**APPENDIX TO
THE CONTRIBUTION OF STRUCTURAL BREAK MODELS TO
FORECASTING MACROECONOMIC SERIES**

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This document contains four appendices to the paper of Bauwens, Koop, Korobilis, and Rombouts (2011). The first three appendices contain details about the implementation of the estimation and forecasting of the structural break models named PPT and KP in the paper. These models are explained in Section 2 of the paper and information about the forecasting implementation of these models is presented in Section 4 of the paper. The fourth appendix contains tables that show detailed results that are summarized and discussed in Section 5 of the paper.

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Appendix A: Technical Details for PPT Approach

In this appendix, we describe posterior and predictive simulation as well as prior elicitation for our implementation of the PPT approach. Bauwens and Rombouts (2010) provide more details for posterior simulation and computing the marginal likelihood, which is used for choosing the number of breaks.

The model is defined by $y_t = Z_t\beta_{s_t} + \sigma_{s_t}\varepsilon_t$ and by the break process which involves $S^T = (s_1, \dots, s_T)'$ where $s_t \in \{1, 2, \dots, K\}$ is a state variable and K is the number of in sample regimes (see Section 2 of the paper). Notice that the last regime is an absorbing state over the sample period, but PPT relax this in the forecast period.

Priors

We use priors of the form:

$$\begin{aligned}\beta_j | \beta_0, B_0 &\sim N_m(\beta_0, B_0), \\ \beta_0 &\sim N_m(\underline{\mu}_\beta, \underline{V}_\beta), \\ B_0^{-1} &\sim Wishart(\underline{\xi}, \underline{B}), \\ \sigma_j^{-2} | v_0, d_0 &\sim Gamma(v_0, d_0), \\ v_0 &\sim Gamma(\underline{\lambda}, \underline{\rho}), \\ d_0 &\sim Gamma(\underline{c}, \underline{d}) \\ p_i &\sim Beta(\underline{a}, \underline{b}).\end{aligned}$$

In particular, in the forecasting exercise we set $\underline{\mu}_\beta = 0$, $\underline{V}_\beta = I_m$, $\underline{B} = 10I_m$, $\underline{\xi} = m + 1$ (where m is the dimension of Z_t), $\underline{\lambda} = 1$, $\underline{\rho} = 0.1$, $\underline{c} = 1$, $\underline{d} = 0.1$, and $\underline{a} = \underline{b} = 1$. This implies that all priors are proper but little informative.

Posterior simulator

The posterior simulation algorithm is a Gibbs sampler. Given initial conditions, the data, and in each block the other parameters, the sampling is done as follows:

1. Draw S^T using Chib's (1998) algorithm.
2. Draw p_i from $Beta(\underline{a} + T_i, \underline{b} + 1)$ for $i = 1, \dots, K$, where T_i is the number of observations in regime i .
3. Draw $\beta_i | \sigma_i^2$ from Normal and $\sigma_i^2 | \beta_i$ from Gamma, for $i = 1, 2, \dots, K$.
4. Draw $\beta_0 | B_0$ from Normal and $B_0^{-1} | \beta_0$ from Wishart.
5. Draw $d_0 | v_0$ from Gamma and $v_0 | d_0$ by numerical evaluation and inversion of its cdf.

Appendix B: Technical Details for KP Approach

In this appendix, we describe posterior and predictive simulation as well as prior elicitation for our implementation of the KP approach.

It is convenient to write the model equation as $y_t = Z_t\beta_{s_t} + \exp(\omega_{s_t}/2)\varepsilon_t$. The transition probabilities between the states are defined in equation (5) of the paper so that the last diagonal element of the transition matrix is equal to p_K rather than one as in the PPT approach.

Priors

We use priors of the form:

$$\begin{aligned}\beta_j &\sim N_m(\beta_{j-1}, B_0) \\ \omega_j &\sim N(\omega_{j-1}, \delta) \\ \beta_0 &\sim N_m(0, \underline{V}_\beta) \\ \omega_0 &\sim N(0, \underline{V}_\omega) \\ B_0^{-1} &\sim \text{Wishart}(\underline{\xi}, \underline{B}) \\ \delta^{-1} &\sim \text{Gamma}(\kappa_1, \kappa_2) \\ p_{i,i} &\sim \text{Beta}(\underline{a}, \underline{b}).\end{aligned}$$

In particular, in the forecasting exercise we set $\underline{V}_\beta = I_m$, $\underline{V}_\omega = 1$, $\underline{B} = 10I_m$, $\underline{\xi} = m + 1$, $\kappa_1 = \kappa_2 = 0.5$, and $\underline{a} = \underline{b} = 1$. This implies that all priors are proper but very little informative.

