TVP-VARs and Factor Models

Introduction

- Why TVP-VARs?
- Example: U.S. monetary policy
- was the high inflation and slow growth of the 1970s were due to bad policy or bad luck?
- Some have argued that the way the Fed reacted to inflation has changed over time
- After 1980, Fed became more aggressive in fighting inflation pressures than before
- This is the "bad policy" story (change in the monetary policy transmission mechanism)
- This story depends on having VAR coefficients different in the 1970s than subsequently.

() TVP-VARs 2 / 49

- Others think that variance of the exogenous shocks hitting economy has changed over time
- Perhaps this may explain apparent changes in monetary policy.
- This is the "bad luck" story (i.e. 1970s volatility was high, adverse shocks hit economy, whereas later policymakers had the good fortune of the Great Moderation of the business cycle – at least until 2008)
- This motivates need for multivariate stochastic volatility to VAR models
- Cannot check whether volatility has been changing with a homoskedastic model

- Most macroeconomic applications of interest involve several variables (so need multivariate model like VAR)
- Also need VAR coefficients changing
- Also need multivariate stochastic volatility
- TVP-VARs are most popular models with such features
- But other exist (Markov-switching VARs, Vector Floor and Ceiling Model, etc.)

Homoskedastic TVP-VARs

- ullet Begin by assuming $\Sigma_t = \Sigma$
- Remember VAR notation: y_t is $M \times 1$ vector, Z_t is $M \times k$ matrix (defined so as to allow for a VAR with different lagged dependent and exogenous variables in each equation).
- TVP-VAR:

$$y_t = Z_t \beta_t + \varepsilon_t$$
$$\beta_{t+1} = \beta_t + u_t$$

- ε_t is i.i.d. $N(0, \Sigma)$ and u_t is i.i.d. N(0, Q).
- ε_t and u_s are independent of one another for all s and t.

() TVP-VARs 5 / 49

- Bayesian inference in this model?
- Already done: this is just the Normal linear state space model of the last lecture.
- MCMC algorithm of standard form (e.g. Carter and Kohn, 1994).
- But let us see how it works in practice in our empirical application
- Follow Primiceri (2005)

Illustration of Bayesian TVP-VAR Methods

- Same quarterly US data set from 1953Q1 to 2006Q3 as was used to illustrate VAR methods
- Three variables: Inflation rate $\Delta \pi_t$, the unemployment rate u_t and the interest rate r_t
- VAR lag length is 2.
- Training sample prior: prior hyperparameters are set to OLS quantities calculating using an initial part of the data
- Our training sample contains 40 observations.
- Data through 1962Q4 used to choose prior hyperparameter values, then Bayesian estimation uses data beginning in 1963Q1.

() TVP-VARs 7 / 49

- β_{OLS} is OLS estimate of VAR coefficients in constant-coefficient VAR using training sample
- $V\left(\beta_{OLS}\right)$ is estimated covariance of β_{OLS} .
- Prior for β_0 :

$$\beta_0 \sim N\left(\beta_{OLS}, 4 \cdot V\left(\beta_{OLS}\right)\right)$$

- Prior for Σ^{-1} Wishart prior with $\nu = M+1, S=I$
- \bullet Prior for Q^{-1} Wishart prior with $\underline{\nu}_{\mathit{Q}}=$ 40, $\underline{\mathit{Q}}=$ 0.0001 \cdot 40 \cdot $V\left(\beta_{\mathit{OLS}}\right)$

) TVP-VARs 8 / 49

- With TVP-VAR we have different set of VAR coefficients in every time period
- So different impulse responses in every time period.
- Figure 1 presents impulse responses to a monetary policy shock in three time periods: 1975Q1, 1981Q3 and 1996Q1.
- Impulse responses defined in same way as we did for VAR
- Posterior median is solid line and dotted lines are 10th and 90th percentiles.

