Appendix 4.2 Market outcomes

In this Appendix we establish that, given the 'ideal' institutional conditions set out in the text of the chapter, a system of markets will bring about the satisfaction of the necessary conditions for efficiency in allocation - the consumption efficiency condition, the production efficiency condition and the product mix condition.

A 4.2.1 Individuals: utility maximisation

Consider an individual consumer, with a fixed money income M and gaining utility from the consumption of two goods, X and Y. The prices of these goods are determined in competitive markets, at the levels P_X and P_Y, and are taken as given by all individuals. With this individual's utility function given by

$U = U(X, Y)$

we can express the problem of maximising utility subject to a budget constraint as

Max U(X, Y)

subject to

$P_X X + P_Y Y = M$

The Lagrangian for this problem is

$L = U(X, Y) + \lambda[P_X X + P_Y Y - M]$

and, using the same notation for the derivatives (the marginal utilities) as previously, the first-order conditions for a maximum are:

$$\frac{\partial L}{\partial X} = U_X + \lambda P_X = 0 \qquad (4.34.a)$$

$$\frac{\partial L}{\partial X} = U_Y + \lambda P_Y = 0 \qquad (4.34.b)$$

From these equations we get

$$U_X = -\lambda P_X$$

$$U_Y = -\lambda P_Y$$

so that

$$\frac{U_X}{U_Y} = \frac{P_X}{P_Y} \qquad (4.35)$$

Equation 4.35 holds for all consumers, all of whom face the same P_X and P_Y, and the left hand side is the marginal rate of utility substitution. So for any two consumers A and B, we have:

$$MRUS^A = MRUS^B = \frac{P_X}{P_Y} \qquad (4.36)$$

The consumption efficiency condition is satisfied, see equation 4.3 in the chapter and equation 4.24 in the previous Appendix, and the marginal rate of utility substitution common to all individuals is equal to the price ratio, as stated in the chapter at equation 4.8.

4.2.2 Firms: profit maximisation

Consider the production of X by firms i = 1, 2............m. All firms face the same selling price, P_X, and all pay the same fixed prices for capital and labour inputs, P_K and P_L. The objective of every firm is to maximise profit, so to ascertain the conditions characterising the behaviour of the ith firm we consider

$$\text{Max} \quad P_X X_i(K_i^X, L_i^X) - P_K K_i^X - P_L L_i^X$$

where the necessary conditions are

$$P_X X_K^i - P_K = 0 \qquad (4.37.a)$$

$$P_X X_L^i - P_L = 0 \qquad (4.37.b)$$

or

$$X_K^i = \frac{P_K}{P_X} \qquad (4.38.a)$$

$$X_L^i = \frac{P_L}{P_X} \qquad (4.38.b)$$

from which

$$\frac{X_K^i}{X_L^i} = \frac{P_K}{P_L} \qquad (4.39)$$

Equation 4.39 holds for all i, and the left hand side is the expression for the marginal rate of technical substitution. Hence, all firms producing X operate with the same MRTS. Further, it is obvious that considering profit maximisation by the jth firm in the industry producing the commodity Y will lead to

$$\frac{Y_K^j}{Y_L^j} = \frac{P_K}{P_L} \qquad (4.40)$$

which with equation 4.39 implies

$$MRTS_X^i = MRTS_Y^j \qquad (4.41)$$

for i = 1,2.....m and j = 1,2,.......n. The production efficiency condition, equation 4.4 in the chapter, is satisfied.

Recall that

$$MRT_K = \frac{Y_K}{X_K} \qquad (4.42.a)$$

and

$$MRT_L = \frac{Y_L}{X_L} \qquad (4.42.b)$$

From equations 4.38.a and 4.38.b, and the corresponding conditions from profit maximisation in the production of Y, omitting the superscripts for firms we have

$$X_K = \frac{P_K}{P_X}, X_L = \frac{P_L}{P_X}, Y_K = \frac{P_K}{P_Y}, Y_L = \frac{P_L}{P_Y}$$

and substituting and cancelling in equations 4.42.a and 4.42.b

$$MRT_K = \frac{P_X}{P_Y} = MRT_L$$

and bringing this together with equation 4.36 gives

$$MRUS^A = MRUS^B = MRT_K = MRT_L \qquad (4.43)$$

which shows that the product mix condition, equation 4.5 in the chapter and 4.26 in the previous Appendix, is satisfied.

In the chapter it was stated that the necessary condition for profit maximisation was the equality of marginal cost with the output price. To establish this let $C(X^i)$ be the firm's cost function and write the profits for the ith firm in the industry producing X as

$$\pi_X^i = P_X X^i - C(X^i)$$

from which the necessary condition for maximisation is

$$\partial \pi_X^i / \partial X^i = P_X - \partial C / \partial X^i = 0$$

which is

$$P_X = \partial C / \partial X^i$$

i.e., price equals marginal cost.