Chapter 12: Financial Volatility

Chapters 8-11 developed several different regression models for time series variables.

For instance: explaining stock or bond returns, exchange rates and yield spreads.

In finance, often interested in the volatility (i.e. variability/variance) of asset prices.

Volatility relates to risk and is important in portfolio management , Capital Asset Pricing Model (CAPM), financial derivatives (e.g. Black-Scholes option price formula), etc etc.

In this chapter, we discuss models and methods for estimating the volatility of a series.

Autoregressive conditional heteroskedasticity (ARCH)

Generalized autoregressive conditional heteroskedasticity (GARCH)

Volatility in Asset Prices: Introduction

Recall the random walk model:

$$Y_t = Y_{t-1} + e_t.$$

or

$$\Delta Y_t = e_t.$$

Random walk with drift

$$\Delta Y_t = \alpha + e_t.$$

Stock prices, on average, increase by α per period, but are otherwise unpredictable.

Volatility in Asset Prices: Introduction (continued)

For now, assume random walk model for an asset price is the correct one.

That is, either:

1. asset price follows a random walk

or

2. it follows a random walk with drift, and that we have taken deviations from the mean (i.e. deviations from mean get rid of the drift, see Chapter 4).

Δy_t = variable with deviations from means taken:

$$\Delta y_t = \Delta Y_t - \Delta \overline{Y},$$

where

$$\Delta \overline{Y} = \frac{\sum \Delta Y_t}{T}$$

Volatility in Asset Prices: Introduction (continued)

Thus, even if the asset price is drifting upwards over time, following is sensible:

$$\Delta y_t = e_t.$$

Note: the ARCH model provides a better definition for volatility.

But to provide some intuition note that it is possible to simply use Δy_t^2 as an estimate of volatility at time t.

Using Squared Stock Returns as Measure of Volatility

Why is Δy_t^2 an estimate of volatility at time t?

High volatility is associated with big changes, either in a positive or in a negative direction.

Large rises or large falls in the price of an asset will imply Δy_t^2 is positive and large.

In stable times the asset price will not be changing much and Δy_t^2 will be small.

Hence, Δy_t^2 will be small in stable times and large in chaotic times.

Using Squared Stock Returns as Measure of Volatility

Also, remember variance is like volatility.

From Chapter 2, variance of ΔY is:

$$\frac{\sum \left(\Delta Y_t - \Delta \bar{Y}\right)^2}{N-1}$$

But this is no good for our purposes since this calculates one single variance estimate for all time periods. We want different variance estimate in each time period.

But for one period, this estimate of the variance basically becomes

$$\Delta y_t^2$$

This is informal motivation for why Δy_t^2 is an estimate of the volatility at time t.

Building a Model with this Measure of Volatility

We can use Δy_t^2 as a dependent variable in any regression. But which one?

Autoregressive models are commonly used to model "clustering in volatility", which is often present in financial time series data.

AR(1) model using volatility as dependent variable:

$$\Delta y_t^2 = \alpha + \phi \Delta y_{t-1}^2 + e_t.$$

Volatility at t depends on volatility at t-1

Building a Model with this Measure of Volatility (continued)

If $\phi > 0$

If volatility was unusually high last period, it will also tend to be unusually high this period.

If volatility was unusually low last period then this period's volatility will also tend to be low.

Clustering in volatility

Can be extended to the AR(p) model.

All of the intuition of AR(1) model (see Chapter) 9 is relevant here. But interpretation relates to the volatility of the series rather than to the series itself.

Provided Δy_t^2 is stationary, then OLS estimates and P-values can be interpreted in the standard way.

