# Chapter 9: Univariate Time Series Analysis

- **In the last chapter we discussed models with only lags of explanatory variables. These can be misleading if:**

1. **The dependent variable $Y_t$ depends on *lags of the dependent variable* as well, possibly, as $X_t$, $X_{t-1},\ldots,X_{t-q}$.**

2. **The variables are nonstationary.**

- **In this chapter and the next, we develop tools for dealing with both issues and define what we mean by "nonstationary".**

- **To simplify the analysis, focus solely on one time series, Y. Hence, *univariate time series analysis.***

- **It is important to understand the properties of each individual series before proceeding to regression modelling involving several series.**

# Example: Stock Prices on the NYSE

**Figure 9.1: natural logarithm of monthly data from 1952 through 1995 on a major stock price index provided by the New York Stock Exchange**

**Aside on logs**

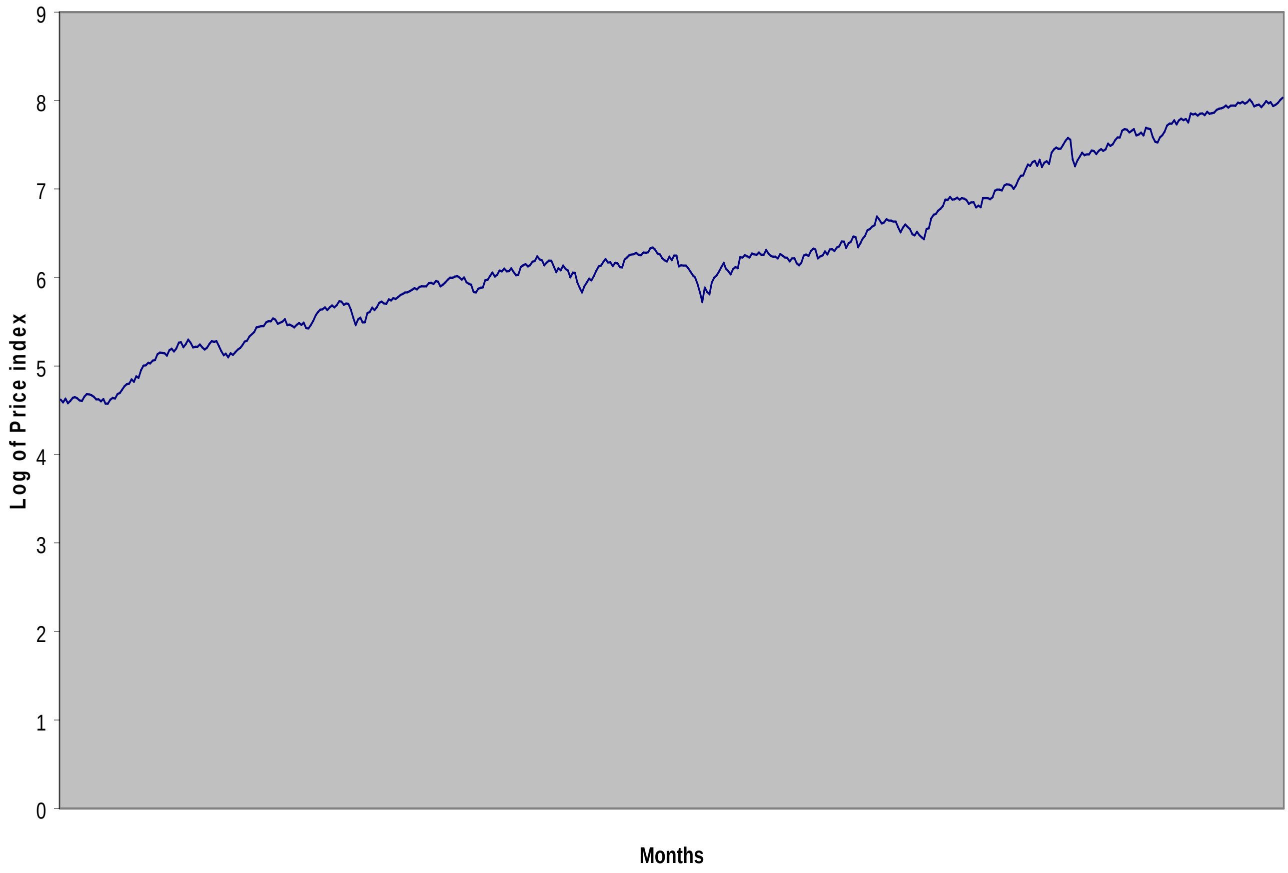
- **It is common to take the natural logarithm of time series which are growing over time (i.e. work with ln(Y) instead of Y). Why?**

- **A time series graph of ln(Y) will often approximate a straight line.**

- **In regressions with logged variables coefficients can be interpreted as elasticities.**

- **$\ln(Y_t)-\ln(Y_{t-1})$ is (approximately) the percentage change in Y between period t-1 and t.**

Figure 9.1: Log of Stock Price Index



# Example: NYSE Data (cont.)

- **Note <u>trend</u> behaviour of personal income series.**

- **Many macroeconomic time series exhibit such trends.**

# Differencing

$$\Delta Y_t = Y_t - Y_{t-1}$$

- **$\Delta Y_t$ measures the change (or growth) in Y between periods t-1 and t.**

- **If $Y_t$ is the log of a variable, then $\Delta Y_t$ is the percentage change.**