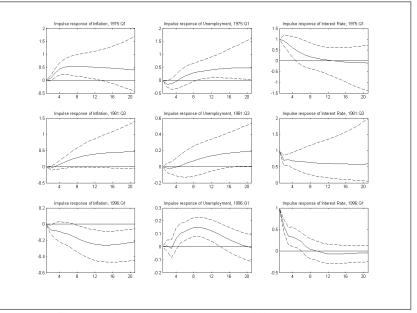


Figure 1: Impulse responses at different times

Combining other Priors with the TVP Prior

- Often Bayesian TVP-VARs work very well in practice.
- In some case the basic TVP-VAR does not work as well, due to over-parameterization problems.
- Previously, we noted worries about proliferation of parameters in VARs, which led to use of priors such as the Minnesota prior or the SSVS prior.
- With many parameters and short macroeconomic time series, it can be hard to obtain precise estimates of coefficients.
- Risk of over-fitting
- Priors which exhibit shrinkage of various sorts can help mitigate these problems.
- With TVP-VAR proliferation of parameters problems is even more severe.
- Hierarchical prior of state equation is big help, but may want more in some cases.

() TVP-VARs 11 / 49

Combining TVP Prior with Minnesota Prior

- E.g. Ballabriga, Sebastian and Valles (1999, JIE), Canova and Ciccarelli (2004, JOE), and Canova (2007, book)
- Replace TVP-VAR state equation by

$$\beta_{t+1} = A_0 \beta_t + (I - A_0) \,\overline{\beta}_0 + u_t$$

- u_t is i.i.d. N(0, Q)
- A_0 , $\overline{\beta}_0$ and Q can be unknown parameters or set to known values
- E.g. Canova (2007) sets $\overline{\beta}_0$ and Q to have forms based on the Minnesota prior and sets $A_0=cI$ where c is a scalar.
- ullet Note if c=1, then $E\left(eta_{t+1}
 ight)=E\left(eta_{t}
 ight)$ (as in TVP-VAR)
- If c=0 then $E\left(\beta_{t+1}\right)=\overline{\beta}_0$ (as in Minnesota prior)
- Q based on prior covariance of Minnesota prior
- c can either be treated as an unknown parameter or a value can be selected for it.

() TVP-VARs 12 / 49

Combining TVP Prior with SSVS Prior

- Same setup as preceding slide
- Set $\overline{\beta}_0 = 0$.
- Let $a_0 = vec(A_0)$
- Use SSVS prior for a₀
- a_{0j} (the j^{th} element of a_0) has prior:

$$a_{0j}|\gamma_{j}\sim\left(1-\gamma_{j}
ight)N\left(0,\kappa_{0j}^{2}
ight)+\gamma_{j}N\left(0,\kappa_{1j}^{2}
ight)$$

- ullet as before, γ_i is dummy variable
- κ_{0j}^2 is very small (so that a_{0j} is constrained to be virtually zero)
- κ_{1j}^2 is large (so that a_{0j} is relatively unconstrained).
- Property: with probability γ_j , a_{0j} is evolving according to a random walk in the usual TVP fashion
- With probability $(1-\gamma_i)$, $a_{0j}\approx 0$

() TVP-VARs 13 / 49

MCMC Algorithm

- I will not provide complete details, but note only:
- These are Normal linear state space models so standard algorithms (e.g. Carter and Kohn) can draw β^T
- For TVP+Minnesota prior this is enough (other parameters fixed)
- For TVP+SSVS simple to adapt MCMC algorithm for SSVS with VAR

Adding Another Layer to the Prior Hierarchy

- Another approach used by Chib and Greenberg (1995, JOE) for SUR model
- Adapted for VARs by, e.g., Ciccarelli and Rebucci (2002)

$$y_{t} = Z_{t}\beta_{t} + \varepsilon_{t}$$

$$\beta_{t+1} = A_{0}\theta_{t+1} + u_{t}$$

$$\theta_{t+1} = \theta_{t} + \eta_{t}$$

- all assumptions as for TVP-VAR, plus η_t is i.i.d. $N\left(0,R\right)$
- Slightly more general that previous Normal linear state space model, but very similar MCMC (so will not discuss MCMC)

() TVP-VARs 15 / 49

Adding Another Layer to the Prior Hierarchy

- Why might this generalization be useful?
- \bullet A_0 can be chosen to reflect some other prior information
- E.g. SSVS prior as above
- E.g. Ciccarelli and Rebucci (2002) is panel VAR application
- G countries and, for each country, k_G explanatory variables exist with time-varying coefficients.
- They set

$$A_0 = \iota_G \otimes I_{k_G}$$

- Implies time-varying component in each coefficient which is common to all countries
- Parsimony: θ_t is of dimension k_G whereas β_t is of dimension $k_G \times G$.

