Appendix 11.2 Markets and intertemporal allocation

Given the ideal circumstances discussed in the chapter, the literature looks at two sorts of market system as potential institutional means for the support of an efficient intertemporal allocation.

A11.2.1 Futures markets

Here it is imagined that contracts are made for all future exchanges at the beginning of period 0. Contracts referring to future periods are futures contracts, and the markets in which they are traded are futures markets. Letting M represent the total value of all future commodity expenditure that an individual can legitimately commit to, the individual's problem is

$$\text{Max } U(X_0, X_1, Y_0, Y_1)$$

subject to

$$M = P_{X0}X_0 + P_{Y0}Y_0 + P_{X1}X_1 + P_{Y1}$$

M could be thought of as the total sum of money paid to the individual in respect of the amounts of labour that they contract to supply in all future periods. The Lagrangian here is

$$L = U(X_0, X_1, Y_0, Y_1)$$
$$+ [M - P_{X0}X_0 - P_{Y0}Y_0 - P_{X1}X_1 - P_{Y1}Y_1]$$

with first-order conditions:

$U_{X0} - \lambda P_{X0} = 0$

$U_{Y0} - \lambda P_{Y0} = 0$

$U_{X1} - \lambda P_{X1} = 0$

$U_{Y1} - \lambda P_{Y1} = 0$

From these we derive for two individuals A and B both facing the same prices:

$$\text{MRUS}^A_{X0,Y0} = \text{MRUS}^B_{X0,Y0} = \frac{P_{X0}}{P_{Y0}} \quad (11.59a)$$

$$\text{MRUS}^A_{X1,Y1} = \text{MRUS}^B_{X1,Y1} = \frac{P_{X1}}{P_{Y1}} \quad (11.59b)$$

$$\text{MRUS}^A_{X0,X1} = \text{MRUS}^B_{X0,X1} = \frac{P_{X0}}{P_{X1}} \quad (11.59c)$$

$$\text{MRUS}^A_{X0,Y1} = \text{MRUS}^B_{X0,Y1} = \frac{P_{X0}}{P_{Y1}} \quad (11.59d)$$

$$\text{MRUS}^A_{Y0,Y1} = \text{MRUS}^B_{Y0,Y1} = \frac{P_{Y0}}{P_{Y1}} \quad (11.59e)$$

$$\text{MRUS}^A_{Y0,X1} = \text{MRUS}^B_{Y0,X1} = \frac{P_{Y0}}{P_{X1}} \quad (11.59f)$$

Comparing these with equations 11.43 shows that the extended consumption efficiency conditions are satisfied.

Consider production first in terms of the firm producing commodity *X*. Its objective is the maximisation of its net receipts at the beginning of period 0, which relate to contracts to supply *X* to individuals in that and all future periods. The way we have set production conditions up means that the firm has no monetary outgoings – we have not explicitly represented any inputs other than capital, and we have had capital accumulation as a process internal to the firm. The firm's net receipts are then its receipts from its sales to consumers. Its decision is how much to sell, and hence to invest, in period 0. That is to

$$\text{Max } \pi = P_{X0}[X_0(K_0^X) - (K_1^X - K_0^X)]$$
$$+ P_{X1}[X_1(K_1^X) - (K_2^X - K_1^X)]$$

by choice of K_1. The first-order necessary condition is

$$\partial \pi / \partial K^X_1 = -P_{X0} + P_{X1}X_{K1} + P_{X1} = 0$$

from which

$$1 + X_{K1} = \frac{P_{X0}}{P_{X1}} \qquad (11.60)$$

Comparing this with 11.59c and using the MRT definition we have

$$\text{MRT}^X_{0,1} = \text{MRUS}^A_{X0,X1} = \text{MRUS}^B_{X0,X1} \qquad (11.61a)$$

and proceeding in the same way for commodity *Y* leads to

$$\text{MRT}^{Y}_{0,1} = \text{MRUS}^{\text{A}}_{Y0,Y1} = \text{MRUS}^{\text{B}}_{Y0,Y1} \tag{11.61b}$$

Equations 11.61 say that for each commodity we have, intertemporally, that MRT equals MRUS, as required for efficiency – see 11.45. When considering 11.45 we showed that, given an assumption, the statement of the required condition in terms of MRUSs and MRTs was equivalent to a statement in terms of consumption discount rates and rates of return. Clearly, the same applies here to the satisfaction of the condition in this system of markets – the overall efficiency condition of a common consumption discount rate equal to a common rate of return will be satisfied.

Regarding the assumption

$$\text{MRUS}_{X0,Y0} = \text{MRUS}_{X1,Y1}$$

note that by 11.59a and 11.59b, this will hold if

$$\frac{P_{X0}}{P_{Y0}} = \frac{P_{X1}}{P_{Y1}}$$

that is if relative prices are the same in both periods.

A11.2.2 Loanable funds market

Suppose that there exists a market in which funds can be borrowed or lent at the interest rate i. All contracts, other than those involving borrowing or lending, refer only to the one period of time at the beginning of which they are made.

The individual's problem is

$$\text{Max } U(X_0, X_1, Y_0, Y_1)$$

subject to

$$M_0 + [1/(1 + i)]M_1 = \{p_{X0}X_0 + p_{Y0}Y_0\}$$
$$+ [1/(1 + i)]\{p_{X1}X_1 + p_{Y1}Y_1\}$$

The left-hand side here is the present value of the sum of the individual's receipts at start of each period. The individual pays for each period's consumption at the start of each period, and the right-hand side is the present value of expenditures. The constraint simply says that the present value of receipts equals the present value of expenditures. Note the different notation here for prices as compared with that used in Section A11.2.1. In the case of futures markets everything is determined at the beginning of period 0, and prices such as P_{X0} refer to money sums then payable. With loanable funds markets, commodity trades take place at the start of each period, and prices such as p_{X0} refer to money sums then payable.