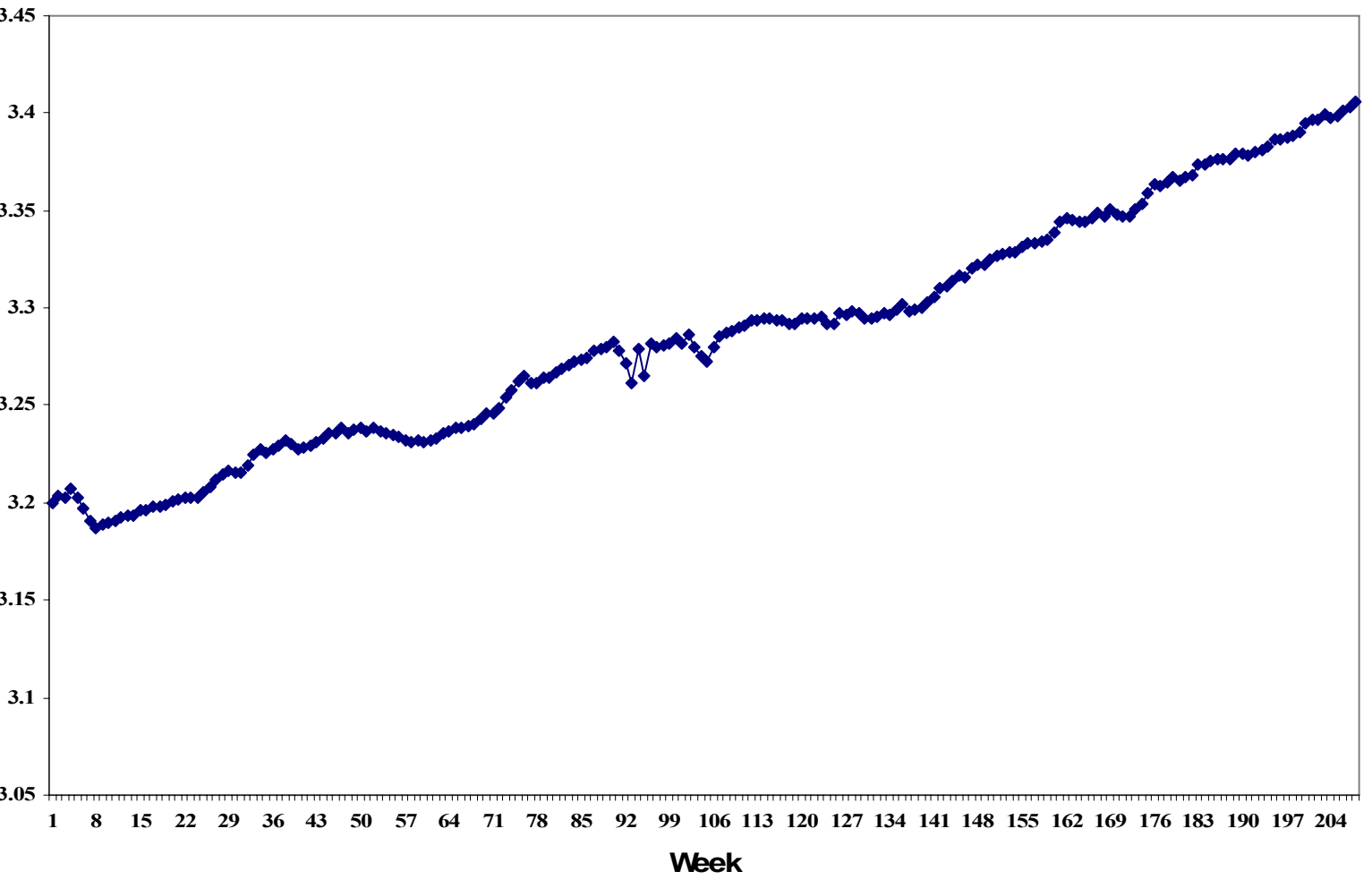**Testing for a unit root in volatility can be conducted using a Dickey-Fuller test.**

Example: Volatility in Stock Prices

Y = the stock price of a company collected each week for four years (i.e. T=208).

The data has been logged.

Figure 12.1 provides a time series plot of this data.

