EC 306: Introductory Econometrics

Class Problem Sheet 1: Understanding What Makes a Good Estimator in Simple Contexts

1. Let Y_i for $i = 1, , , , N$ be a random sample from a probability distribution with mean μ and variance 1 (i.e. $E(Y_i) = \mu$ and $var(Y_i) = 1$ for $i = 1, .., N$). Interest centres on estimating μ and five different possible estimators are considered:

$$\widehat{\mu}_1 = \frac{\sum_{i=1}^{N} Y_i}{N}$$

$$\widehat{\mu}_2 = \frac{Y_1 + Y_N}{2}$$

$$\widehat{\mu}_3 = \frac{1}{2}\left[\frac{\sum_{i=1}^{N_1} Y_i}{N_1} + \frac{\sum_{i=N_1+1}^{N} Y_i}{N - N_1}\right] \text{ where } 1 < N_1 < N$$

$$\widehat{\mu}_4 = \frac{\sum_{i=1}^{N} Y_i}{N - 1}$$

i) Which of these is an unbiased estimator for μ?

ii) The term "asymptotically unbiased" means that an estimator becomes unbiased as N goes to ∞. Which of these is an asymptotically unbiased estimator of μ?

iii) Derive the variance of each of the estimators that you found to be unbiased in part i).

iv) Use the concept of "efficiency" and your results from part iii) to discuss which estimator you think is best.

v) Show that the setup in this question is equivalent to the linear regression model with only an intercept (or equivalently, the simple regression model discussed in lectures with $X_i = 1$ for $i = 1, .., N$) where μ plays the role of the intercept (and the error variance in the regression model is set to 1).

vi) Using part v), what does the Gauss Markov theorem say is the best linear unbiased estimator for μ?

2. The simple linear regression model without intercept was discussed in the lectures and is given by:

$$Y_i = \beta X_i + \varepsilon_i,$$

where X_i is a scalar. Assume the classical assumptions hold.

i) (Review of lecture material). The ordinary least squares estimator is given by:

$$\widehat{\beta} = \frac{\sum X_i Y_i}{\sum X_i^2}.$$

Show that $E\left(\widehat{\beta}\right) = \beta$ (and, thus, that $\widehat{\beta}$ is unbiased) and $var\left(\widehat{\beta}\right) = \frac{\sigma^2}{\sum X_i^2}$.

ii) Consider another estimator for β:

$$\widetilde{\beta} = \frac{\sum Y_i}{\sum X_i}.$$

Is this an unbiased estimator for β?

iii) Calculate $var\left(\widetilde{\beta}\right)$.

iv) Use the concept of "efficiency" and your results from the previous parts of the question to discuss which estimator you think is best.