- **$\Delta Y_t$ is the difference of Y (or first difference).**

- **$\Delta Y_t$ is often called "delta Y".**

# Example: NYSE Data (cont.)

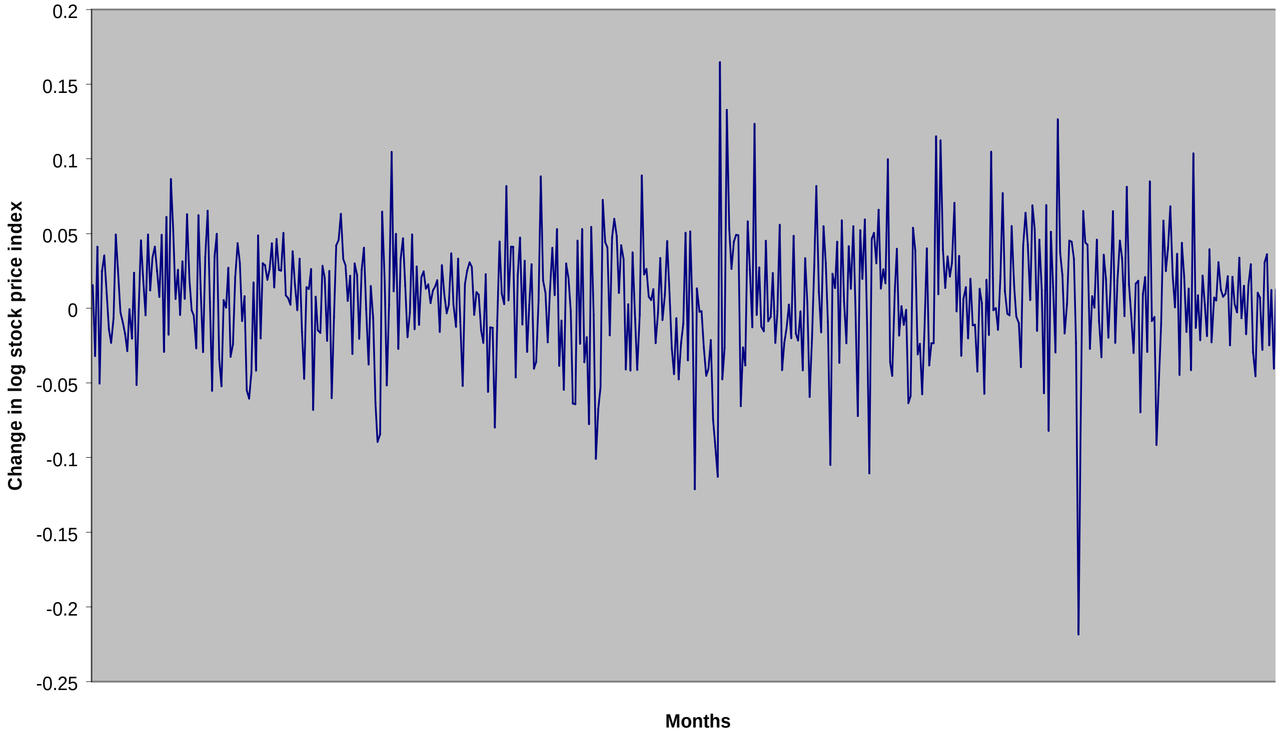
**See Figure 9.2.**

- **$\Delta Y$ = % change in personal income**

- **Not trending, very erratic.**

- **The differences/growth rates/returns of many financial time series have such properties.**

**Figure 9.2: Stock Price Return**

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# The Autocorrelation Function

- **Correlation between Y and lags of itself shed important light of the properties of Y.**

- **Relates to the idea of a trend (discussed above) and nonstationarity (not discussed yet).**

**<u>Example</u>: Y = NYSE stock price**

- **Correlation between $Y_t$ and $Y_{t-1}$ is .999!**

- **Correlation between $\Delta Y_t$ and $\Delta Y_{t-1}$ is .0438.**

- **These are *autocorrelations* (i.e. correlations between a variable and lags of itself).**

# The Autocorrelation Function: Notation

- **$r_1$ = correlation between Y and Y lagged one period.**

- **$r_p$ = correlation between Y and Y lagged p periods.**

- ***Autocorrelation function*** **treats $r_p$ as a function of p.**

# Example: NYSE Data (cont.)

**Autocorrelation functions of Y and ΔY**

| Lag length (p) | Stock Price | Change in Stock Price |
|---|---|---|
| 1 | .9990 | .0438 |
| 2 | .9979 | -.0338 |
| 3 | .9969 | .0066 |
| 4 | .9958 | .0297 |
| 5 | .9947 | .0925 |
| 6 | .9934 | -.0627 |
| 7 | .9923 | -.0451 |
| 8 | .9912 | -.0625 |
| 9 | .9902 | -.0113 |
| 10 | .9893 | -.0187 |
| 11 | .9885 | -.0119 |
| 12 | .9876 | .0308 |

- **Y is highly correlated with lags of itself, but the change in Y is not.**

- **Information could also be presented on bar charts. See Figures 9.3 and 9.4.**

