# Appendix 13.2 Irreversibility, development and risk

Consider first the case of a risk-neutral decision-maker where there is no irreversibility. With the two possible period 2 benefit functions as $F^1_2(A_2)$ and $F^2_2(A_2)$, where the respective probabilities are $p$ and $q$ with $q = 1 - p$, the decision maker's problem is to

$$\text{Max } F_1(A_1) + \text{E}[F_2(A_2)]$$

$$= F_1(A_1) + pF^1_2(A_2) + qF^2_2(A_2)$$

for which the necessary conditions are:

$$\partial F_1/\partial A_1 = 0 \quad (13.50a)$$

$$p(\partial F^1_2/\partial A_2) + q(\partial F^2_2/\partial A_2) = 0 \quad (13.50b)$$

For the linear MNB functions of Figure 13.5, we have

$$\partial F_1/\partial A_1 = \alpha - \beta A_1 \quad (13.51a)$$

$$\partial F^1_2/\partial A_2 = \alpha - \beta A_2 \quad (13.51b)$$

$$\partial F^2_2/\partial A_2 = k\alpha - \beta A_2 \quad (13.51c)$$

and substituting these into equations 13.50 gives the necessary conditions as:

$$\alpha - \beta A_1 = 0 \quad (13.52a)$$

$$p(\alpha - \beta A_2) + q(k\alpha - \beta A_2) = 0 \quad (13.52b)$$

From equation 13.52a

$$A_1^{RNI} = \alpha/\beta \qquad (13.53)$$

and from equation 13.52b

$$A_2^{RNI} = (\alpha/\beta)(p + qk) \qquad (13.54)$$

give the period 1 and 2 levels of $A$, where the superscript RNI is for 'risk, no irreversibility'. The effect of the introduction of risk alone can be seen by comparing equations 13.53 and 13.54 with equations 13.36 from Appendix 13.1. There is no effect on the period 1 level of $A$. The size of the effect on the second-period level depends on the sizes of $p$ and $q$, but except for $p = 0$ it is the case that $A_2^{RNI} < A_2^{NI}$. This is because equation 13.54 can be rewritten, using $q = 1 - p$, as

$$A_2^{NRI} = k(\alpha/\beta) + p(\alpha/\beta)(1 - k)$$

where the second term is negative.

With irreversibility incorporated into the decision problem it becomes

$$\text{Max } F_1(A_1) + \mathrm{E}[F_2(A_2)] \text{ subject to } A_1 = A_2$$

for which the Lagrangian is

$$L = F_1(A_1) + pF^1_2(A_2) + qF^2_2(A_2) + \lambda[A_1 - A_2]$$

giving as necessary conditions

$$\partial F_1/\partial A_1 + \lambda = 0 \tag{13.55a}$$

$$p(\partial F^1{}_2/\partial A_2) + q(\partial F^2{}_2/\partial A_2) - \lambda = 0 \tag{13.55b}$$

$$A_1 = A_2 \tag{13.55c}$$

Substituting for the derivatives in equations 13.55 from 13.51 and solving leads to

$$A_1^{IR} = A_2^{IR} = \{\alpha\,(1 + p + qk)\}/2\beta \tag{13.56}$$

where the superscript IR stands for 'irreversibility, risk'. In Appendix 13.1 the result for the case where there is irreversibility but perfect future knowledge was established as

$$A^I_1 = A^I_2 = (\alpha/\beta)\{(1 + k)/2\} \tag{13.57}$$

Consider the first-period levels. $A_1^{IR}$ is less than $A^I_1$ if

$$\alpha\,(1 + p + qk)/2\beta < (\alpha/\beta)\{(1 + k)/2\} \tag{13.58}$$

which, using $q = 1 - p$, reduces to

$$p < pk$$

which follows from $k > 1$, for any $p > 0$. So, $A_1^{IR}$ is less than $A^I_1$, as shown in Figure 13.5.

The condition for $A_2^{IR} < A^{I}_2$, as shown in Figure 13.5, is also the expression 13.58, so that is also established for $k > 1$, $p > 0$.

Figure 13.5 also shows $A_1^{IR} > A_1^{NI}$ and $A_2^{IR} > A'_2$. From 13.46 in Appendix 13.1 and 13.56 the condition for $A_1^{IR} > A_1^{NI}$ is

$$\{\alpha (1 + p + qk)\}/2\beta > \alpha/\beta \qquad (13.59)$$

which, using $q = 1 - p$, reduces to

$$k > p(k - 1)$$

which is true for $k > 1$ and $0 < p < 1$. The condition for $A_2^{IR} > A'_2$ is also expression 13.59.