Appendix 14.3 Optimal and efficient extraction or harvesting of a renewable or non-renewable resource in the presence of resource extraction costs

In this appendix, we derive the optimal solution to the exhaustible resource depletion problem discussed in Part 2 of this chapter. We allow for the resource to be either renewable or non-renewable, and its extraction or harvesting to be costly. Once again, we use the solution method outlined in Appendix 14.1.

Utility is a function of the level of consumption:

$$U_t = U(C_t)$$

The objective function to be maximised is:

$$W = \int_{t=0}^{t=\infty} U(C_t)\mathrm{e}^{-\rho t} dt$$

There are two state variables in this problem: S_t, the resource stock at time t, and K_t, the capital stock at time t. Associated with each state variable is a shadow price, P (for the resource stock) and ω (for the capital stock). The two state equations of motion are:

$$\dot{S}_t = G(S_t) - R_t \qquad (14.28)$$

$$\dot{K}_t = Q(K_t,\ R_t) - C_t - \Gamma(R_t,\ S_t) \qquad (14.29)$$

There are several things to note about these equations of motion:

- If the environmental resource being used is a non-renewable resource, $G(S) = 0$ and so equation 14.28 collapses to the special case $\dot{S}_t = -R_t$.
- Equation 14.29 incorporates resource extraction costs, which are modelled as reducing the amount of output available for either consumption or addition to the stock of capital.
- Equation 14.29 incorporates a production function of the same form as in Appendix 14.2.

There are two control variables in this problem: C_t (consumption) and R_t (the rate of resource extraction). Initial and terminal state conditions are identical to those in Appendix 14.2. The current-value Hamiltonian is

$$H_C = U(C_t) + P_t(G(S_t) - R_t) + \omega_t(Q\{K_t,\ R_t\} \\ -C_t - \Gamma(R_t,\ S_t))$$

Ignoring time subscripts and the subscript $_C$ on the expression for the current-value Hamiltonian, the necessary conditions for a maximum are:

$$\frac{\partial H_C}{\partial C} = U_C - \omega = 0 \qquad (14.30a)$$

$$\frac{\partial H_C}{\partial R} = -P + \omega Q_R - \omega \Gamma_R = 0 \qquad (14.30b)$$

$$\dot{P} = -\frac{\partial H}{\partial S} + \rho P \Leftrightarrow \dot{P} = \rho P - PG_S + \omega \Gamma_s \qquad (14.30c)$$

$$\dot{\omega} = -\frac{\partial H}{\partial K} + \rho \omega \Leftrightarrow \dot{\omega} = \rho \omega - Q_K \omega \qquad (14.30d)$$

Special cases of these conditions

Let us first concentrate on the simplifications which take place in the Hotelling efficiency condition for the shadow price of the environmental resource (equation 14.30c) when some special cases are considered.

(a) Non-renewable resources. For non-renewables, as noted above, $G(S) = 0$. The Hamiltonian does not, therefore, contain the term G_S. This implies that condition 14.30c simplifies to

$$\dot{P} = \rho P + \Gamma_S \omega$$

which is identical to the Hotelling rule given in the text for optimal depletion of a non-renewable resource that incurs extraction costs (equation 14.20c).

(b) Extraction costs do not depend on the size of the resource stock. Next suppose that we are considering a non-renewable resource for which extraction costs are zero or, more generally, are positive but do not depend on the size of the remaining resource stock. In this case, we have either $\Gamma = 0$ or $\Gamma = \Gamma(R)$. In both cases, $\Gamma_S = 0$. Therefore, the final term in equation 14.30c is zero and so (for non-renewable resources) the Hotelling rule collapses to equation 14.14c, the one we used in Part 1 of the chapter. That is, $\dot{P} = \rho P$.