Posterior simulator

The posterior simulation algorithm is a Gibbs sampler. Given initial conditions, the data, and in each block the other parameters, the sampling is done as follows:

1. Draw S^T using Chib's (1998) algorithm.
2. Draw p_i from $\text{Beta}(\underline{a} + T_i, \underline{b} + 1)$ for $i = 1, \dots, K$, where T_i is the number of observations in regime i .
3. Draw $[\beta_{s_t}]_{t=1}^T$ using the modified Kalman filter algorithm (see below).
4. Draw $[\omega_{s_t}]_{t=1}^T$ using the modified Kalman filter algorithm, after writing the model in appropriate linear state space form using the Kim, Shephard and Chib (1998) algorithm.
5. Draw B_0^{-1} and δ^{-1} , conditional on the draws of β_t and ω_t , using standard expressions.

Modified Kalman filter algorithm

Consider a state-space model of the following form:

$$y_t = z_t a_{s_t} + \varepsilon_t \tag{1a}$$

$$a_j = a_{j-1} + \eta_{s_t} \tag{1b}$$

$$\varepsilon_t \sim N(0, \gamma_1^2), \eta_j \sim N(0, \gamma_2^2)$$

conditional on knowing s_t , where (1a) is the measurement equation and (1b) is the state equation, with observed data y_t and unobserved state a_{s_t} . If the errors ϵ_t, η_t are *iid* and uncorrelated with each other, we can use the Kalman filter to estimate the state a .

Let $a_{t|s}$ denote the expected value of a_t and $P_{t|s}$ its corresponding variance, using data up to time s . Given starting values $a_{0|0}$ and $P_{0|0}$, the Kalman filter recursions provide us with initial filtered estimates:

$$\begin{aligned} a_{t|t-1} &= a_{t-1|t-1} \\ P_{t|t-1} &= \begin{cases} P_{t-1|t-1} + \gamma_2^2 & , \text{ if } s_{t-1} \neq s_t \\ P_{t-1|t-1} & , \text{ otherwise} \end{cases} \end{aligned} \quad (2)$$

$$\begin{aligned} K_t &= P_{t|t-1} z_t' (z_t P_{t|t-1} z_t + \gamma_1^2)^{-1} \\ a_{t|t} &= a_{t|t-1} + K_t (y_t - z_t a_{t|t-1}) \\ P_{t|t} &= P_{t|t-1} - K_t z_t P_{t|t-1}. \end{aligned} \quad (3)$$

Once we reach the last period ($t = T$) we take the standard draw $a_{s_T} \sim N(a_{T|T}, P_{T|T})$. If $s_T = T$ then a break occurs in each observation and we have a full tvp model, so that the Carter and Kohn smoother applies to all observations t . However with structural breaks models it will be the case that $s_T \ll T$ (i.e. the number of breaks is smaller than the number of observations, i.e. we do not have a full tvp model), we can only simulate a_j for $j = s_T + 1, \dots, T$ (i.e. the “out-of-sample breaks” in a) using equation (1b). For $j = 1, \dots, s_T$ we can use a standard smoother to get smoothed estimates. To do that, we run the backward recursions for $t = T - 1, \dots, 1$:

$$\begin{aligned} a_{t|t+1} &= a_{t|t} + P_{t|t} P'_{t+1|t} (a_{t+1} - a_{t|t}), \text{ iff } s_{t+1} \neq s_t \\ P_{t|t+1} &= P_{t|t} - P_{t|t} P'_{t+1|t} P_{t+1|t}, \text{ iff } s_{t+1} \neq s_t \end{aligned}$$

and draw $a_{s_t} \sim N(a_{t|t+1}, P_{t|t+1})$ iff $s_{t+1} \neq s_t$.

