Appendix 6.1 The least-cost theorem and pollution control instruments

This appendix is structured as follows. In Part 1, we define the notation used and set the scene for what follows. Then in Part 2 we derive a necessary condition for pollution control to be cost-effective: that is, to attain any given target at least cost. An EPA has several instruments available for attaining a pollution (or pollution abatement) target. Here we consider three classes of instrument: quantitative regulations (a variant of command and control) in Part 3; an emissions tax (Parts 4 and 5); an emissions abatement subsidy (Part 6); and transferable emissions permits (Part 7). Collectively, Parts 3 to 7 take the reader through what an EPA would need to know, and how it could operate each of those instruments, in order to achieve a target at least cost. Finally in Part 8, we generalise previous results to the case of a non-uniformly-mixing pollutant.

Part 1 Introduction

There are N polluting firms, indexed $i = 1,..., N$. Each firm faces a fixed output price and fixed input prices, and maximises profits by an appropriate choice of output level (Q_i) and emission level (M_i). Emissions consist of a uniformly mixing pollutant, so that the source of the emission is irrelevant as far as the pollution damage is concerned.

Let $\hat{\Pi}_i$ be the maximised profit of the ith firm in the absence of any control over its emission level and in the absence of any charge for its emissions. This is its unconstrained maximum profit level. At this unconstrained profit maximum the firm's emission level is $\hat{M}_i$.

Let Π_i^* be the maximised profit of the *i*th firm when it is required to attain a level of emissions M_i^* < $\hat{M}_i$. This is its constrained maximum level of profits. To reduce emissions, some additional costs will have to be incurred or the firm's output level must change (or both). The constrained profit level will, therefore, be less than the unconstrained profit level. That is, $\Pi_i^* < \hat{\Pi}_i$.

We next define the firm's abatement costs, *C*, as unconstrained minus constrained profits:

$$C_i = \hat{\Pi}_i - \Pi_i^*$$

Abatement costs will be a function of the severity of the emissions limit the firm faces; the lower is this limit, the greater will be the firm's abatement costs. Let us suppose that this abatement cost function is quadratic. That is

$$C_i = \alpha_i - \beta_i M_i^* + \delta_i M_i^{*2} \tag{6.4}$$

We illustrate this abatement cost function in Figure 6.14. Note that that the abatement cost function is defined only over part of the range of the quadratic function. Abatement costs are zero when the emission limit is set at $\hat{M}_i$, the level the firm would have itself chosen to emit in the absence of control. Abatement costs are maximised when M_i^* = 0, and so the firm is prohibited from producing any emissions.

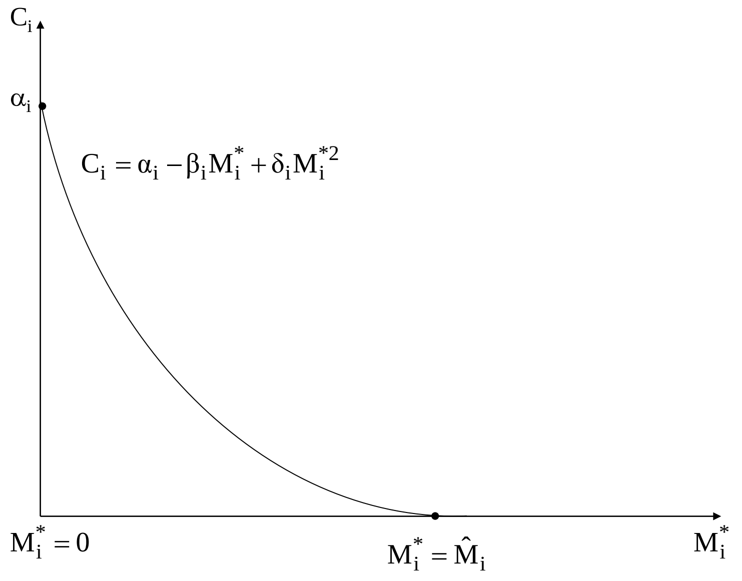

Figure 6.14 The firm's abatement cost function.

Two things should be said about equation 6.4. First, as each parameter is indexed by *i*, abatement costs are allowed to vary over firms. Second, the arguments that follow do not depend on the abatement cost function being quadratic. We have chosen that functional form for expositional simplicity only.

