EC 306: Introductory Econometrics

The best strategy for learning this material is to work through this problem sheet to the best of your ability before the tutorial. Then, in the tutorial, the tutor will go through the problem sheet with you and you can ask him to pay particular attention to any points you have had trouble with.

Class Problem Sheet 2

1. The simple linear regression model without intercept was discussed in the lectures and is given by:

$$Y_i = \beta X_i + \varepsilon_i,$$

where X_i is a scalar. For this question, we will free up one of the classical assumptions to allow for heteroskedasticity. In particular, the classical assumptions hold except we now assume

$$var\left(\varepsilon_i\right) = \sigma^2 \omega_i^2.$$

i) (Review of lecture material) The ordinary least squares estimator is given by:

$$\widehat{\beta} = \frac{\sum X_i Y_i}{\sum X_i^2}.$$

Show that $E\left(\widehat{\beta}\right) = \beta$ and, thus, that the OLS estimator is unbiased. What is $var\left(\widehat{\beta}\right)$?

ii) The generalized least squares estimator for this model is given by:

$$\widehat{\beta}_{GLS} = \frac{\sum \left(\frac{X_i}{\omega_i}\right)\left(\frac{Y_i}{\omega_i}\right)}{\sum \left(\frac{X_i}{\omega_i}\right)^2}$$

Show that $E\left(\widehat{\beta}_{GLS}\right) = \beta$ and, thus, that the OLS estimator is unbiased. What is $var\left(\widehat{\beta}_{GLS}\right)$? Is $var\left(\widehat{\beta}_{GLS}\right) \leq var\left(\widehat{\beta}\right)$?

iii) Assume that you know what $var\left(\varepsilon_i\right)$ is and that $\widehat{\beta}_{GLS}$ is Normally distributed. Derive a 95% confidence interval involving the GLS estimator using your results form part ii).

iv) Assume that you know what $\sigma^2 \omega_i^2$ is and that $\widehat{\beta}$ is Normally distributed. Derive a 95% confidence interval involving the OLS estimator using your results from part i).

v) Now consider two possible 95% confidence intervals for β. They are the (correct) OLS and GLS confidence intervals given in parts iii) and iv). Compare these two intervals. Which one is wider?

2. (Measurement error in the dependent variable). Consider the regression model

$$Y_i = \beta X_i + \varepsilon_i.$$

This regression satisfies the classical assumptions. However, you cannot run this regression since do not observe Y_i, but instead observe:

$$Y_i^* = Y_i + v_i,$$

where v_i is i.i.d. with mean zero, variance σ_ν^2 and is independent of ε_i. Show that OLS is BLUE in the regression of Y^* on X.