**Figure 9.3: Autocorrelation Function for Stock Prices**

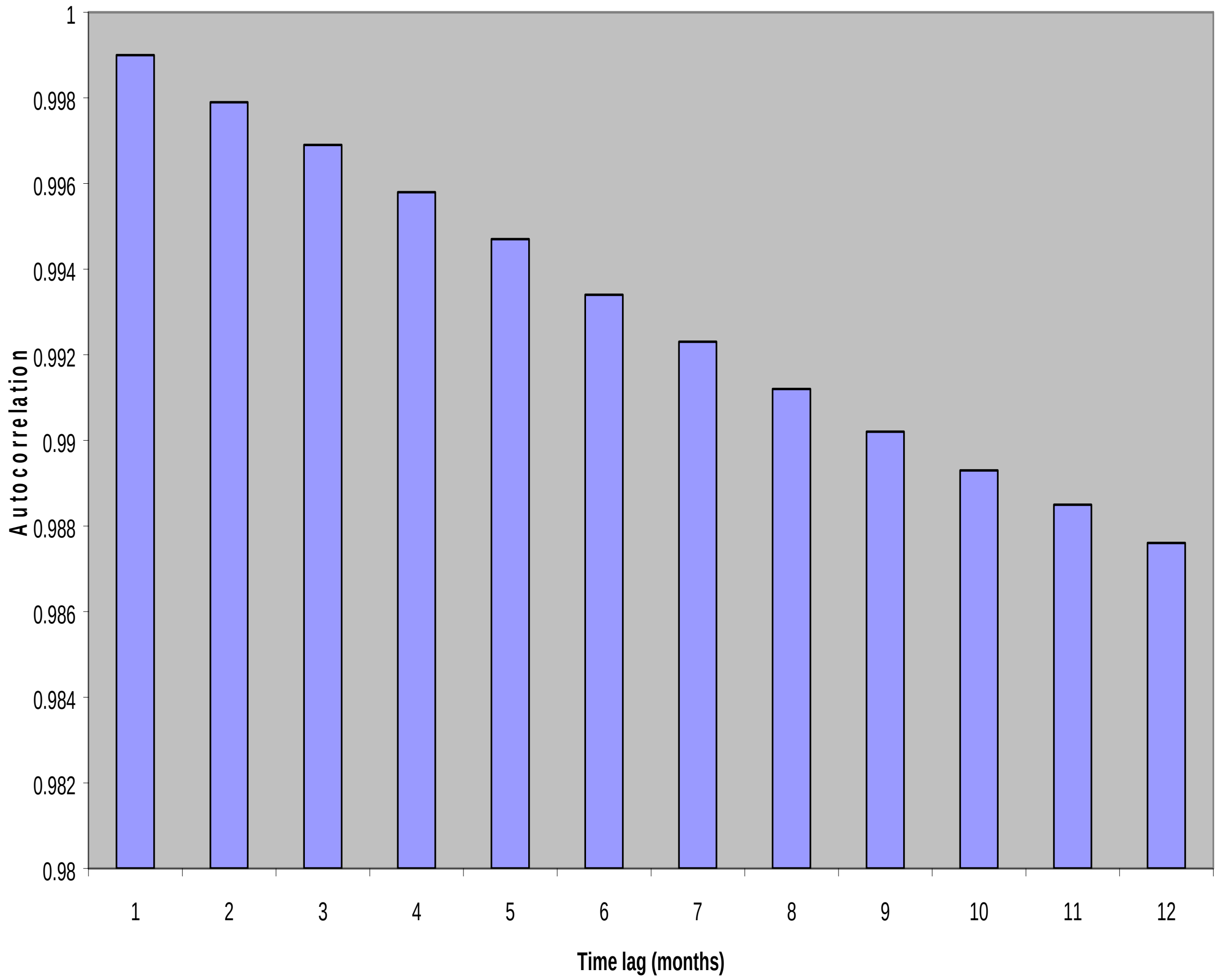
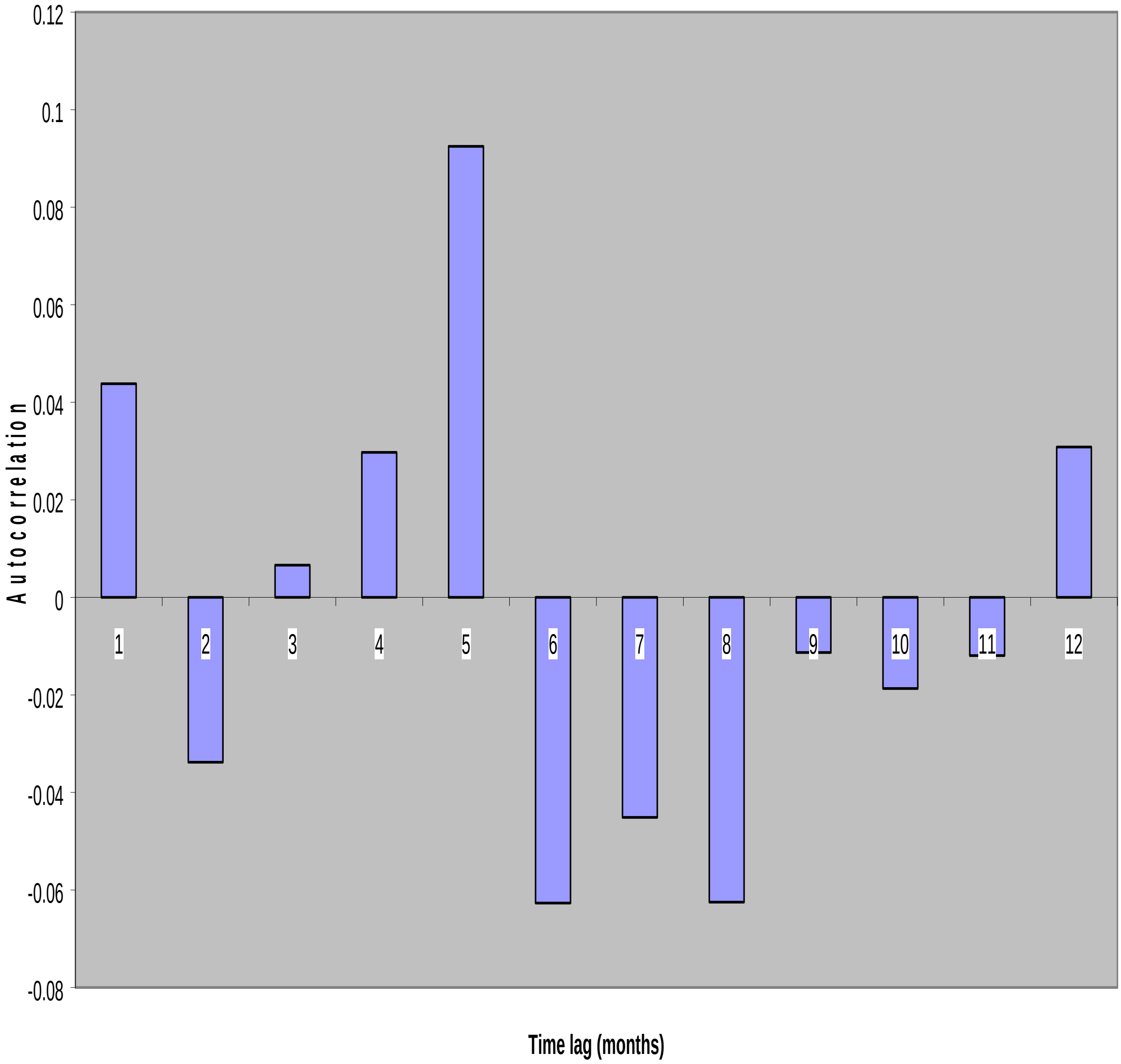