Figure 12.1: Log of Stock Price

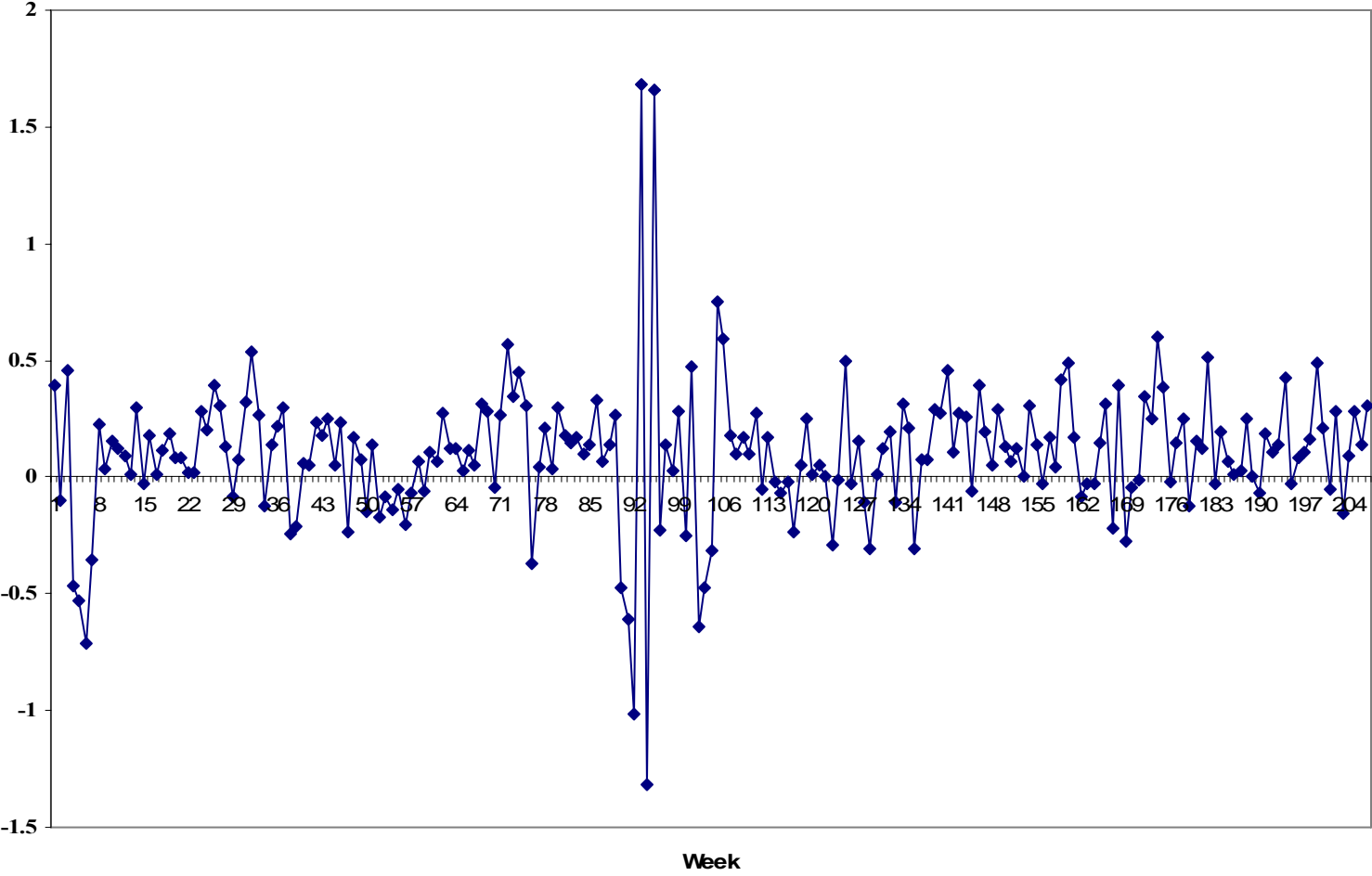# Example: Volatility in Stock Prices (continued)

Figure 12.2 plots ΔY, the percentage change in Y.

Note: $100 \times [\ln(Y_t) - \ln(Y_{t-1})]$ is the percentage change in the stock price.

