EC 306: Introductory Econometrics

The best strategy for learning this material is to work through this problem sheet to the best of your ability before the tutorial. Then, in the tutorial, the tutor will go through the problem sheet with you and you can ask him to pay particular attention to any points you have had trouble with.

Class Problem Sheet 3: Deriving Some Results for Time Series Models

1. Assume that a time series variable, Y_t, follows an autoregressive process:

$$Y_t = \rho Y_{t-1} + \varepsilon_t$$

and ε_t satisfies the classical assumptions (i.e. it is i.i.d. with $E(\varepsilon_t) = 0$, $var(\varepsilon_t) = \sigma^2$ and $cov(\varepsilon_t, \varepsilon_s) = 0$ for $s \neq t$). Assume that $\sigma^2 < \infty$. In addition, you can assume that the time series has been running from period $-\infty$ (but we only observe it for time $t = 1, .., T$).

i) Assuming $-1 < \rho < 1$ work out the $var(Y_t)$ and the autocovariance function (i.e. work out $cov(Y_t, Y_{t-s})$ for $s = 1, 2, 3, ...$) and the autocorrelation function (i.e. work out $corr(Y_t, Y_{t-s})$ for $s = 1, 2, 3, ...$ where $corr$ means correlation).

ii) Discuss what happens to your derivations in part i) when $\rho = 1$. What happens when $\rho > 1$?

iii) The formal definition of a stationary time series is as follows: Y_t is stationary if a) $E(Y_t) = \mu$ for all t (i.e. the time series has a constant mean which is that same at all times), b) $var(Y_t) < \infty$ and c) $cov(Y_t, Y_{t-s}) = \gamma_s$ (i.e. the correlation between two values of the series s periods only depends on s and not on t). In light of your answers to parts i) and ii), under what condition is Y_t stationary?

2. Consider the autoregressive distributed lag (ADL) model:

$$Y_t = \alpha + \phi_1 Y_{t-1} + .. + \phi_p Y_{t-p} + \beta_0 X_t + \beta_1 X_{t-1} + ... + \beta_q X_{t-q} + \varepsilon_t.$$

Show that the ADL can be rewritten as:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + .. + \gamma_{p-1} \Delta Y_{t-p+1} + \theta X_t + \omega_1 \Delta X_t + ... + \omega_q \Delta X_{t-q+1} + \varepsilon_t.$$