Figure 9.4: Autocorrelation Function for Stock Returns



# Autocorrelation: Intuition

- **Y is highly correlated over time. ΔY does not exhibit this property.**

- **If you knew past values of stock price, you could make a very good estimate of what stock price was this month. However, knowing past values of the change in stock price will not help you predict the change in stock price this month (not change in stock price is return, exclusive of dividends).**

- **Y "remembers the past". ΔY does not.**

- **Y is a nonstationary series while ΔY is stationary. (Note: These words not formally defined yet.)**

# The Autoregressive Model

- **Previous discussion has focussed on graphs and correlations, now we go on to regression.**

- **Autoregressive model of order 1 is written as AR(1) and given by:**

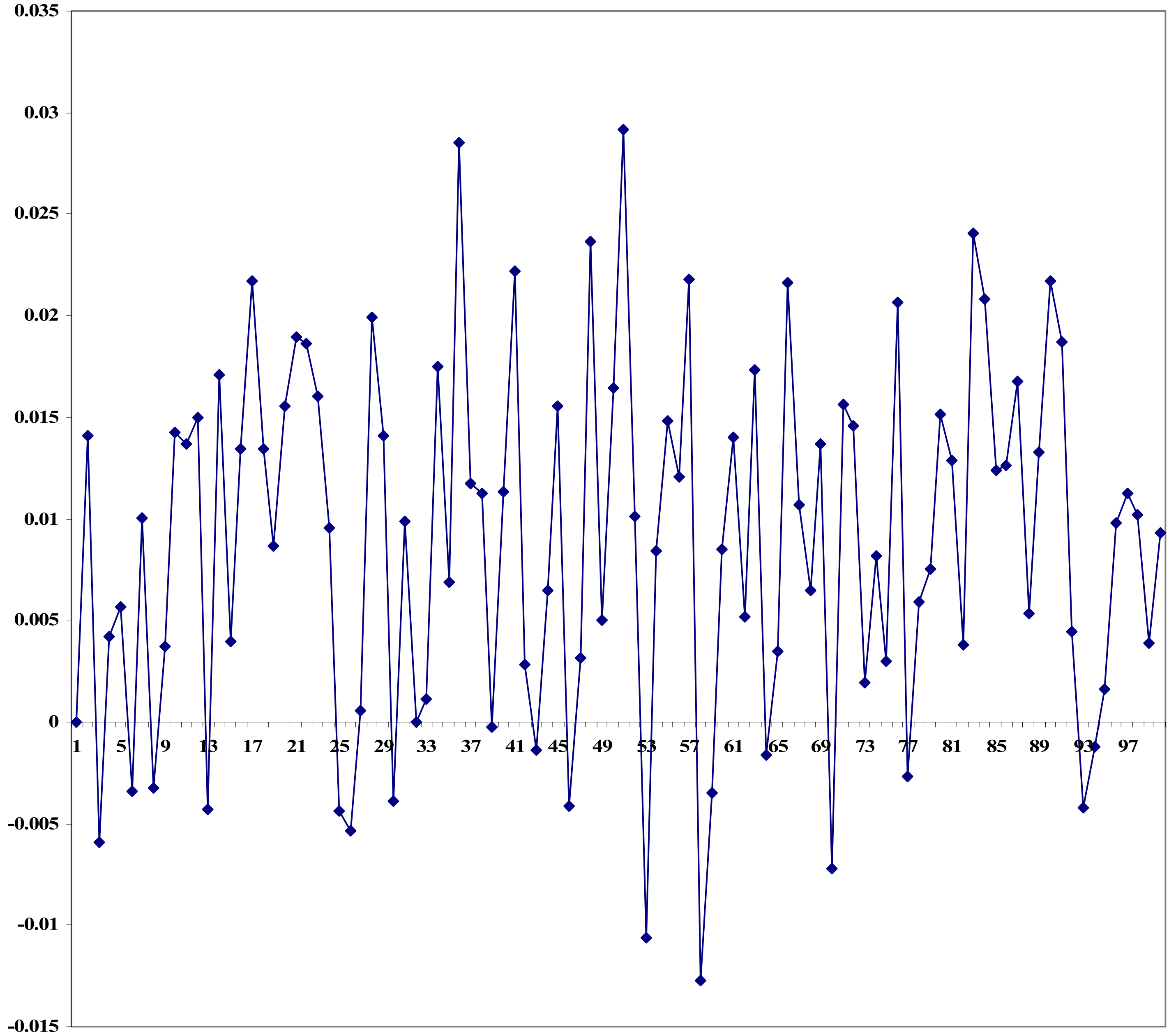
$$Y_t = \alpha + \phi Y_{t-1} + e_t$$

- **Figures 9.5, 9.6 and 9.7 indicate the types of behaviour that this model can generate.**