Remember, this is the stock return (exclusive of dividends)

Figure 12.2: Percentage Change in Stock Price

Example: Volatility in Stock Prices (continued)
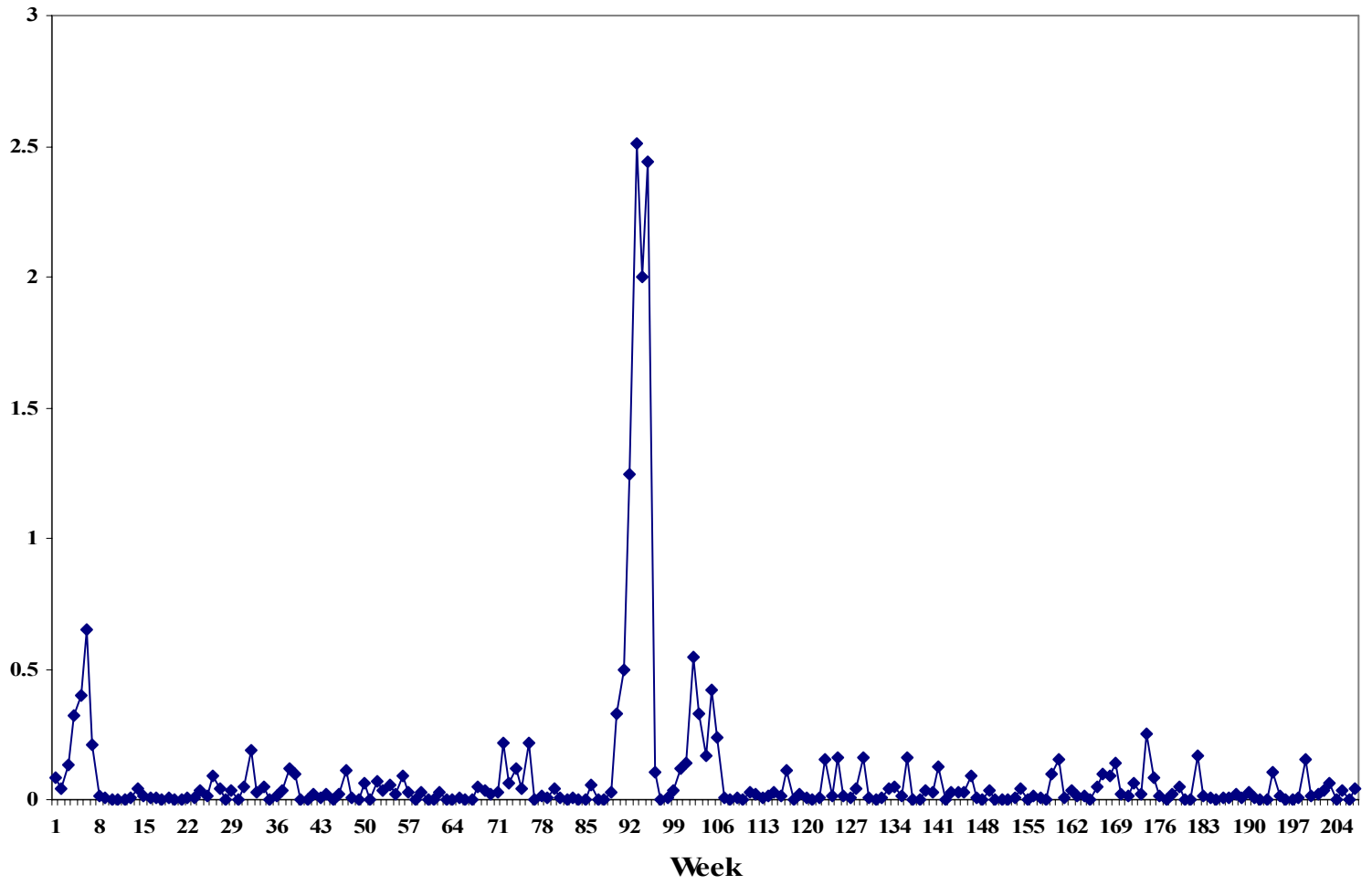
To investigate volatility calculate Δy_t^2

That is

i) calculate the average change in stock price, .099%
ii) subtract this number from every stock price change
iii) square the result.