Appendix C: Predictive Simulator for PPT and KP models

Forecasting with no breaks out-of-sample (PPT model)

Since the PPT model implies that observations following T (the last sample date) are generated from $y_{T+h}|Y_{T+h-1}, \theta_K$ where $\theta_K = (\beta_K, \sigma_K^2)$, i.e. under the last operating regime, we can compute predictive densities as follows:

$$p(y_{T+h}|s_{T+h} = K, s_T = K, Y_T) = \int \dots \int \prod_{j=0}^{h-1} p(y_{T+h-j}|Y_{T+h-1-j}, \theta_K) p(\theta_K|\theta_0, S_T, Y_T) p(\theta_0|S_T, Y_T, \underline{A}) p(S_T|Y_T) dy_{T+h-1} \dots dy_{T+1} d\theta_K d\theta_0 dS_{T-1}, \quad (4)$$

where the integration is done with respect to S_{T-1} rather than S_T since $s_T = K$. This is implemented by simulation within the Gibbs sampler for the posterior density: for each Gibbs draw of θ_K , θ_0 and S_{T-1} , we generate sequentially future values $y_{T+1}, \dots, y_{T+h}$, each from $y_{T+h-j} \sim p(y_{T+h-j}|Y_{T+h-1-j}, \theta_K)$, and we keep y_{T+h} as a draw of the corresponding predictive density $p(y_{T+h}|s_{T+h} = K, s_T = K, Y_T)$. Doing this for e.g. $h = 4$ provides also the draws of the predictive densities for $h \leq 4$.

Forecasting with breaks out-of-sample (PPT & KP models)

The previous discussion does not allow for a break to occur in the forecast period. In order to allow in the PPT for the possibility of occurrence of one new regime after T , we lift the restriction $p_K = 1$ (something already done in the KP model) and extend the transition matrix to

$$\begin{pmatrix} p_1 & 1-p_1 & 0 & \dots & 0 & 0 & 0 \\ 0 & p_2 & 1-p_2 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & p_{K-1} & 1-p_{K-1} & 0 \\ 0 & 0 & 0 & \dots & 0 & p_K & 1-p_K \\ 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}.$$

Additional regimes can be added by extending further the transition matrix, but here we consider the predictive density subject to one break occurring after date T . Assume that the break occurs at date $T+d$ where d can take any value in the set $\{1, 2, \dots, h\}$. For the predictive simulation of y_{T+h} with $h < d$ (the no-post-sample break case), we proceed as above. For $h \geq d$, the break occurrence implies that $y_{T+h} \sim p(y_{T+h}|Y_{T+h-1}, \theta_{K+1})$ where θ_{K+1} is the parameter characterizing the new regime, and is drawn from its hierarchical prior density $p(\theta_{K+1}|\theta_0)$. The observed sample does not provide information about θ_{K+1} and thus does not directly update this prior, but it does so indirectly by updating the prior information about θ_0 since this is drawn from its posterior distribution in the Gibbs sampler.

Assume first that $h = d = 1$. Then, given θ_0 (drawn in the Gibbs sampler), θ_{K+1} is drawn from $p(\theta_{K+1}|\theta_0)$ and given this draw, y_{T+1} is drawn from $p(y_{T+1}|Y_T, \theta_{K+1})$. This procedure is repeated at each iteration of the Gibbs sampler and delivers a sample of draws from the predictive density $p(y_{T+1}|s_{T+1} = K+1, s_T = K, Y_T)$.

Next assume that $h = 2$ and $d = 1$: y_{T+1} is simulated as explained just above, and y_{T+2} is drawn from $p(y_{T+2}|y_{T+1}, Y_T, \theta_{K+1})$ where y_{T+1} is set at its simulated value and θ_{K+1} is maintained to be the value used for drawing this y_{T+1} . For h larger than 2, one proceeds sequentially in the same way, i.e. freezing θ_{K+1} and using the simulated lagged values y_{T+h-j} ($j = 1, 2, \dots, h-1$) in the conditioning of $p(y_{T+h}|Y_{T+h-1}, \theta_{K+1})$.

Finally if $h \geq d \geq 1$, the values y_{T+j} for $j = 1, 2, \dots, d-1$ are sequentially simulated as in the no-post-sample break case. Then for $j = d$, θ_{K+1} is drawn from $p(\theta_{K+1}|\theta_0)$ and given this draw, y_{T+j} for $j = d, d+1, \dots, h$ are drawn sequentially. The next formula validates this simulation procedure for known break date $|\tau_K$ equal to $T+d$:

$$\begin{aligned} p(y_{T+h}|\tau_K = T+d, s_{T+h} = K+1, s_T = K, Y_T) = \\ \int \dots \int \prod_{j=0}^{h-1} p(y_{T+h-j}|Y_{T+h-1-j}, \theta_{K+1}1_{\{h \geq d\}} + \theta_K 1_{\{h < d\}}) \\ p(\theta_{K+1}|\theta_0, S_T, Y_T) p(\theta_K|\theta_0, S_T, Y_T) p(\theta_0|S_T, Y_T, \underline{A}) p(S_T|Y_T) \\ dy_{T+h-1} \dots dy_{T+1} d\theta_{K+1} d\theta_K d\theta_0 dS_{T-1} \end{aligned} \quad (5)$$