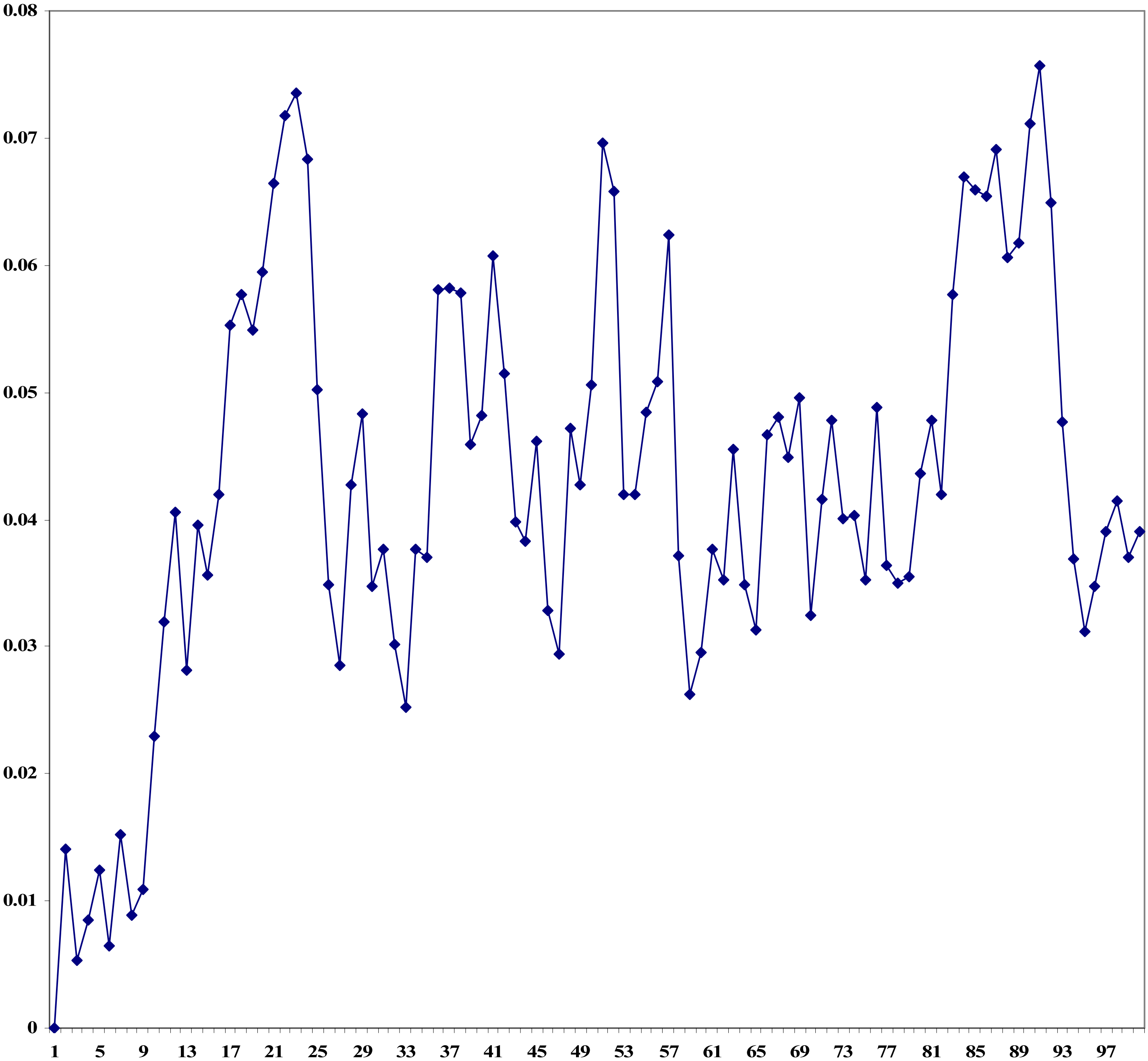
- **$\phi$ = 1 generates trending behaviour typical of financial time series.**