() TVP-VARs 16 / 49

Imposing Inequality Restrictions on the VAR Coefficients

- Another way of ensuring shrinkage
- E.g. restrict β_t to be non-explosive (i.e. roots of the VAR polynomial defined by β_t lie outside the unit circle)
- Sometimes (given over-fitting and imprecise estimates) can get posterior weight in explosive region
- Even small amount of posterior probability in explosive regions for β_t can lead to impulse responses or forecasts which have counter-intuitively large posterior means or standard deviations.
- Koop and Potter (2009, on my website) discusses how to do this. I will not present details, but outline basic idea

() TVP-VARs 17 / 49

- With unrestricted TVP-VAR, took draws $p\left(\beta^T|y^T, \Sigma, Q\right)$ using MCMC methods for Normal linear state space models
- One method to impose inequality restrictions involves:
- Draw β^T in the unrestricted VAR. If any drawn β_t violates the inequality restriction then the entire vector β^T is rejected.
- Problem: this algorithm can get stuck, rejecting virtually every β^T (all you need is a single drawn β_t to violate inequality and entire β^T is rejected)
- Note: algorithms like Carter and Kohn are "multi-move algorithms" (draw β^T all at same time).
- Alternative is "single move algorithm": drawing β_t for t=1,..,T one at a time from $p\left(\beta_t | y^T, \Sigma, Q, \beta_{-t}\right)$ where $\beta_{-t} = \left(\beta_1',..,\beta_{t-1},\beta_{t+1},...,\beta_T'\right)'$

() TVP-VARs 18 / 49

- Koop and Potter (2009) suggest using single move algorithm
- Reject β_t only (not β^T) if it violates inequality restriction
- Usually multi-move algorithms are better than single-move algorithms since latter can be slow to mix.
- I.e. they produce highly correlated series of draws which means that, relative to multi-move algorithms, more draws must be taken to achieve a desired level of accuracy.
- But if multi-move algorithm gets stuck, single move might be better.

Dynamic Mixture Models

- Remember: Normal linear state space model depends on so-called system matrices, Z_t , Q_t , T_t , W_t and Σ_t .
- ullet Suppose some or all of them depend on an s imes 1 vector \widetilde{K}_t
- Suppose \widetilde{K}_t is Markov random variable (i.e. $p\left(\widetilde{K}_t | \widetilde{K}_{t-1},...,\widetilde{K}_1\right) = p\left(\widetilde{K}_t | \widetilde{K}_{t-1}\right)$ or independent over t
- Particularly simple if \widetilde{K}_t is a discrete random variable.
- Result is called a dynamic mixture model
- Gerlach, Carter and Kohn (2000, JASA) have an efficient MCMC algorithm

() TVP-VARs 20 / 49

- Why are dynamic mixture models useful in empirical macroeconomics?
- E.g. TVP-VAR:

$$y_t = Z_t \beta_t + \varepsilon_t$$
$$\beta_{t+1} = \beta_t + u_t$$

- ε_t is i.i.d. $N(0, \Sigma)$
- BUT: u_t is i.i.d. $N\left(0, \widetilde{K}_t Q\right)$.
- Let $\widetilde{K}_t \in \{0,1\}$ with hierarchical prior:

$$egin{aligned}
ho\left(\widetilde{K}_t=1
ight) &= q. \
ho\left(\widetilde{K}_t=0
ight) &= 1-q \end{aligned}$$

• where q is an unknown parameter.

() TVP-VARs 21 / 49

- Property:
- If $\widetilde{K}_t = 1$ then usual TVP-VAR:

$$\beta_{t+1} = \beta_t + u_t$$

• If $\widetilde{K}_t = 0$ then VAR coefficients do not change at time t:

$$\beta_{t+1} = \beta_t$$

- Parsimony.
- This model can have flexibility of TVP-VAR if the data warrant it (i.e. can select $\widetilde{K}_t = 1$ for t = 1, ..., T).
- But can also select a much more parsimonious representation.
- An extreme case: if $\widetilde{K}_t = 0$ for t = 1, ..., T then back to VAR without time-varying parameters.