The Lagrangian for this problem is

$$L = U(X_0, X_1, Y_0, Y_1) + \lambda\, [M_0 + [1/(1 + i)]M_1$$

$$- \{p_{X0}X_0 + p_{Y0}Y_0\} - [1/(1 + i)]\{p_{X1}X_1 + p_{Y1}Y_1\}$$

where the first-order conditions are:

$$U_{X0} - \lambda p_{X0} = 0$$

$$U_{Y0} - \lambda p_{Y0} = 0$$

$$U_{X1} - \lambda\, [1/(1 + i)]p_{X1} = 0$$

$$U_{Y1} - \lambda\, [1/(1 + i)]p_{Y1} = 0$$

From these we derive for two individuals A and B both facing the same prices:

$$\text{MRUS}^A_{X0,Y0} = \text{MRUS}^B_{X0,Y0} = \frac{P_{X0}}{P_{Y0}} \qquad (11.62a)$$

$$\text{MRUS}^A_{X1,Y1} = \text{MRUS}^B_{X1,Y1} = \frac{P_{X1}}{P_{Y1}} \qquad (11.62b)$$

$$\text{MRUS}^A_{X0,X1} = \text{MRUS}^B_{X0,X1} = \frac{P_{X0}}{[1/1(1+i)]P_{X1}} \qquad (11.62c)$$

$$\text{MRUS}^A_{X0,Y1} = \text{MRUS}^B_{X0,Y1} = \frac{P_{X0}}{[1/1(1+i)]P_{Y1}} \qquad (11.62d)$$

$$\text{MRUS}^A_{Y0,Y1} = \text{MRUS}^B_{Y0,Y1} = \frac{P_{Y0}}{[1/1(1+i)]P_{Y1}} \qquad (11.62e)$$

$$\text{MRUS}^A_{Y0,X1} = \text{MRUS}^B_{Y0,X1} = \frac{P_{Y0}}{[1/1(1+i)]P_{X1}} \qquad (11.62f)$$

Comparing equations 11.62 with equations 11.43 we see that the consumption conditions for efficiency are satisfied.

In the context of a loanable funds market the problem for the firm producing *X*, for example, is to maximise the present value of net receipts by choice of investment level in period 0. That is to

$$\text{Max } \pi = p_{X0}[X_0(K^X_0) - (K^X_1 - K^X_0)]$$
$$+ [1/(1 + i)]p_{X1}[X_1(K^X_1) - (K^X_2 - K^X_1)]$$

by choice of K_1. The first-order necessary condition is

$$\partial \pi / \partial K^X_1 = -p_{X0} + [1/(1 + i)]\{p_{X1}X_{K1} + p_{X1}\} = 0$$

from which

$$1 + X_{K1} = \frac{P_{X0}}{[1/(1+i)P_{X1}} \qquad (11.63)$$

Comparing this with 11.62c and using the MRT definition we have

$$\text{MRT}^X_{0,1} = \text{MRUS}^A_{X0,X1} = \text{MRUS}^B_{X0,X1} \qquad (11.64a)$$

and proceeding in the same way for commodity *Y* leads to

$$\text{MRT}^{Y}_{0,1} = \text{MRUS}^{\text{A}}_{Y0,Y1} = \text{MRUS}^{\text{B}}_{Y0,Y1} \tag{11.64b}$$

The intertemporal efficiency conditions are satisfied.

Assuming that $p_{X0} = p_{X1}$, 11.63 becomes

$$1 + X_{K1} = 1 + i$$

which is

$$1 + \delta_X = 1 + i$$

or

$$\delta_X = i \tag{11.65a}$$

and in the same way for $p_{Y0} = p_{Y1}$

$$\delta_Y = i \tag{11.65b}$$

which establishes that equality of rates of return with the market rate of interest condition is satisfied. Given the same assumptions about relative prices over time, equations 11.62c to 11.62e have all the commodity consumption discount rates equal to the rate of interest determined in the market for loanable funds.

The discussion in the text of the chapter was conducted in terms of aggregate consumption. Suppose that there are $j = 1, 2, \ldots, J$ individuals, each determining period 0 and 1 consumption levels, given receipts M_0 and M_1, according to

$$\text{Max } U^j(C^j_0, C^j_1)$$

subject to

$$M_0^j + \left(\frac{1}{1+j}\right) M_1^j = C_0^j + \left(\frac{1}{1+i}\right) C_1^j$$

Then, as you can readily confirm, for each individual

$$r_j = i \qquad (11.66)$$

i.e. the consumption discount rate is equal to the market rate of interest.

Now suppose that each of these individuals is the owner of a firm producing the consumption/capital good. In that role, each individual's problem is to invest so as to maximise the present value of their firm. From

$$\text{Max } [Q^j_0(K^j_0) - (K^j_1 - K^j_0)]$$

$$+[1/(1+i)][Q^j_1(K^j_1) - (K^j_2 - K^j_1)]$$

we get

$$1 + Q^j_{K1} = 1 + i$$

which is the same as

$$\delta^j = i \qquad (11.67)$$

which says that rates of return will be equalised and equal to the market interest rate, as efficiency requires.

In their roles as owners of firms, individuals choose a level of investment such that the rate of return equals the interest rate, and thus fix receipts in each period, M^j_0 and M^j_1. Given these, each individual determines, given their preferences and the market rate of interest, C^j_0 and C^j_1 as set out above. Those for whom $C^j_0 > M^j_0$ become period 0 borrowers, and those for whom $C^j_0 < M^j_0$ become period 0 lenders.