- **$\phi$ = 0 looks more like change in financial time series.**

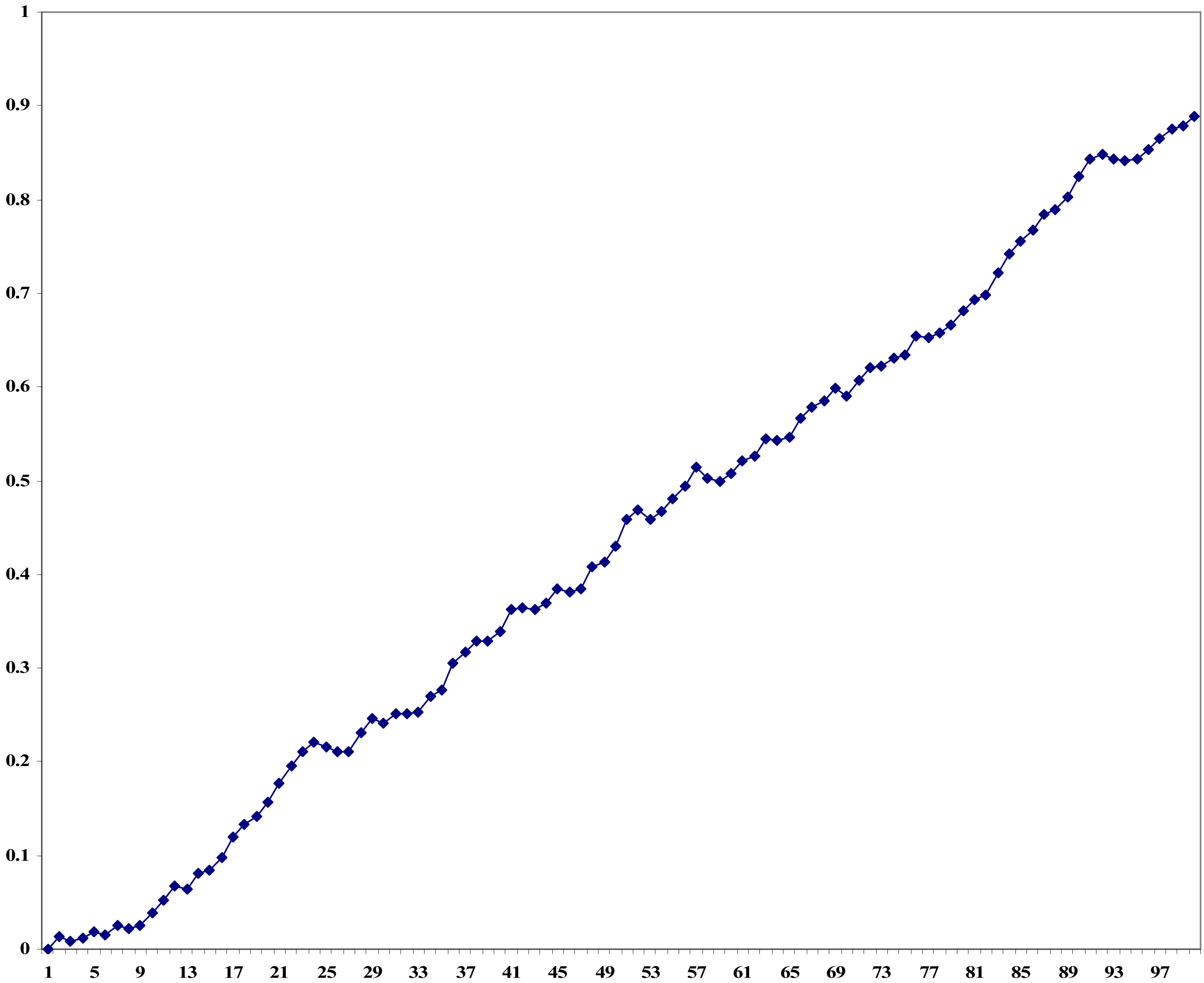
**Figure 9.5: AR(1) Time Series with Phi=0**



# Figure 9.6: AR(1) Time Series with Phi=.8



# Figure 9.7: AR(1) Time Series with Phi=1



# Nonstationary vs. Stationary Time Series

- **Formal definitions require difficult statistical theory. Some intuition will have to suffice.**

- **"Nonstationary" means "anything which is not stationary".**

- **Focus on a case of great empirical relevance: <u>unit root</u> nonstationarity.**

# Ways of Thinking about Whether Y is Stationary or has a Unit Root

1. **If $\phi = 1$, then Y has a unit root. If $|\phi|<1$ then Y is stationary.**

2. **If Y has a unit root then its autocorrelations will be near one and will not drop much as lag length increases.**

3. **If Y has a unit root, then it will have a long memory. Stationary time series do not have long memory.**

4. **If Y has a unit root then the series will exhibit trend behaviour.**

5. **If Y has a unit root, then $\Delta Y$ will be stationary. Hence, series with unit roots are often referred to as *difference stationary*.**

# More on the AR(1) Model

$$Y_t = \alpha + \phi Y_{t-1} + e_t$$

- **Can rewrite as:**

$$\Delta Y_t = \alpha + \rho Y_{t-1} + e_t$$

$$\text{where} \quad \rho = \phi - 1$$

- **If $\phi = 1$ (unit root) then $\rho = 0$ and:**

$$\Delta Y_t = \alpha + e_t$$

- **Intuition: if Y has a unit root, can work with differenced data --- differences are stationary.**

# More on the AR(1) Model

- **Test if $\rho = 0$ to see if a unit root is present.**

- **$-1<\phi<1$ is equivalent to $-2<\rho<0$. This is called the *stationarity condition*.**

**<u>Aside: The Random Walk with drift model</u>:**

$$Y_t=\alpha+Y_{t-1} + e_t$$

- **This is thought to hold for many financial variables such as stock prices, exchange rates.**

- **Intuition: Changes in Y are unpredictable, so no arbitrage opportunities for investors.**

# Extensions of the AR(1) Model

- **AR(p) model:**

$$Y_t = \alpha + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + e_t,$$

- **Properties similar to the AR(1) model.**

- **Alternative way of writing AR(p) model:**

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + e_t.$$

- **Coefficients in this alternative regression ($\rho$, $\gamma_1,\dots,\gamma_{p-1}$) are simple functions of $\phi_1,\dots,\phi_p$.**

# The AR(p) Model

- **AR(p) is in the form of a regression model.**

- **$\rho$=0 implies that the time series Y contains a unit root (and $-2<\rho<0$ indicates stationarity).**

- **If a time series contains a unit root then a regression model involving only $\Delta Y$ is appropriate (i.e. if $\rho = 0$ then the term $Y_{t-1}$ will drop out of the equation).**

- **"If a unit root is present, then you can difference the data to induce stationarity."**

# More Extensions: Adding a Deterministic Trend

- **Consider the following model:**

$$Y_t = \alpha + \delta t + e_t .$$

- **The term $\delta t$ is a *deterministic trend* since it is an exact (i.e. deterministic) function of time.**

