Appendix 19.5 El Serafy's method for the estimation of the depreciation of natural capital

The principle underlying El Serafy's rule for the calculation of a user cost measure of depreciation for a non-renewable resource is as follows. From the net receipts from sales a certain proportion is assumed to be set aside and invested at a constant rate of return in order to yield a constant level of income indefinitely. User cost is then defined as the difference between net receipts and that constant income, which is regarded as the true, or sustainable, income from resource depletion. Given some specialising assumptions, this principle leads to a very simple rule for calculating depreciation for non-renewable resources as user cost, and the corresponding true income from resource extraction.

Assuming that receipts accrue at the end of the period, and using the same notation as in the text, the value of a resource deposit at the start of the period is given by:

$$\begin{aligned} V_0 &= \{R_1(P_1 - C_1)/(1+r)\} + \{R_2(P_2 - C2)/ \\ &\quad (1+r)^2\} + \ldots + \{R_T(P_T - C_T)/(1+r)^T\} \\ &= \sum_{t=1}^{T} R_1(P_t - C_1)/(1+r)^t \end{aligned} \quad (19.64)$$

If we use X for a constant perpetual income stream, the present value of that stream at the start of the period is given by:

$$\begin{aligned} W_0 &= X/(1+r) + X/(1+r)^2 + X/(1+r)^3 + \ldots \\ &= \sum_{t=1}^{\infty} X/(1+r)^1 \end{aligned} \quad (19.65)$$

Now assume $R_t = R$, $P_t = P$ and $C_t = C$ for all t, and use $N = R(P - C)$ for net receipts and $d = 1/(1 + r)$ so that equation 19.64 can be written as

$$V_0 = N[d + d^2 + . . . + d^T] \tag{19.66}$$

and equation 19.65 as

$$W_0 = X[d + d^2 + d^3 + ...] \tag{19.67}$$

where the term inside the brackets on the right-hand side of equation 19.67 is an infinite series. Note that $d < 1$ for $r > 0$.

Multiplying both sides of equation 19.66 by d gives

$$dV_0 = N[d^2 + d^3 + ...+ d^{T+1}] \tag{19.68}$$

and subtracting equation 19.68 from equation 19.66 gives

$$V_0 - dV_0 = N[d - d^{T+1}]$$

so that

$$V_0 = N[d - d^{T+1}]/[1 - d] \tag{19.69}$$

Rewrite equation 19.67 for a finite time horizon, n, as

$$W_0^{'} = X[d + d^2 + ... + d^n]$$

where n is some finite number, and proceeding as we did above we get

$$W_0^{'} = X[d - d^{n+1}]/1 - d]$$

where letting $n \to \infty$ makes d^{n+1} vanish in the limit, so that we have

$$W_0 = \lim_{n\to\infty}(W_0^{'}) = dX/[1 - d] \qquad (19.70)$$

for the present value of the perpetual income stream.

Now, if this perpetual income stream is to be solely based on the ownership of the resource deposit, assuming a fully competitive economy, we must have $V_0 = W_0$; that is, the value of the mine at the start of the period must be equal to the present value of the perpetual income stream. Using equations 19.69 and 19.70 this gives

$$N[d - d^{T+1}]/[1 - d] = dX/[1 - d]$$

which on collecting terms and substituting 1/(1 + r) for d is

$$X = N[1 - \{1/(1 + r)^T\}]$$

or

$$N - X = N/(1 + r)^T$$

where substituting for $N = R(P - C)$ we get

$$R(P - C) - X = R(P - C)/(1 + r)^T$$

for user cost, so that for this measure of depreciation we have

$$D_N = R(P - C)/(1 + r)^T \qquad (19.71)$$

as in the text of the chapter.

User cost/depreciation as a proportion of net receipts is

$$\{R(P - C) - X\}/\{R(P - C)\} = 1/(1 + r)^T \qquad (19.72)$$

and depends only on the lifetime of the resource stock and the interest rate. Sustainable or true income as a share of net receipts is simply one minus the share of user cost. Table 19.18 gives the user cost share for different values for resource lifetime and the interest rate. With low interest rates and short lifetimes, user cost is nearly 100% of net receipts (the income share is close to zero). With long lifetimes and high interest rates, nearly all net receipts count as income. For any given asset lifetime, notice the importance of the choice of interest rate.

Table 19.18 User cost share of receipts from sales of non-renewable natural resources

Lifetime of resource at current extraction rates (years)	Discount rate (%)				
	1	3	5	7	10
1	99	97	95	93	91
5	95	86	78	71	62
10	91	74	61	51	39
25	78	48	30	18	9
50	61	23	9	3	1
100	37	5	1	0	0