where $1_{\{h \geq d\}}$ is equal to 1 if $h \geq d$ and 0 otherwise, and $1_{\{h < d\}} = 1 - 1_{\{h \geq d\}}$. To marginalize this density with respect to the break date d , we sum over all values of d as follows:

$$\begin{aligned} \sum_{d=1}^h p(y_{T+h}|\tau_K = T+d, s_{T+h} = K+1, s_T = K, Y_T) \\ \times \Pr[\tau_K = T+d|s_{T+h} = K+1, s_T = K, Y_T] \end{aligned} \quad (6)$$

with $\Pr[\tau_K = T+d|s_{T+h} = K+1, s_T = K, Y_T] = p_K^{d-1}(1-p_K)/(1-p_K^h)$. Finally, we can integrate $p(y_{T+h}|s_{T+h}, s_T = K, Y_T)$ with respect to the number of post-sample breaks (0 or 1): $p(y_{T+h}|s_T = K, Y_T) =$

$$\begin{aligned} p(y_{T+h}|s_{T+h} = K, s_T = K, Y_T) p(s_{T+h} = K|s_T = K, Y_T) \\ p(y_{T+h}|s_{T+h} = K+1, s_T = K, Y_T) [1 - p(s_{T+h} = K|s_T = K, Y_T)] \end{aligned} \quad (7)$$

where $p(s_{T+h} = K|s_T = K, Y_T) = p_{KK}^h$. This is simulated by drawing s_{T+h} from its discrete distribution, and then y_{T+h} from (4) if $s_{T+h} = K$ and from (6) if $s_{T+h} = K+1$. To sample the discrete distribution, we need a value of p_K . This is simulated in the Gibbs sampler from its full conditional posterior density, which is $\text{Beta}(\underline{a} + T_K, \underline{b} + 1)$, where T_K is the number of observations in regime K according to the sampled S_T vector.

As an example, to implement the simulation of y_{T+1} , we substitute (4), (6) and (5) in (7) and obtain $p(y_{T+1}|s_T = K, Y_T) =$

$$\begin{aligned} p_K \int \dots \int p(y_{T+1}|Y_T, \theta_K) p(\theta_K|\theta_0, S_T, Y_T) p(\theta_0|S_T, Y_T, \underline{A}) p(S_T|Y_T) \\ d\theta_K d\theta_0 dS_{T-1} \\ + (1-p_K) \int \dots \int p(y_{T+1}|Y_T, \theta_{K+1}) p(\theta_{K+1}|\theta_0, S_T, Y_T) p(\theta_K|\theta_0, S_T, Y_T) \\ p(\theta_0|S_T, Y_T, \underline{A}) p(S_T|Y_T) d\theta_{K+1} d\theta_K d\theta_0 dS_{T-1}. \end{aligned}$$

This formula shows that the simulation for one predictive draw in the KP model, and the PPT model with the possibility of breaks occurring out-of-sample, is performed as follows:

1. Draw S_T , θ_0 and θ_K from the posterior (i.e. use a draw of the Gibbs sampler once it has converged).
2. Draw $p_K \sim \text{Beta}(\underline{a} + T_K, \underline{b} + 1)$.
3. Draw $s_{T+1} = K$ or $K+1$ with respective probabilities $(p_K, 1-p_K)$.
4. If $s_{T+1} = K$, draw $y_{T+1} \sim p(y_{T+1}|Y_T, \theta_K)$. If $s_{T+1} = K+1$, draw $\theta_{K+1} \sim p(\theta_{K+1}|\theta_0, S_T, Y_T)$ and $y_{T+1} \sim p(y_{T+1}|Y_T, \theta_{K+1})$.

If this is repeated as many times as one iterates in the Gibbs sampler, one obtains as many draws of the predictive of y_{T+1} . Generalizing this algorithm to $h \geq 2$ is not difficult but requires lengthy formulas.

Appendix D: Additional Tables

These tables are providing detailed results on which Tables 5 and 6 in the paper are based. For each series in Table 1 of the paper, we provide in the following tables the RMSE and APL values from the recursive forecasting exercise described in Section 5 of the paper. For one-step ahead forecasts, see Tables 1 (RMSE) and 3 (APL) and for four-step ahead forecasts see Tables 2 and 4. We report the relative values, with the model in the last column (UC-SV) serving as reference. This reference model is just chosen for convenience.

