Appendix 19.3 Theory for an imperfect economy

This appendix is based on the formal analysis in the appendix to Dasgupta (2001), which has production using a renewable resource rather than a non-renewable resource as here. Substituting non-renewable for renewable makes no essential difference to the analysis. Arrow et al (2003) extends the analysis to the case of many assets.

We use the notation of the two previous appendices, and consider an economy where

$$\dot{K}_t = Q(K_t, R_t) - C_t \tag{19.53}$$

and

$$\dot{S}_t = -R_t \tag{19.54}$$

These are the equations of motion for the simple exhaustible resource dependent closed economy that we looked at in Appendix 19.1. As there, population is assumed constant, and there is no technological progress. Instead of assuming a government that can implement an optimal programme, it is now assumed that there is a resource allocation mechanism α that maps (K_0, S_0) into an economic programme which is a set of time paths for C_t, K_t and S_t that satisfy equations 1 and 2 and that this α is known to the government so that it can forecast those time paths.

At time t discounted current and future social wellbeing is:

$$V_t = \int_{\tau=t}^{\infty} U(C_\tau) e^{-\rho(\tau - t)} d\tau \tag{19.55}$$

At t the economy is experiencing sustainable development if

$$\frac{dV_t}{dt} \geq 0 \qquad (19.56)$$

At time t the future is determined by K_t, S_t and the resource allocation mechanism, so that:

$$V_t(\alpha, K_t, S_t) = \int_{\tau=t}^{\infty} U(C_\tau\{\alpha, K_\tau, S_\tau\})e^{-\rho(\tau-t)}d\tau \qquad (19.57)$$

Then

$$\frac{\partial V_t(\alpha, K_t, S_t)}{\partial K_t} = \int_{\tau=t}^{\infty} \left[\frac{\partial U(C_\tau\{\alpha, K_\tau, S_\tau\})}{\partial K_\tau}\right]e^{-\rho(\tau-t)}d\tau \qquad (19.58a)$$

and

$$\frac{\partial V_t(\alpha, K_t, S_t)}{\partial S_t} = \int_{\tau=t}^{\infty} \left[\frac{\partial U(C_\tau\{\alpha, K_\tau, S_\tau\})}{\partial S_\tau}\right]e^{-\rho(\tau-t)}d\tau \qquad (19.58b)$$

Accounting prices are defined as

$$p_t \equiv \frac{\partial V_t(\alpha, K_t, S_t)}{\partial K_t} \qquad (19.59a)$$

and

$$q_t \equiv \frac{\partial V_t(\alpha, K_t, S_t)}{\partial S_t} \qquad (19.59b)$$

Then, from

$$\frac{dV_t}{dt} = \left[\frac{\partial V_t(\alpha, K_t, S_t)}{\partial K_t}\right]\frac{dK_t}{dt} + \left[\frac{\partial V_t(\alpha, K_t, S_t)}{\partial S_t}\right]\frac{dS_t}{dt}$$

we get

$$\frac{dV_t}{dt} = p_t \frac{dK_t}{dt} + q_t \frac{dS_t}{dt} \qquad (19.60)$$

With

$I_t^K = p_t \frac{dK_t}{dt}$ for (net) investment in reproducible capital, and

$I_t^S = q_t \frac{dS_t}{dt}$ for the depreciation of the resource stock,

define

$$I_t^G \equiv I_t^K + I_t^S \qquad (19.61)$$

as genuine investment, otherwise known as genuine saving.

From equations 19.60 and 19.61

$$I_t^G = \frac{dV_t}{dt}$$

so that $I_t^G \geq 0$ as $dVt/dt \geq 0$ and the expression 19.56 above can be replaced as an indicator of sustainable development by:

$$I_t^G \geq 0 \qquad (19.62)$$

If genuine investment/saving is non-negative at t, the economy is on a sustainable path at t.

In this analysis, V is in units which are utils. In that case national income as the sum of consumption and total net investment is

$$EDP_t^{'} = U_C C_t + I_t^K + I_t^S = U_C C_t + I_t^G \qquad (19.63)$$

where we use EDP' with a prime to stay with the environmentally adjusted domestic product terminology used in the chapter, while signalling the difference. From equation 19.63 it is clear that $U_C C_t \leq EDP_t^{'}$ implies $I_t^G \geq 0$ which corresponds to the consumption less than properly measured national income condition for sustainability discussed in the chapter.