Figure 12.3 plots the resulting series which is our measure of volatility.

Can see that volatility varies over time

Figure 12.3: Volatility of Stock Price

Example: Volatility in Stock Prices (continued)

Formal evidence on volatility can be found by building an AR(p) model for Δy_t^2

Sequential testing procedure (see Chapter 9) yields the following AR(1) model.

AR(1) Model using Δy_t^2 as Dependent Variable

	Coeff	t Stat	P-value	Lower 95%	Upper 95%
Inter.	.024	1.624	.106	-.005	.053
Δy_{t-1}^2	.737	15.552	1.7E-36	.643	.830

Last week's volatility has strong explanatory power for this week's volatility, since its coefficient is strongly statistically significant.

R^2=.54, so 54% of the variation in volatility can be explained by last period's volatility.

Autoregressive Conditional Heteroskedasticity (ARCH)

ARCH (including extensions) are the most popular ways of modelling financial volatility.

To explain what ARCH is, begin with the multiple regression model:

$$Y_t = \alpha + \beta_1 X_{1t} + \beta_2 X_{2t} + \ldots + \beta_k X_{kt} + e_t.$$

This general model contains many of the other models we have been working with:

If $X_{jt}=Y_{t-j}$ then this is an AR model.

If there are no explanatory variables at all (i.e. $\alpha=\beta_1=\ldots=\beta_k=0$) and $\Delta y_t = \Delta Y_t - \Delta \overline{Y}$, then have model of financial volatility analogous to that used in the first half of this chapter.

ARCH (continued)

ARCH model relates to the variance (or volatility) of e_t.

To adopt a very common notation in financial econometrics) let:

$$\sigma_t^2 = \text{var}(e_t).$$

σ_t^2 is our notation for volatility (crucial in many financial applications)

ARCH model with p lags (ARCH(p)):

$$\sigma_t^2 = \gamma_0 + \gamma_1 e_{t-1}^2 + .. + \gamma_p e_{t-p}^2$$

where $\gamma_1, ..,\gamma_p$ are coefficients that can be estimated in many statistical software packages.

ARCH (continued)

Note: If no explanatory variables and the dependent variable is Δy_t, we have

$$\sigma_t^2 = \gamma_0 + \gamma_1 \Delta y_{t-1}^2 + .. + \gamma_p \Delta y_{t-p}^2 ,$$

and ARCH volatility depends on recent values of Δy_t^2 – just like in first half of chapter.

Closely related to the autoregressive model (which accounts for the "AR" part of the name ARCH)

ARCH models have similar properties to AR models – except that these properties relate to the volatility of the series.

Example: Volatility in Stock Prices (continued)

Note: with ARCH models, don't worry about subtracting the mean off of stock returns as in the first half of the chapter (by including an intercept in regression we are allowing for a random walk with drift).

Take our logged stock price data and take the first difference to create ΔY_t .

Estimate ARCH(1) model with ΔY_t as the dependent variable and an intercept in the regression equation.

Example: Volatility in Stock Prices (continued)

Computer software packages produce tables similar to following.

ARCH(1) Model using Stock Returns Data

	Coeff.	P-value	Lower 95%	Upper 95%
ΔY_t				
Intercept	.105	.000	.081	.129
ARCH				
Lag 1	.660	.000	.302	1.018
Intercept	.024	.000	.016	.032

Example: Volatility in Stock Prices (continued)

How to interpret this table?

Upper part of this table refers to the coefficients in the regression equation.

In this case, we have only included an intercept (labeled α in the regression equation).

The lower part of the table refers to the ARCH equation.

Since we are working with an ARCH(1) model, the equation includes an intercept (labeled γ_0 in the ARCH equation) and one lag of the errors squares (labeled γ_1 in the ARCH equation and "Lag 1" in the table).