- **Unit root series contain a so-called *stochastic trend*.**

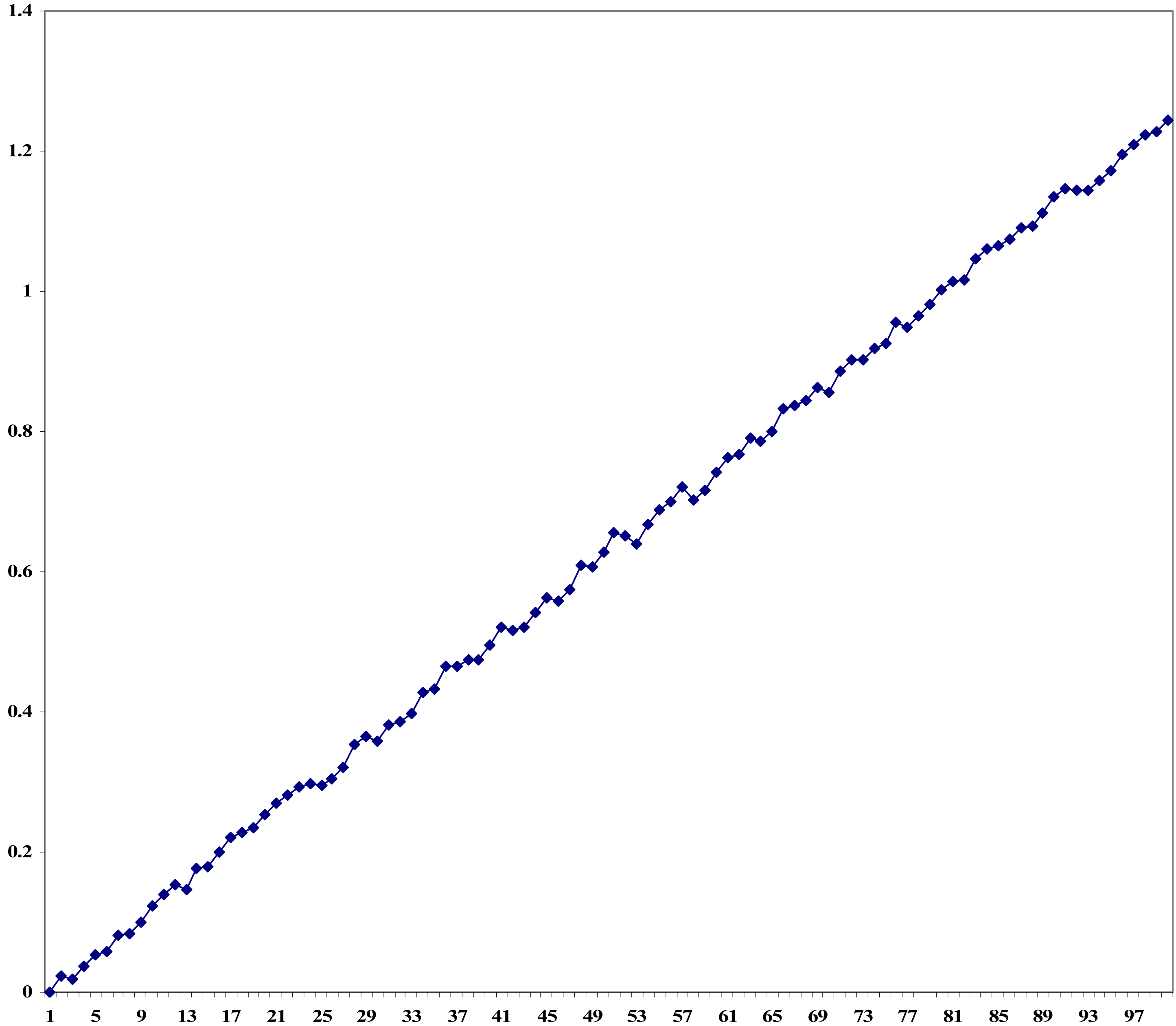
- **Combine with the AR(1) model to obtain:**

$$Y_t = \alpha + \phi Y_{t-1} + \delta t + e_t .$$

- **Can generate behaviour that looks similar to unit root behaviour <u>even if $|\phi|<1$.</u> (i.e. even if they are stationary).**

- **See Figure 9.8.**

# Figure 9.8: Trend Stationary Series



# Summary

- **The nonstationary time series variables on which we focus are those containing a unit root. These series contain a stochastic trend. If we difference these time series, the resulting time series will be stationary. For this reason, they are also called difference stationary.**

- **The stationary time series on which we focus have $-2<\rho<0$. But these series may exhibit trend behaviour through the incorporation of a deterministic trend. If this occurs, they are also called trend stationary.**

# AR(p) with Deterministic Trend Model

- **Most general model we use:**

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \ldots + \gamma_p \Delta Y_{t-p+1} + \delta t + e_t \,.$$

- **Why work with this form of the model?**

1. **A unit root is present if $\rho = 0$. Easy to test.**

2. **The specification is less likely to run into multicollinearity problems. Remember: in finance we often find Y is highly correlated with lags of itself but ΔY is not.**

# Estimation of the AR(p) with Deterministic Trend Model

- **OLS can be done in usual way.**

**<u>Example:</u> Y = NYSE stock price**

- **$\Delta Y$ is the dependent variable in the regression below.**

**<u>AR(4) with Deterministic Trend Model</u>**

| | Coeff | t Stat | P-value | Lower 95% | Upper 95% |
|---|---|---|---|---|---|
| Inter | .082 | 2.074 | .039 | .004 | .161 |
| $Y_{t-1}$ | -.016 | -1.942 | .053 | -.033 | .0002 |
| $\Delta Y_{t-1}$ | .051 | 1.169 | .243 | -.035 | .138 |
| $\Delta Y_{t-2}$ | -.027 | -.623 | .534 | -.114 | .059 |
| $\Delta Y_{t-3}$ | .015 | .344 | .731 | -.071 | .101 |
| time | 1E-4 | 1.979 | .048 | 7E-7 | 0.0002 |