The RMSE/APL values for the reference model are reported to fix their order of magnitude. For example, in Table 1, we see that for the UC-SV model and the first series, the RMSE is equal to 0.608, whereas the relative RMSE for PPT10 is 0.989, implying that PPT11 has a RMSE 1.1 percent lower than the UC-SV model. For each series, the smallest (for RMSE) or largest (for APL) value across all models is in bold. If this global minimum is in the set of break models, the value in italics is the minimum across the no-break models (see Table 2 of the paper for the definition of break and no-break models). If the global minimum is in the latter group, the value in italics is the minimizer across the break models.

Table 1: Root mean squared errors at horizon 1
on last thirty percent of sample

	series	PPT10	PPT11	PPT40	PPT41	KP1	KP4	TVP1	TVP4	ROW1	ROW4	REC1	REC4	UC-SV
1	GDPC96	0.989	0.991	0.947	0.945	0.994	0.967	0.999	0.965	1.048	1.019	1.007	<i>0.964</i>	0.608
2	GDPCDEF	1.015	1.011	0.969	0.964	1.075	0.983	1.000	0.944	1.021	1.058	1.082	<i>0.969</i>	0.261
3	PCECC96	1.139	1.135	0.962	0.965	1.126	0.983	1.157	1.024	1.070	0.997	1.137	<i>0.989</i>	0.419
4	PCECTPI	1.027	1.030	1.007	1.003	1.029	1.029	0.942	0.963	<i>0.953</i>	0.969	1.037	1.008	0.439
5	GPDIC96	1.011	1.017	0.996	0.998	1.006	0.999	1.024	1.009	1.011	1.043	1.018	<i>0.999</i>	0.354
6*	OPHPBS	0.831	0.836	0.804	0.802	0.844	0.803	0.858	0.837	0.900	0.911	0.831	0.800	0.844
7	ULCNFB	0.896	0.898	0.824	0.823	0.941	0.829	0.800	0.804	<i>0.813</i>	0.826	0.930	0.827	1.275
8	CPIAUCSL	0.702	0.700	0.725	0.719	0.689	1.253	0.699	0.675	0.714	0.707	0.698	<i>0.676</i>	0.986
9	PPIFCG	0.730	0.730	0.705	0.698	0.721	1.301	0.722	0.671	0.754	0.728	0.729	<i>0.677</i>	2.499
10	TB3MS	0.937	0.939	0.936	0.936	0.919	1.004	1.033	1.019	<i>0.976</i>	1.057	1.051	1.068	0.381
11	GS10	0.844	0.848	0.804	0.808	0.849	0.808	0.849	0.830	0.847	0.810	0.847	<i>0.807</i>	0.433
12	M1SL	0.712	0.714	0.867	0.823	0.755	0.722	0.711	0.713	0.751	0.827	<i>0.713</i>	0.719	1.709
13	M2SL	0.759	0.757	0.672	<i>0.671</i>	0.760	1.290	0.762	0.690	0.781	0.704	0.759	0.668	0.957
14*	UTL11	0.817	0.812	0.528	0.529	0.768	0.549	0.631	0.687	0.793	0.552	0.771	<i>0.547</i>	0.151
15*	SP500	0.926	0.922	0.935	0.930	0.924	1.235	0.928	0.945	0.944	1.045	<i>0.923</i>	0.949	0.798
16	INDPRO	0.925	0.923	1.062	1.034	0.935	0.911	0.992	0.963	0.970	1.011	0.980	<i>0.925</i>	1.035
17*	HOUST	0.844	0.849	0.794	0.791	0.851	0.805	0.663	1.003	0.863	0.851	0.851	<i>0.806</i>	0.090
18	AHEMAN	0.884	0.885	0.925	0.923	0.972	0.911	0.881	0.879	<i>0.886</i>	0.911	1.123	0.958	0.319
19	UNRATE	0.966	0.966	0.967	0.969	0.976	0.942	1.117	1.135	1.001	1.063	<i>0.959</i>	0.974	0.235
20	PAYEMS	0.859	0.857	0.789	0.788	0.880	0.769	0.956	0.927	0.897	0.860	0.889	<i>0.866</i>	0.273
21	EXUSUK	0.898	0.898	<i>0.876</i>	0.879	0.900	0.880	0.906	0.903	0.921	0.902	0.894	0.874	0.406
22*	PMI	0.821	0.818	0.731	0.732	0.829	0.721	0.796	0.745	0.864	0.784	0.822	<i>0.722</i>	0.379
23*	NAPMNOI	0.866	0.863	0.823	0.821	0.864	<i>0.801</i>	0.879	0.820	0.909	0.892	0.863	0.799	0.568

See Table 4 of the paper for model definitions. Values in the last column are the RMSE for the UC-SV model. Values in other columns are the RMSE values for each model in the column header, divided by the value for the UC-SV model. For each series, the smallest value across all models is in bold. If this global minimum is in the category PPT+KP+TVP, the value in italics is the minimum across all other models. If the global minimum is in these other models, the value in italics is the minimizer across the PPT+KP+TVP models.