Example: Volatility in Stock Prices (continued)

Numbers can be read in the same manner as in the tables in earlier regression chapters.

Numbers in the column labeled "Coeff" are estimates of the coefficients

Note: not OLS estimates, but rather sophisticated estimates for ARCH models

Numbers in the columns labeled "P-value" are P-values for testing the hypothesis that the corresponding coefficient equals zero.

Since the P-values are all less than .05, we can conclude all the variables are statistically significant at the 5% level.

Example: Volatility in Stock Prices (continued)

Final two columns are lower and upper bounds for a 95% confidence interval.

Estimate of γ_1 is .66:

volatility this month depends strongly on the errors squared last month.

Example: Volatility in Stock Prices (continued)

Lag length selection in ARCH models done in the same manner as with any time series model.

You can use an information criterion (see Chapter 11) to select a model.

Or look at P-values for whether coefficients equal zero (and, if they do seem to be zero, then variables can be dropped).

E.g. ARCH(2) model

ARCH(2) Model using Stock Returns Data

	Coeff	P-value	Lower 95%	Upper 95%
ΔY_t				
Intercept	.109	.000	.087	.131
ARCH				
Lag 1	.717	.000	.328	1.107
Lag 2	-.043	.487	-.165	.079
Intercept	.025	.000	.016	.033

Example: Volatility in Stock Prices (continued)

Note: coefficient on "Lag 2" (i.e. γ_2) is not significant (P-value is greater than .05).

Thus, ARCH(1) model is adequate.

Note: For many purposes (e.g. pricing financial derivatives), an estimate of σ_t^2 is required for every time period. This is provided by software package.

Extensions of the ARCH model

E.g. Stata lists seven different variants of the ARCH model with acronyms like GARCH, SAARCH, TARCH, AARCH, NARCH and NARCHK.

Another popular model stochastic volatility.

Here we will discuss the most popular of these extensions: Generalized ARCH or GARCH.

GARCH also includes lags of the volatility measure itself (instead of just adding lags of squared errors).

GARCH

GARCH(p,q) and has volatility equation:

$$\sigma_t^2 = \gamma_0 + \gamma_1 e_{t-1}^2 + .. + \gamma_p e_{t-p}^2 + \lambda_1 \sigma_{t-1}^2 + .. + \lambda_q \sigma_{t-q}^2.$$

Properties of GARCH model similar to ARCH.

But GARCH model is much more flexible, much more capable of matching a wide variety of patterns of financial volatility.

Example: Volatility in Stock Prices (continued)

GARCH(1,1) Model using Stock Returns Data

	Coeff	P-value	Lower 95%	Upper 95%
ΔY_t				
Intercept	.109	.000	.087	.131
ARCH				
ARCH Lag 1	.714	.000	.327	1.101
GARCH Lag 1	-.063	.457	-.231	.104
Intercept	.026	.000	.015	.038

Table interpreted in the same manner as for the ARCH tables.

But an extra row labeled "GARCH Lag 1" which contains results for λ_1 (i.e. the lagged volatility).

Lagged volatility is insignificant Thus the extension to a GARCH(1,1) model does not seem necessary.

ARCH(1) model adequate for this data set.

Chapter Summary

1. **Many time series variables, particularly asset prices, seem to exhibit random walk behavior. For this reason, it is hard to predict how they will change in the future. However, such variables often do exhibit predictable patterns of volatility.**

2. **The square of the change in an asset price is a measure of its volatility.**

3. **Standard time series methods can be used to model the patterns of volatility in asset prices. The only difference is that volatility of the asset price is used as the dependent variable.**

4. **ARCH models are a more formal way of measuring volatility. They contain two equations. One is a standard regression equation. The second is a volatility equation, where volatility is defined as being the (time varying) variance of the regression error.**

5. **ARCH models share similarities with AR models, except that the "AR" part relates to the volatility equation.**

6. **There are many extensions of ARCH, of which GARCH is the most popular.**

7. **ARCH and GARCH models can be estimated using many common statistical software packages.**