, the tot al weight of the spanning tree, and then a list of the edges in the spanning tree *in the order in which they were chosen* and with the weight of each edge written below it. For example, the solution might look like this:

   $$\begin{array}{lll} 947654102, & \text{weight} = 8, & \begin{array}{cccccc} 13 & 34 & 45 & 56 & 26 & 27 \\ 2 & 1 & 1 & 1 & 0 & 3 \end{array} \end{array}$$

   Finally, draw the spanning tree, with vertices positioned as in the figure in the margin.



BONUS PROBLEM

3. Must two MSTs for the same graph have the same collection of edge weights?

   Note: To get credit for a Bonus problem the solution must be very clear and convincing. You are welcome to discuss these with me, but you may not use any external resources to aid you, only your own thinking. Handing in a solution to a Bonus problem is a declaration that you have adhered to this. Credit for Bonus problems will be added to your mark, although not beyond 100% for the assignments in this part of the course.