# Testing in AR(p) with Deterministic Trend Model

- **For everything except $\rho$, testing can be done in usual way using t-statistics and P-values.**

- **Hence, can use standard tests to decide whether to include deterministic trend.**

**<u>Lag length selection</u>**

- **A common practice: begin with an AR(p) model and look to see if the last coefficient, $\gamma_p$ is significant. If not, estimate an AR(p-1) model and see if $\gamma_{p-1}$ is significant. If not, estimate an AR(p-2), etc.**

# Example: Y = NYSE Stock Price Data

- **Sequential testing strategy leads us to drop the deterministic trend and go all the way back to a model with one lag, an AR(1).**

- **$\Delta$Y is the dependent variable in the regression.**

| | Coeff | t Stat | P-value | Lower 95% | Upper 95% |
|---|---|---|---|---|---|
| Inter | .00763 | 0.6188 | .5363 | -.0166 | .0318 |
| $Y_{t-1}$ | -.00012 | -0.0631 | .9497 | -.0039 | -.0037 |

**Note that it looks like the coefficient on $Y_{t-1}$ is insignificant – but this coefficient is $\rho$ (so don't throw out $Y_{t-1}$ yet!)**

# Testing for a Unit Root

- **You might think you can test $\rho = 0$ in the same way (i.e. look at P-value and, if it is less than .05, reject the unit root hypothesis, if not accept the unit root).**

- **THIS IS INCORRECT!**

- **Justification: Difficult statistics.**

- **Essentially: The t-statistic correct, but the P-value (and standard error) is wrong.**

- **A correct test is the <u>Dickey-Fuller Test</u>, which uses the t-statistic and compares it to a critical value.**

# Practical Advice on Unit Root Testing

- **Most computer packages will do the unit root test for you and provide a critical value or a P-value for the Dickey-Fuller test**

- **If the t-statistic is less negative than the Dickey-Fuller critical value then accept the unit root hypothesis.**

- **Else reject the unit root and conclude the variable is stationary (or trend stationary if your regression includes a deterministic trend)**

- **Alternatively, if you are using software which does not do the Dickey-Fuller test (e.g. Excel), use the following rough rule of thumb which should be okay if sample size is moderately large (e.g. T>50).**

# Testing for a Unit Root: An Approximate Strategy

1. **Use the sequential testing strategy outlined above to estimate the AR(p) with deterministic trend model. Record the t-stat corresponding to ρ (i.e. the coefficient on $Y_{t-1}$).**

2. **If the final version of your model includes a deterministic trend, the Dickey-Fuller critical value is approximately –3.45. If the t-stat on ρ is more negative than –3.45, reject the unit root hypothesis and conclude that the series is stationary. Otherwise, conclude that the series has a unit root.**

3. **If the final version of your model does not include a deterministic trend, the Dickey-Fuller critical value is approximately –2.89. If the t-stat on ρ is more negative than this, reject the unit root hypothesis and conclude that the series is stationary. Otherwise, conclude that the series has a unit root.**

# Example: Y = NYSE Stock Price Data (continued)

**The final version of the AR(p) model did not include a deterministic trend.**

**The t-stat on $\rho$ is –0.063, which is <u>not</u> more negative than –2.89.**

**Hence we can <u>accept</u> the hypothesis that NYSE stock prices contain a unit root and are, in fact, a random walk.**

# Example: Y = Long term interest rates

**Use preceding strategy on a data set containing long term interest rate data**

**Beginning with pmax=4 and sequentially deleting insignificant lagged variables, we end up with an AR(1) model:**

$$\Delta Y_t = \alpha + \rho Y_{t-1} + e_t \,.$$

**OLS estimation results for this model are given in the following table.**

| | Coeff | t Stat | P-value | Lower 95% | Upper 95% |
|---|---|---|---|---|---|
| Intercept | .039 | 2.682 | .008 | .010 | .067 |
| $Y_{t-1}$ | -.004 | -2.130 | .035 | -.0077 | -.0003 |

**A researcher who did not know about the Dickey-Fuller test would incorrectly say: "Since the P-value for ρ is less than .05, we can conclude that ρ is significant. Thus, the long term interest rate variable does not contain a unit root".**

**The correct researcher says:**

**"The final version of the AR(p) model I used did not include a deterministic trend. Hence, I must use the Dickey-Fuller critical value of –2.89. The t-stat on ρ is –2.13, which is <u>not</u> more negative than –2.89. Hence we can <u>accept</u> the hypothesis that long term interest rates contain a unit root."**

# Chapter Summary

1. **Many financial time series exhibit trend behavior, while their differences do not exhibit such behavior.**

2. **The autocorrelation function is a common tool for summarizing the relationship between a variable and lags of itself.**

3. **Autoregressive models are regression models used for working with time series variables and can be written with $Y_t$ or $\Delta Y_t$ as the dependent variable.**

4. **The distinction between stationary and non-stationary models is a crucial one.**

5. **Series with unit roots are the most common type of non-stationary series considered in financial research.**

6. **If $Y_t$ has a unit root then the AR(p) model with $\Delta Y_t$ as the dependent variable can be estimated using OLS. Standard statistical results hold for all coefficients except the coefficient on $Y_{t-1}$.**

7. **The Dickey-Fuller test is a test for the presence of a unit root. It involves testing whether the coefficient on $Y_{t-1}$ is equal to zero (in the AR(p) model with $\Delta Y_t$ being the dependent variable).**