Table 2: Root mean squared errors at horizon 4
on last thirty percent of sample

	series	PPT10	PPT11	PPT40	PPT41	KP1	KP4	TVP1	TVP4	ROW1	ROW4	REC1	REC4	UC-SV
1	GDPC96	0.849	0.852	0.848	0.847	0.857	0.872	0.832	0.824	<i>0.863</i>	0.900	0.864	0.856	0.823
2	GDPCDEF	1.116	1.105	1.032	1.028	1.172	1.020	1.046	1.012	1.148	1.011	1.208	1.035	0.268
3	PCECC96	1.031	1.041	0.948	0.944	1.035	0.972	1.000	0.984	1.066	1.057	1.040	<i>0.989</i>	0.550
4	PCECTPI	1.039	1.023	0.991	0.995	1.019	0.999	0.855	0.871	<i>0.857</i>	0.865	1.037	1.000	0.511
5	GPDIC96	0.779	0.779	0.789	0.789	0.782	0.784	0.781	0.781	0.823	1.010	<i>0.785</i>	0.795	0.497
6*	OPHPBS	0.824	0.822	0.820	0.822	<i>0.817</i>	0.817	0.854	0.860	0.850	0.857	0.819	0.816	0.869
7	ULCNFB	0.947	0.951	0.957	0.959	0.983	0.961	0.921	0.939	<i>0.925</i>	1.008	1.033	0.967	1.105
8	CPIAUCSL	0.836	0.840	0.920	0.884	0.847	0.842	0.837	0.837	<i>0.843</i>	0.871	0.838	0.843	0.886
9	PPIFCG	0.795	0.795	0.780	0.799	0.788	0.771	0.791	<i>0.777</i>	0.793	0.810	0.792	0.775	2.419
10	TB3MS	0.776	0.776	0.776	<i>0.772</i>	0.781	0.781	0.795	0.793	0.807	0.885	0.782	0.771	0.592
11	GS10	0.753	0.754	0.750	0.750	0.759	0.761	0.758	0.759	0.770	0.773	0.754	<i>0.752</i>	0.474
12	M1SL	0.855	0.852	1.003	0.917	0.876	0.893	0.854	0.875	0.860	0.889	<i>0.854</i>	0.888	1.508
13	M2SL	0.835	0.834	0.835	0.833	0.832	1.538	0.834	0.834	0.835	0.846	<i>0.833</i>	0.838	0.875
14*	UTL11	1.133	1.093	0.937	0.939	0.952	0.943	1.116	1.213	1.105	1.015	0.961	<i>0.952</i>	0.377
15*	SP500	0.824	0.828	1.085	0.953	0.829	0.832	0.824	0.829	0.843	0.885	<i>0.828</i>	0.830	0.949
16	INDPRO	0.813	0.810	1.028	0.848	0.834	0.806	0.773	0.770	0.964	0.893	<i>0.784</i>	0.785	1.868
17*	HOUST	0.981	0.980	1.009	1.006	0.983	1.040	1.226	1.395	1.405	1.374	<i>0.989</i>	1.051	0.230
18	AHEMAN	<i>0.816</i>	0.835	0.890	0.892	0.961	0.894	0.836	0.867	0.814	0.813	1.426	0.972	0.350
19	UNRATE	0.865	0.863	0.862	0.860	0.887	0.908	0.865	0.881	0.995	1.066	<i>0.845</i>	0.870	0.386
20	PAYEMS	0.994	0.994	0.969	0.962	1.009	0.959	0.932	0.926	1.151	1.100	<i>0.962</i>	1.020	0.559
21	EXUSUK	0.870	0.866	0.850	0.855	0.852	0.857	0.845	0.846	0.865	0.926	<i>0.850</i>	0.850	0.469
22*	PMI	0.825	0.817	0.754	0.761	0.815	<i>0.745</i>	1.026	0.989	0.893	0.905	0.805	0.743	0.743
23*	NAPMNOI	0.800	0.800	0.757	0.787	0.787	<i>0.723</i>	0.992	0.952	0.854	0.724	0.777	0.711	1.022

See Table 4 of the paper for model definitions. Values in the last column are the RMSE for the UC-SV model. Values in other columns are the RMSE values for each model in the column header, divided by the value for the UC-SV model. For each series, the smallest value across all models is in bold. If this global minimum is in the category PPT+KP+TVP, the value in italics is the minimum across all other models. If the global minimum is in these other models, the value in italics is the minimizer across the PPT+KP+TVP models.