() TVP-VARs 22 / 49

- I will not present details of MCMC algorithm since it is becoming a standard one
- See also the Matlab code on my website
- Dynamic mixture models used to model structural breaks, outliers, nonlinearities, etc.
- E.g. Giordani, Kohn and van Dijk (2007, JoE).

TVP-VARs with Stochastic Volatility

- In empirical work, you will usually want to add multivariate stochastic volatility to the TVP-VAR
- But this can be dealt with quickly, since the appropriate algorithms were described in the lecture on State Space Modelling
- Remember, in particular, the approaches of Cogley and Sargent (2005) and Primiceri (2005).
- MCMC: need only add another block to our algorithm to draw Σ_t for t=1,...,T.
- Homoskedastic TVP-VAR MCMC: $p\left(Q^{-1}|y^T, \beta^T\right)$, $p\left(\beta^T|y^T, \Sigma, Q\right)$ and $p\left(\Sigma^{-1}|y^T, \beta^T\right)$
- $\begin{array}{l} \bullet \ \ \text{Heteroskedastic TVP-VAR MCMC: } p\left(Q^{-1}|y^T,\beta^T\right), \\ p\left(\beta^T|y^T,\Sigma_1,..,\Sigma_T,Q\right) \ \ \text{and} \ \ p\left(\Sigma_1^{-1},..,\Sigma_T^{-1}|y^T,\beta^T\right) \end{array}$

() TVP-VARs 24 / 49

Empirical Illustration of Bayesian Inference in TVP-VARs with Stochastic Volatility

- Continue same illustration as before.
- All details as for homoskedastic TVP-VAR
- Plus allow for multivariate stochastic volatility as in Primiceri (2005).
- Priors as in Primiceri
- Can present empirical features of interest such as impulse responses
- But (for brevity) just present volatility information
- Figure 2: time-varying standard deviations of the errors in the three equations (i.e. the posterior means of the square roots of the diagonal element of Σ_t)

() TVP-VARs 25 / 49

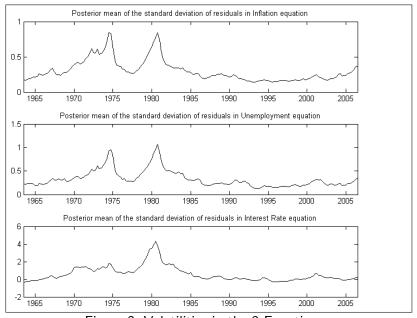


Figure 2: Volatilities in the 3 Equations

Summary of TVP-VARs

- TVP-VARs are useful for the empirical macroeconomists since they:
- are multivariate
- allow for VAR coefficients to change
- allow for error variances to change
- They are state space models so Bayesian inference can use familiar MCMC algorithms developed for state space models.
- They can be over-parameterized so care should be taken with priors.
- Recently there has been interest in large TVP-VARs

() TVP-VARs 27 / 49

Factor Methods

- In past VARs and TVP-VAR usually have small number of dependent variables (e.g. 3 or 4 and rarely more than 10)
- Increasingly researchers working with large VARs (and even large TVP-VARs)
- But before large VARs, factor methods were dominant answer to question:
- How to extract information in data sets with many variables but keep model parsimonious?

() TVP-VARs 28 / 49

Static Factor Model

- y_t is $M \times 1$ vector of time series variables
- M is very large
- yit denote a particular variable.
- Simplest static factor model:

$$y_t = \lambda_0 + \lambda f_t + \varepsilon_t$$

- ullet f_t is q imes 1 vector of unobserved latent factors (where q << M)
- Factors contain information extracted from all the M variables.
- Same f_t occurs in every equation for y_{it} for i = 1, ..., M
- But different coefficients (λ is an $M \times q$ matrix of so-called factor loadings).

() TVP-VARs 29 / 49

- Note that restrictions are necessary to identify the model
- Common to say ε_t is i.i.d. N(0, D) where D is diagonal matrix.
- Implication: ε_{it} is pure random shock specific to variable i, co-movements in the different variables in y_t arise only from the factors.
- Note also that $\lambda f_t = \lambda CC^{-1}f_t$ which shows we need identification restriction for factors too.
- Different models arise from different treatment of factors.
- Simplest is: $f_t \sim N(0, I)$
- ullet This can be interpreted as a state equation for "states" f_t
- Factor models are state space models so our MCMC tools of Lecture 3 can be used.

