Appendix 15.3 A worked numerical example

Let us take 1990 as the 'initial year' of the study. In 1990, the oil price was P_0 = \$20 per barrel, and oil output was R_0 = 21.7 billion barrels. From our demand function (equation 15.8)

$$P_0 = K\mathrm{e}^{-aR_0}$$

we obtain

$$R_0 = \frac{\text{In } K}{a} - \frac{1}{a} \text{In } P_0$$

The price elasticity of the initial year is, therefore,

$$\varepsilon_0 = \frac{\mathrm{d}R_0}{\mathrm{d}P_0}\frac{P_0}{R_0} = \left[-\frac{1}{aP_0}\right]\frac{P_0}{R_0} = -\frac{1}{aR_0}$$

Assume that $\varepsilon = -0.5$; then we can estimate a:

$$a = -\frac{1}{\varepsilon R_0} = \frac{1}{0.5 \times 21.7} \approx 0.1$$

We can also estimate the parameter K as follows:

$$K = P_0 \exp(aR_0) = 20 \exp(0.1 \times 21.7) \approx 175$$

The global oil reserve stock is S = 1150 billion barrels. The optimal oil extraction programme under the assumptions of a discount rate ρ = 3% and perfect competition are given by the following.

The optimal exhaustion time is:

$$T^* = \sqrt{\frac{2Sa}{\rho}} = \sqrt{\frac{2\times 1150\times 0.1}{0.03}} = 87.5 \text{ years}$$

The optimal initial oil output is

$$R_0^* = \sqrt{\frac{2\rho S}{a}} = \sqrt{\frac{2\times 0.03\times 1150}{0.1}}$$
$$= 26.26 \text{ billion barrels}$$

The corresponding optimal initial oil price is

$$P_0{*} = K \exp(-aR_0{*}) = 175 \exp(-0.1 \times 26.26)$$
$$= \$12.7/\text{barrel}$$

The optimal oil output is obviously higher than the actual output in 1990, and the optimal price is lower than the actual one. So there is apparent evidence of distortion (inefficiency) in the world oil market.