

## Appendix 12.2 Weak complementarity with observable Marshallian demand curves

The indirect utility function  $v$  yields the maximum amount of utility  $u$  given a particular price vector  $p$  for marketed goods, an income level  $y$  and quantity  $e$  of the non-marketed environmental good  $e$  such that  $u=v(p,y,e)$ . Indirect utility functions and Marshallian demand curves are linked via what is known as Roy's Theorem. Roy's Theorem states that the derivative of the indirect utility function with respect to the price of a good divided by the derivative of the indirect utility function with respect to the level of income yields the negative of the Marshallian demand function,  $x(p,y,e)$ . It is now easy to demonstrate how demand dependency assumptions can be invoked to value environmental goods. By the fundamental theorem of calculus

$$\frac{v(p^*,y,e)}{\lambda} - \frac{v(p,y,e)}{\lambda} = \int_p^{p^*} \frac{v_p(p,y,e)}{\lambda} dp \quad (12.78)$$

Notice that by Roy's Theorem the expression on the right hand side of equation 12.78 is actually equal to the market demand curve for the weakly complementary good given  $\lambda$  is the marginal utility of income. Now differentiate both sides with respect to the level of the environmental good  $e$  to obtain

$$\frac{v_e(p^*,y,e)}{\lambda} - \frac{v_e(p,y,e)}{\lambda} = -\int_p^{p^*} x_e(p,y,e) dp \quad (12.79)$$

At this point we are once more ready to invoke weak complementarity and the idea that there exists a price so high that marginal changes in the level of the environmental good have no further effect on utility. The consequence of the assumption of weak complementarity is that the leading term on the left hand side of the equation is equal to zero so that

$$\frac{v_e(p,y,e)}{\lambda} = \int_p^{p^*} x_e(p,y,e) dp \quad (12.80)$$

Finally we integrate between the limits of the change in the level of the environmental good

$$\frac{v(p, y, e^1)}{\lambda} - \frac{v(p, y, e^0)}{\lambda} = \int_p^{p^*} x(p, y, e^1) dp - \int_p^{p^*} x(p, y, e^0) dp$$

(12.81)

The left hand side of this expression gives a monetary valuation of welfare change associated with a change in the level of  $e$  assuming that the marginal utility of income is constant (this is where the approximation comes in). The right hand side is composed of two terms. The first term is the market (Marshallian) demand curve for the weakly complementary good observed when the environmental good takes the level  $e^1$  whilst the second is the market demand curve when the public good takes the level  $e^0$ . Hence the value of the change in the level of the environmental good is equal to the change in the areas between two market demand curves which is illustrated in Figure 12.6. This area is in principle an estimable quantity provided that we possess data on market demands exhibiting variation in both prices and the level of the environmental good.

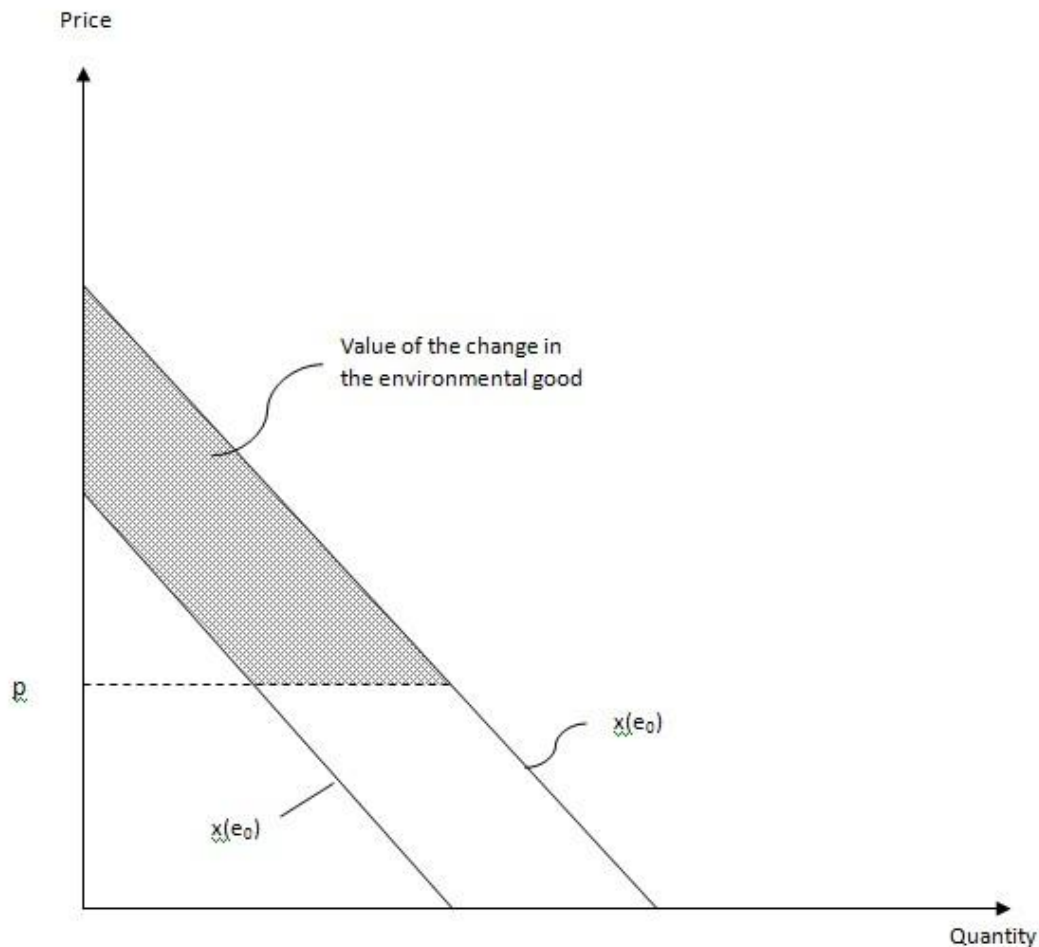


Figure 12.6 Measuring welfare changes with weak complementarity

How useful are weak complementarity and weak substitutability as an empirical strategy? It is often hard to think of marketed commodities which possess this property.<sup>1</sup> And whether or not these restrictions hold cannot be empirically tested. And if observable Marshallian demand curves are used the technique yields only an approximation to the true CS monetary measure of the utility change with errors of uncertain sign and magnitude (Bockstael and McConnell, 1993). Notwithstanding this, environmental economists

<sup>1</sup> One example could be groundwater quality and the price of bottled water. If the price of bottled water is so low that no-one drinks groundwater no-one cares about the quality of groundwater. This is an example of weak substitutability.

continue to use methods based on Marshallian demand functions in the hope that the errors involved, with respect to the true surplus measures, are not too great. Finally, non-use values cannot be estimated by methods based on demand dependency. Indeed, according to Freeman (1993) the very definition of non-use values is those values that cannot be estimated from expenditures on marketed goods. Furthermore all revealed preference techniques are actually applications of demand dependency.