() TVP-VARs 30 / 49

The Dynamic Factor Model (DFM)

- In macroeconomics, usually need to extend static factor model to allow for the dynamic properties which characterize macroeconomic variables.
- A typical DFM:

$$y_{it} = \lambda_{0i} + \lambda_i f_t + \varepsilon_{it}$$

$$f_t = \Phi_1 f_{t-1} + ... + \Phi_p f_{t-p} + \varepsilon_t^f$$

- f_t is as for static model
- λ_i is $1 \times q$ vector of factor loadings.
- Each equation has its own intercept, λ_{0i} .
- ε_{it} is i.i.d. $N(0, \sigma_i^2)$
- f_t is VAR with ε_t^f being i.i.d. $N\left(0, \Sigma^f\right)$
- Note: usually ε_{it} is autocorrelated (easy extension, omitted here for simplicity)

() TVP-VARs 31 / 49

Replacing Factors by Estimates: Principal Components

- Proper Bayesian analysis of the DFM treats f_t as vector of unobserved latent variables.
- Before doing this, we note a simple approximation.
- The DFM has similar structure to regression model:

$$y_{it} = \lambda_{0i} + \widetilde{\lambda}_{0i} f_t + .. + \widetilde{\lambda}_{pi} f_{t-p} + \widetilde{\varepsilon}_{it}$$

- If f_t were known we could use Bayesian methods for the multivariate Normal regression model to estimate or forecast with the DFM.
- ullet Principal components methods to can be used to approximate f_t .
- Precise details of how principal components is done provided many places

() TVP-VARs 32 / 49

Treating Factors as Unobserved Latent Variables

- DFM is a Normal linear state space model so use Bayesian methods for state space models discussed in Lecture 3.
- A bit more detail on MCMC algorithm:
- Conditional on the model's parameters, Σ^f , Φ_1 , ..., Φ_p , λ_{0i} , λ_i , σ_i^2 for i=1,...,M, use (e.g.) Carter and Kohn algorithm to draw f_t
- Conditional on the factors, measurement equations are just *M* Normal linear regression models.
- Since ε_{it} is independent of ε_{it} for $i \neq j$, posteriors for λ_{0i} , λ_i , σ_i^2 in the M equations are independent over i
- Hence, the parameters for each equation can be drawn one at a time (conditional on factors).
- Finally, conditional on the factors, the state equation is a VAR
- Any of the methods for Bayesian VARs of Lecture 2 can be used.

() TVP-VARs 33 / 49

The Factor Augmented VAR (FAVAR)

- DFMs are good for forecasting (extract all information in huge number of variables)
- VARs are good for macroeconomic policy (e.g. impulse responses).
- Why not combine DFMs and VARs together to get model which can do both?
- FAVAR results
- Bernanke, Boivin and Eliasz (2005, QJE) is pioneering paper

() TVP-VARs 34 / 49

Impulse Response Analysis in DFM

With VARs impulse responses based on structural VAR:

$$C_0 y_t = c_0 + \sum_{j=1}^{p} C_j y_{t-j} + u_t$$

- u_t is i.i.d. N(0, I) and C_0 chosen to give shocks structural interpretation
- If $C(L) = C_0 \sum_{i=1}^{p} C_i L^p$ impulse responses obtained from VMA:

$$y_t = C(L)^{-1} u_t$$

• With the DFM, can obtain VMA representation for y_t by substituting in factor equation:

$$y_t = \varepsilon_t + \lambda \Phi (L)^{-1} \varepsilon_t^f$$

= $B(L) \eta_t$

• But η_t combines ε_t and ε_t^f — cannot isolate "shock to interest rate equation" as monetary policy shock and do impulse response analysis in standard way.