Part 2 The least-cost theorem

We now consider the problem of an environmental protection agency (EPA) meeting some standard for total emissions (from all *N* firms) at the least cost. Let M^* denote the predetermined total emission target. In the expressions that follow, the M_i^* variables are to be interpreted as endogenous, the values for which are not predetermined but emerge from the optimising exercise being undertaken. The problem can be stated as

$$\text{Min} \sum_{i=1}^{N} C_i \text{ subject to } M^* = \sum_{i=1}^{N} M_i^* \qquad (6.5)$$

The Lagrangian for this problem is

$$L = \sum_{i=1}^{N} C_i - \mu\left(M^* - \sum_{i=1}^{N} M_i^*\right)$$
$$= \sum_{i=1}^{N}\left(\alpha_i - \beta_i M_i^* + \delta_i M_i^{*\,2}\right) - \mu\left(M^* - \sum_{i=1}^{N} M_i^*\right) \qquad (6.6)$$

The necessary conditions for a least-cost solution are

$$\frac{\partial L}{\partial M^*{}_i} = -\beta_i + 2\delta_i M_i^* + \mu^* = 0,\ \ i = 1, 2, \ldots, N \qquad (6.7)$$

and

$$\frac{\partial L}{\partial \mu} = -M^* + \sum_{i=1}^{N} M_i{}^* = 0 \qquad (6.8)$$

Equations 6.7 and 6.8 give *N* + 1 equations in *N* + 1 unknowns. Solving these simultaneously gives each firm's emission limit, M_i^* (which now should be regarded as the *optimised* emissions limit for the firm), and the optimised shadow price of the pollution constraint (the Lagrange multiplier) μ*. Since μ* is constant over all firms, it can be seen from equation 6.7 that a least-cost pollution abatement programme requires that the marginal cost of abatement be equal over all firms.

There is a tricky issue relating to signs in equation 6.7. Notice that an increase in M_i^* corresponds to a relaxation of a pollution target (a decrease in required abatement) so the term ($-\beta_i + 2\delta_i M_i^*$) is the marginal cost of a *reduction in pollution abatement* being required of firm *i*. It will therefore be a negative quantity. This can be verified by looking at the slope of the C_i function in Figure 6.14.

By multiplying equation 6.7 through by minus one, we obtain

$$\beta_i - 2\delta_i M_i^* = \mu^* \qquad (6.7')$$

Here the term on the left-hand side ($\beta_i - 2\delta_i M_i^*$) is the firm's marginal cost of an increase in pollution abatement, a positive quantity. It follows from 6.7′ that μ* is also a positive quantity. This is consistent with the text of this chapter and the previous one.

Part 3 Least-cost pollution control using quantitative regulation

If the EPA knew each firm's abatement cost function (that is, it knew C_i for i = 1,..., N), then for any total emission standard it seeks, M^*, the system of equations 6.7 and 6.8 could be solved for M_i^* for each firm. The EPA could then tell each firm how much it could emit. The total quantity of emissions would, from equation 6.8, be reached exactly, and the target would, as the above theorem shows, be attained at least cost.

Part 4 Least-cost pollution control using an emissions tax

As an alternative to setting quantitative emissions controls on each firm, an emission tax could be used. If the EPA knew each firm's abatement cost function, then for any total emission standard it seeks, M^*, the system of equations 6.7 and 6.8 could be solved for the value of the shadow price of the pollution constraint, μ*. Note that, unlike M_i^*, this shadow price is constant for each firm. The EPA could then set a tax at a rate of t* per unit of emissions and charge each firm this tax on each unit of pollution it emitted. Profit-maximising behaviour would then lead each firm to produce M_i^* emissions, the least-cost solution.

To see why this should be so, note that in the absence of any quantity constraint on emissions, profit-maximising behaviour in the face of an emissions tax implies that the firm will minimise the sum of its abatement costs and pollution tax costs. That is, the firm chooses M_i to minimise CT_i, the total of its abatement and tax costs:

$CT_i = C_i + tM_i = \alpha_i - \beta_i M_i + \delta_i M_i^2 + t^* M_i$

The necessary condition is

$$\frac{\partial CT_i}{\partial M_i} = -\beta_i + 2\delta_i M_i^* + t^* = 0,\ i = 2,\ldots,N \qquad (6.9)$$

Clearly, if t^* in equation 6.9 is set equal to μ^* in equation 6.7, the necessary conditions 6.7 and 6.9 are identical, and so the tax instrument achieves the total emissions target at least cost.

Part 5 What role is there for a tax instrument where each firm's abatement cost functions are not known?

In general, the EPA will not know abatement costs. However, if an arbitrarily chosen tax rate, say $\overline{t}$, is selected, and each firm is charged that rate on each unit of emission, then *some* total quantity of emissions, say $\overline{M}$, will be realised at least cost. Of course, that amount $\overline{M}$ will in general be different from M^*. Only if $\overline{t} = t^*$ will $\overline{M}$ be identical to M^*. An iterative, trial-and-error process of tax rate change may enable the EPA to find the necessary tax rate to achieve a specific target.

Part 6 Least-cost pollution control using an emissions-abatement subsidy

Another method of obtaining a least-cost solution to an emissions target is by use of abatement subsidies. Suppose a subsidy of s^* is paid to each firm on each unit of emissions reduction below its unconstrained profit-maximising level, $\hat{M}_i$. Then profit-maximising behaviour implies that the firm will maximise total subsidy receipts less abatement costs. That is, the firm maximises

$CS_i = s(\hat{M}_i - M_i) - C_i = s(\hat{M}_i - M_i) - (\alpha - \beta_i M_i + \delta_i M_i^2)$

The necessary condition is

$$\frac{\partial \mathrm{CS}_i}{\partial M_i} = \beta_i - 2\delta_i M_i^* - s = 0, \quad i = 1, 2, \ldots, N \tag{6.10}$$

which, after multiplying through by –1, is identical to equation 6.9 if $s = t$. So, once again, if s in equation 6.10 is set equal to μ^* in equation 6.7, the necessary conditions 6.7 and 6.10 are identical, and so the subsidy instrument achieves the total emissions target at least cost. Moreover, this result demonstrates that in terms of their effects on emissions, a tax rate of t per unit of emissions is identical to a subsidy rate of s per unit of emissions abatement, provided $s = t$.