Table 3: Average predictive likelihoods at horizon 1 on last thirty percent of sample

	series	PPT10	PPT11	PPT40	PPT41	KP1	KP4	TVP1	TVP4	ROW1	ROW4	REC1	REC4	UC-SV
1	GDPC96	1.753	1.734	1.680	1.663	1.008	1.678	1.724	1.661	1.747	1.657	1.327	1.348	0.289
2	GDPDEF	<i>3.068</i>	3.019	2.858	2.821	2.970	2.957	2.060	2.085	3.306	3.280	2.505	2.630	0.423
3	PCECC96	1.415	1.403	1.488	1.474	1.441	1.607	1.528	1.692	1.783	<i>1.932</i>	1.425	1.487	0.333
4	PCECTPI	2.264	2.247	2.408	2.391	2.370	<i>2.476</i>	1.893	1.916	2.704	2.586	2.328	2.490	0.388
5	GPDIC96	1.925	1.923	1.864	1.857	<i>1.978</i>	1.941	1.804	1.754	2.354	2.251	1.859	1.893	0.371
6*	OPHPBS	1.509	1.502	1.551	1.552	1.521	<i>1.576</i>	1.575	1.533	1.626	1.583	1.525	1.588	0.230
7	ULCNFB	1.403	1.408	1.370	1.355	1.337	1.393	1.454	<i>1.479</i>	1.563	1.492	1.275	1.375	0.218
8	CPIAUCSL	2.200	2.201	2.333	2.303	2.287	2.422	1.927	2.068	2.519	2.608	2.282	2.499	0.288
9	PPIFCG	1.558	1.561	1.597	1.599	1.582	1.666	1.738	<i>1.675</i>	1.508	1.545	1.615	1.690	0.157
10	TB3MS	2.142	2.117	2.106	2.083	<i>2.193</i>	2.022	2.131	2.016	2.211	2.097	1.178	1.218	0.378
11	GS10	<i>1.952</i>	1.927	1.932	1.900	1.783	1.871	1.619	1.708	2.110	2.167	1.882	1.922	0.335
12	M1SL	1.672	1.672	1.571	1.561	1.546	1.614	<i>1.712</i>	1.608	1.561	1.467	1.712	1.742	0.177
13	M2SL	1.796	1.796	1.942	<i>1.926</i>	1.812	1.146	1.876	2.109	1.934	2.054	1.808	1.983	0.249
14*	UTL11	6.347	6.268	<i>6.856</i>	6.687	4.916	6.191	1.562	1.568	6.744	8.336	4.846	6.199	0.466
15*	SP500	1.626	1.616	1.564	1.567	1.645	1.571	1.528	1.520	1.608	1.556	<i>1.634</i>	1.588	0.268
16	INDPRO	1.357	1.346	1.217	1.196	1.311	1.130	1.439	<i>1.437</i>	1.335	1.245	0.895	0.972	0.257
17*	HOUST	7.475	7.386	<i>7.583</i>	7.548	7.558	7.811	1.587	1.523	9.639	9.497	7.603	7.818	0.485
18	AHEMAN	2.552	2.519	2.266	2.240	2.361	2.255	1.772	1.856	<i>2.550</i>	2.513	1.616	1.941	0.374
19	UNRATE	2.785	2.749	2.622	2.603	2.845	<i>2.856</i>	2.166	2.172	3.328	3.135	2.674	2.637	0.433
20	PAYEMS	3.423	<i>3.368</i>	3.136	3.064	3.161	2.860	2.172	2.063	3.370	3.244	2.375	2.464	0.419
21	EXUSUK	2.008	1.998	1.864	1.866	<i>2.127</i>	1.928	1.900	1.902	2.181	2.042	2.031	2.019	0.388
22*	PMI	2.131	2.122	2.307	2.275	2.334	<i>2.403</i>	1.533	1.597	2.596	2.775	2.147	2.389	0.371
23*	NAPMNOI	1.873	1.875	1.894	1.887	1.891	<i>1.993</i>	1.430	1.418	1.989	1.915	1.895	2.004	0.310

See Table 4 of the paper for model definitions. Values in the last column are the average predictive likelihoods (APL) for the UC-SV model. Values in other columns are the APL values for each model in the column header, divided by the value for the UC-SV model. For each series, the largest value across all models is in bold. If this global maximum is in the category PPT+KP+TVP, the value in italics is the maximum across all other models. If the global maximum is in these other models, the value in italics is the maximizer across the PPT+KP+TVP models.