() TVP-VARs 35 / 49

FAVAR modifies DFM by adding other explanatory variables:

$$y_{it} = \lambda_{0i} + \lambda_i f_t + \gamma_i r_t + \varepsilon_{it}$$

- r_t is $k_r \times 1$ vector of observed variables of key interest.
- E.g. Bernanke, Boivin and Eliasz (2005) set r_t to be the Fed Funds rate (a monetary policy instrument)
- All other assumptions are same as for the DFM.
- Note: by treating r_t in this way, we can isolate a "monetary policy shock" and calculate impulse responses

• FAVAR state equation extends DFM state equation to include r_t:

$$\left(\begin{array}{c} f_t \\ r_t \end{array}\right) = \widetilde{\Phi}_1 \left(\begin{array}{c} f_{t-1} \\ r_{t-1} \end{array}\right) + \ldots + \widetilde{\Phi}_p \left(\begin{array}{c} f_{t-p} \\ r_{t-p} \end{array}\right) + \widetilde{\varepsilon}_t^f$$

- where all assumptions are same as DFM with extension that $\widetilde{\epsilon}_t^f$ is i.i.d. $N\left(0,\widetilde{\Sigma}^f\right)$
- MCMC is very similar to that for the DFM and will not be described here.
- Similar ideas:
- Normal linear state space algorithms can draw f_t
- Measurement equation is series of regressions (conditional on factors)
- The state equation is a VAR (conditional of factors)

() TVP-VARs 37 / 49

Impulse Response Analysis in FAVAR

• FAVAR model can be written:

$$\begin{pmatrix} y_t \\ r_t \end{pmatrix} = \begin{bmatrix} \lambda & \gamma \\ 0 & 1 \end{bmatrix} \begin{pmatrix} f_t \\ r_t \end{pmatrix} + \widetilde{\varepsilon}_t$$

$$\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \widetilde{\Phi}_1 \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + \dots + \widetilde{\Phi}_p \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \widetilde{\varepsilon}_t^f$$

- where $\widetilde{\varepsilon}_t = (\varepsilon_t', 0)'$
- VMA obtained by substituting second equation in first and re-arranging

$$\begin{pmatrix} y_t \\ r_t \end{pmatrix} = \begin{bmatrix} \lambda & \gamma \\ 0 & 1 \end{bmatrix} \widetilde{\Phi}(L)^{-1} \widetilde{\varepsilon}_t^f + \widetilde{\varepsilon}_t$$
$$= \widetilde{B}(L) \eta_t$$

• Now last k_r elements of η_t are solely associated with original VAR-like equations for r_t and impulse responses with conventional interpretation can be done (e.g. "shock to interest rate equation" can be "monetary policy shock")

() TVP-VARs 38 / 49

The TVP-FAVAR

- With VARs: began with constant parameter model
- then we said it is good to allow the VAR coefficients to vary over time: homoskedastic TVP-VAR
- then we said good to allow for multivariate stochastic volatility: heteroskedastic TVP-VAR
- Recent research (e.g. working papers: Del Negro and Otrok (2008, NYFed) and Korobilis (2013, OBES)) is doing the same with FAVARs
- Note: just as with TVP-VARs, TVP-FAVARs can be over-parameterized and careful incorporation of prior information or the imposing of restrictions (e.g. only allowing some parameters to vary over time) can be important in obtaining sensible results.

() TVP-VARs 39 / 49

 A TVP-FAVAR is just like a FAVAR but with t subscripts on parameters:

$$y_{it} = \lambda_{0it} + \lambda_{it}f_t + \gamma_{it}r_t + \varepsilon_{it},$$

 $\left(egin{array}{c} f_t \ r_t \end{array}
ight) = \widetilde{\Phi}_{1t} \left(egin{array}{c} f_{t-1} \ r_{t-1} \end{array}
ight) + ... + \widetilde{\Phi}_{pt} \left(egin{array}{c} f_{t-p} \ r_{t-p} \end{array}
ight) + \widetilde{arepsilon}_t^f$

ullet All each $arepsilon_{it}$ to follow univariate stochastic volatility process

•

- $var\left(\widetilde{\varepsilon}_{t}^{f}\right) = \widetilde{\Sigma}_{t}^{f}$ has multivariate stochastic volatility process of the form used in Primiceri (2005).
- Finally, the coefficients (for i=1,..,M) $\lambda_{0it},\lambda_{it},\gamma_{it},\widetilde{\Phi}_{1t},..,\widetilde{\Phi}_{pt}$ are allowed to evolve according to random walks (i.e. state equations of the same form as in the TVP-VAR complete the model).
- All other assumptions are the same as for the FAVAR.