Part 7 Least-cost pollution control using transferable emissions permits

Suppose that the EPA issues to each firm licences permitting L_i^0 units of emissions. Firms are allowed to trade with one another in permits. The *i*th firm will trade in permits so as to minimise the sum of abatement costs and trade-acquired permits:

$$\begin{aligned} CL_i &= C_i + P\left(L_i - L_i^0\right) \\ &= \alpha_i - \beta_i M_i + \delta_i M_i^2 + P\left(L_i - L_i^0\right) \end{aligned} \tag{6.11}$$

where P is the market price of one emission permit. Given that L_i is the quantity of emissions the firm will produce after trade we can write this as

$$\begin{aligned} \mathrm{CL}_i &= C_i + P(L_i - L_i^0) \\ &= \alpha_i - \beta_i L_i + \delta_i L_i^2 + P(L_i - L_i^0) \end{aligned} \tag{6.12}$$

The necessary condition for minimisation is

$$\frac{\partial CL_i}{\partial L_i} = -\beta_i + 2\delta_i L_i^* + P = 0, \quad i = 1, 2, ..., N \tag{6.13}$$

which can be interpreted as the firm's demand function for permits.

If the EPA sets a total emissions target of M^* then M* is the total supply of permits

$$\text{and } M^* = \sum_{i=1}^{N} L_i^0 = \frac{\partial L}{\partial P} = \sum_{i=1}^{N} L_i \tag{6.14}$$

Now compare equations 6.13 and 6.14 with equations 6.7 and 6.8. These are identical if $P = \mu^*$ (remembering that $L_i = M^*_i$). Moreover, comparison of equation 6.13 with equations 6.9 and 6.10 shows that $P = t = s$. So by an initial issue of permits (distributed in any way) equal to the emissions target, the EPA can realise the target at least cost. Moreover, it can do so without knowledge of individual firms' abatement cost functions.

Part 8 Least-cost abatement for a non-uniformly-mixing pollutant

The target of the EPA is now in terms of ambient pollution levels rather than emission flows. Specifically the EPA requires that

$$A_j = \sum_{i=1}^{N} d_{ji} M_i \leq A_j^* \text{ for } j = 1, ... j \tag{6.15}$$

The problem for the EPA is to attain this target at least cost. We deal with the case where the same ambient target is set for each receptor area. This problem can be stated as

$$\begin{aligned} &\text{Min} \sum_{i=1}^{N} C_i \text{ subject to } A_j = \sum_{i=1}^{N} d_{ji} M \leq A^* \\ &\text{for } j = 1, ..., j \end{aligned} \tag{6.16}$$

The Lagrangian for this problem is

$$L = \sum_{i=1}^{N} C_i - \mu_1 \left(A^* - \sum_{i=1}^{N} d_{1i} M_i \right) - \ldots$$
$$- \mu_J \left(A^* - \sum_{i=1}^{N} d_{1i} M_i \right) \quad (6.17)$$

where $C_i = \alpha_i - \beta_i M_i + \delta_i M_i^2$

The necessary conditions for a least-cost solution are

$$\frac{\partial L}{\partial M_l^*} = -\beta_i + 2\delta_i M_i + \sum_{j=1}^{j=J} (\mu_j^* d_{ji}) = 0,$$
$$i = 1, 2, \ldots, N \quad (6.18)$$

and

$$\frac{\partial L}{\partial \mu_j} = -A^* + \sum_{i=1}^{N} d_{ji} M_i = 0 \text{ for } j = 1, \ldots, J \quad (6.19)$$

The system of equations 6.18 and 6.19 consists of $N + J$ equations which can be solved for the $N + J$ unknowns (M^*_i, i = 1,..., N and μ_j^*, j = 1,..., J).

Equation 6.18 can be written as

$$-\beta_i + 2\delta_i M_i = -\sum_{j=1}^{j=J} (\mu_j^* d_{ji}), \ i = 1, 2, \ldots, N \quad (6.20)$$

Then after multiplying through by –1, using MC_i to denote the *i*th firm's marginal cost of abatement, and expanding the sum on the right-hand side, we obtain

$$\text{MC}_i = \mu_1^* d_{1i} + \mu_2^* d_{2i} + \ldots \mu_J^* d_{Ji},$$
$$i = 1, 2, \ldots, N \quad (6.21)$$

The pair of equations 6.20 and 6.21 can be compared with the solution for the uniformly mixing pollution case, equation 6.7 multiplied by –1.