Table 4: Average predictive likelihoods at horizon 4 on last thirty percent of sample

	series	PPT10	PPT11	PPT40	PPT41	KP1	KP4	TVP1	TVP4	ROW1	ROW4	REC1	REC4	UC-SV
1	GDPC96	<i>2.590</i>	2.437	2.405	2.134	2.366	2.194	2.581	2.424	2.601	2.399	1.971	1.958	0.182
2	GDPCDEF	3.584	3.229	3.712	3.340	3.207	<i>3.771</i>	2.486	2.663	3.995	4.399	2.580	3.301	0.256
3	PCECC96	2.185	2.106	2.127	2.020	2.183	2.184	<i>2.368</i>	2.246	2.628	2.448	2.210	2.223	0.198
4	PCECTPI	2.540	2.449	2.873	2.675	2.720	<i>2.885</i>	2.382	2.371	3.670	3.734	2.575	2.919	0.228
5	GPDIC96	3.231	3.127	3.156	2.945	<i>3.324</i>	3.191	3.119	3.070	3.992	3.762	3.187	3.189	0.211
6*	OPHPBS	2.070	2.023	2.093	1.956	<i>2.101</i>	2.087	2.055	1.936	2.286	2.161	2.092	2.113	0.165
7	ULCNFB	1.889	1.860	1.752	1.670	1.667	1.803	<i>1.973</i>	1.971	2.066	1.918	1.661	1.812	0.151
8	CPIAUCSL	<i>3.242</i>	3.167	3.142	2.960	3.269	3.216	2.733	2.793	3.425	3.217	3.345	3.275	0.184
9	PPIFCG	2.009	1.974	1.961	1.874	2.033	2.026	2.186	2.035	1.970	1.912	<i>2.064</i>	2.060	0.113
10	TB3MS	3.099	2.864	2.941	2.647	2.911	2.682	3.051	2.915	<i>2.900</i>	2.535	1.996	1.999	0.211
11	GS10	<i>3.047</i>	2.936	2.913	2.632	2.761	2.762	2.672	2.651	3.278	3.278	2.977	2.933	0.209
12	M1SL	<i>2.051</i>	2.001	1.857	1.749	1.889	1.904	2.030	1.973	1.932	1.781	2.101	2.106	0.137
13	M2SL	2.651	2.564	2.644	2.507	2.632	1.556	2.768	<i>2.845</i>	2.878	2.752	2.668	2.703	0.168
14*	UTL11	<i>4.959</i>	4.252	3.866	3.425	4.201	3.560	0.852	0.739	5.042	4.621	4.235	3.564	0.247
15*	SP500	2.567	2.485	2.450	2.312	2.550	2.505	2.315	2.275	2.494	2.222	<i>2.565</i>	2.530	0.164
16	INDPRO	1.830	1.729	1.667	1.506	1.720	1.566	1.969	1.883	<i>1.795</i>	1.641	1.361	1.382	0.142
17*	HOUST	6.753	6.181	5.865	5.341	<i>6.777</i>	5.882	0.911	0.865	7.298	7.843	6.859	5.959	0.254
18	AHEMAN	<i>4.375</i>	3.985	3.510	3.156	3.889	3.460	2.756	2.720	4.405	4.317	2.100	2.857	0.215
19	UNRATE	3.970	3.792	3.783	3.559	<i>4.300</i>	3.883	3.625	3.592	5.090	4.492	3.919	3.838	0.241
20	PAYEMS	3.506	3.214	2.877	2.542	3.106	2.657	2.106	1.885	<i>3.284</i>	2.788	2.474	2.292	0.246
21	EXUSUK	3.090	3.124	2.986	2.822	3.248	2.966	<i>3.381</i>	3.291	3.365	3.221	3.308	3.242	0.225
22*	PMI	2.289	2.152	2.216	2.034	<i>2.509</i>	2.334	1.065	1.054	2.760	2.277	2.381	2.392	0.196
23*	NAPMNOI	2.270	2.122	2.198	2.332	2.332	<i>2.376</i>	1.115	1.108	2.474	2.213	2.393	2.472	0.158

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