() TVP-VARs 40 / 49

Bayesian Inference in the TVP-FAVAR

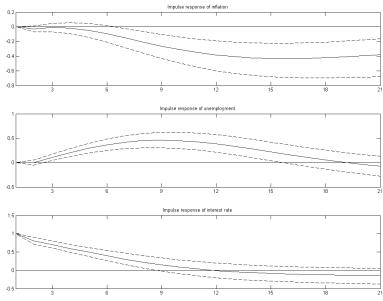
- I will not provide details of MCMC algorithm
- Note only it adds more blocks to the MCMC algorithm for the FAVAR.
- These blocks are all of forms discussed in previous lecture.
- E.g. error variances in measurement equations drawn using the univariate stochastic volatility algorithm of Kim, Shephard and Chib (1998).
- Multivariate stochastic volatility algorithm of Primiceri (2005) can be used to draw $\widetilde{\Sigma}_t^f$.
- The coefficients λ_{0it} , λ_{it} , γ_{it} , $\widetilde{\Phi}_{1t}$, ..., $\widetilde{\Phi}_{pt}$ are all drawn using algorithm for Normal linear state space model

() TVP-VARs 41 / 49

Empirical Illustration of the FAVAR and TVP-FAVAR

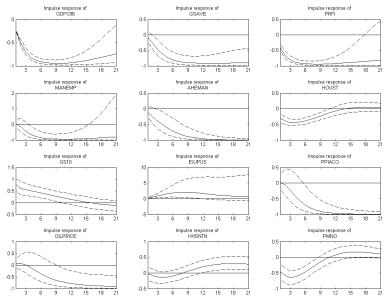
- 115 quarterly US macroeconomic variables spanning 1959Q1 though 2006Q3.
- Transform all variables to be stationarity.
- What variables to put in r_t ?
- Inflation, unemployment and the interest rate.
- ullet FAVAR is same as VAR from previous illustrations, but augmented with factors, f_t
- We use 2 factors and 2 lags in state equation
- Identify impulse responses as in our VAR empirical illustration plus Bernanke, Boivin and Eliasz (2005).

() TVP-VARs 42 / 49



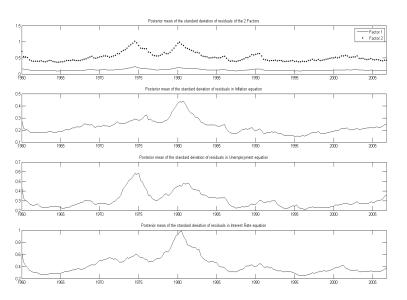
Posterior of impulse responses of main variables to monetary policy shock

() TVP-VARs 43 / 49



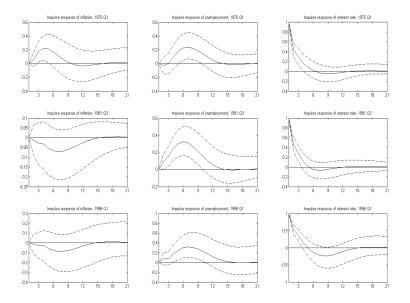
Posterior of impulse responses of selected variables to monetary policy shock

- Now TVP-FAVAR
- Illustrate time varying volatility of equations for r_t and factor equations
- Impulse responses at three different time periods

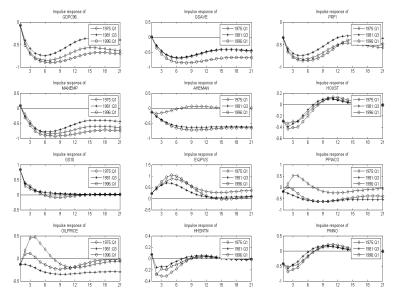


Time-varying volatilities of errors in five key equations of the TVP-FAVAR

() TVP-VARs 46 / 49



Posterior of impulse responses of main variables to monetary policy shock at different times



Posterior means impulse responses of selected variables to monetary policy shock at different times

Summary of Factor Methods

- Factor methods are an attractive way of modelling when the number of variables is large
- DFMs are attractive for forecasting
- FAVARs attractive for macroeconomic policy (e.g. to do impulse response analysis)
- Recently TVP versions of these models have been developed
- Bayesian inference in TVP-FAVAR puts together MCMC algorithm involving blocks from several simple and familiar algorithms.

() TVP-VARs 49 / 49