Appendix 8.1 A general framework for environmental input–output analysis

Figure A8.1 below (where the graphic is wrongly labelled as Figure 9.1) sets out schematically a general input–output system for analysing the interconnections between economic activity and the natural environment. The basis is the recognition that there are three types of linkage between the economy and the environment, which should properly be treated jointly. First, economic agents extract or exploit natural resources, including obvious forms of exploitation such as extraction of ores and minerals, fish harvesting and so on, but also in less obvious ways such as the 'consumption' of fresh air and landscape.

		(industries)		(abatement sectors)	(final buyers) n+m+1 h
		1,2	n	n+1n+m	
Stocks of natural resources	\rightarrow 1		П	v	VIII
	r				
		(extractive or primary industries)	I	IV	VII
	r+n	1000			
	r+n+1		ш	vi	IX
	(residuals)			YOU'S MUS	1999339986
	rinim			÷	

Figure 9.1 An extended input-output system

Second, the processing and consumption of these environmental resources yields residuals which are returned to the environment, and which may have undesirable economic, social or health effects, such as air pollution, soil degradation and loss of habitat. Attempts to eliminate, mitigate or compensate for these effects lead to the third type of economy–environment link, namely activities devoted to abatement or environmental renewal.

In Figure A8.1, the submatrices I and VII correspond to the conventional input–output table: I recording flows of goods and services between the *n* intermediate sectors of the economy, and VII recording deliveries to final buyers or users (private and government consumption, investment, exports). For simplicity, we assume here, as we did in the chapter itself, that each 'industry' produces a unique homogeneous 'commodity', thus avoiding the need for a more complex system of accounts which links industries and commodities.

Submatrix II records the extraction or direct use of natural resources by industries, involving a reduction in the vector of stocks of natural resources. The cell *ij* of submatrix II records the amount or volume of resource *i*, measured in physical units, used or consumed by industry *j* during a particular time period, say one year. Thus if resource *i* is water and industry *j* is water supply, the entry in cell *ij* records the volume of water collected and processed by the water supply sector: subsequent sales or deliveries of water to industry and households would appear in row r + j of submatrices I and VII.

Following conventional input–output modelling practice, if we assume a constant proportional relation between inputs of resources and outputs of industries, we can derive a submatrix of resource input coefficients in which the typical coefficient r_{ij} indicates the amount of resource *i* (in physical units) required per unit of output (typically measured in value units) of industry *j*. Pursuing the example above, r_{ij} would record the number of gallons of water required per million dollars output of the water supply industry.

Many of the cells in submatrix II will be zero, since only a limited number of industries are engaged in the direct extraction or harvesting of natural resources. Processed natural resources will be classified as industrial products and distributed along the rows of submatrices I, IV and VII.

Submatrix III records residual wastes generated by each industry, there being a separate row for each type of residual: thus an entry in cell gk in III records the amount of residual g generated by industry k in the accounting period concerned. Again following standard input–output practice, if we assume a constant proportional relationship between industry output and residuals generation, we can derive a submatrix of waste coefficients in which the typical element w_{gk} indicates the amount of waste element g produced per million dollars output of industry k. Note that although the elements of submatrix III are outputs rather than inputs, they are treated here in an identical way to the input flows in submatrices II and I. Obvious examples of this type of waste production are pollutants generated by industrial production and distribution, an example of which will be considered later in this appendix.

Columns n + 1 to n + m (submatrices IV, V and VI) represent residuals abatement or treatment activities. Note that although abatement activities are here accorded the status of separate industries, in practice such activities may be undertaken by and within the industries which are responsible for generating the residual concerned. For instance a firm which generates waste water may undertake

water purification 'on site' before discharging the water back into the environment. In the accounting system here, the mainstream production and purification activities would be recorded separately. Note also that in this schema, certain abatement/treatment activities may operate at zero levels.

Like other industries, abatement industries purchase goods and services from other industries (submatrix IV), and may also absorb natural resources directly (submatrix V, though this submatrix could well be empty). Moreover, like other industries the abatement sectors may themselves generate residual wastes (submatrix VI).

The output of the abatement industries may be expressed in value terms, as are typically the other industries in the table, or in physical units, as the amount of residual treated or eliminated. In the latter case the input coefficients (submatrix IV) would measure (constant) dollar inputs per ton of residual treated or eliminated. Again, for these industries we assume the Leontief technology of fixed proportional input coefficients.

The final columns of the table record sales or deliveries to final buyers, typically household (private) consumption, government consumption, investment, changes in stocks, exports and (a negative column of) imports, but each of these categories may be further disaggregated. One possibility is to disaggregate the investment column to separately identify capital expenditures directed towards the renewal of natural resources, such as reforestation, soil regeneration, fish stocks renewal, and so on. These activities then provide a link to the vector of stocks of natural resources at the beginning of the environment–economy–environment sequence in Figure 8.1, and are a step towards closure of the model system.

Submatrix VIII allows for the possibility of direct extraction or use of natural resources by final buyers (for example, fresh air, untreated water, fish caught for personal consumption, and so on), while submatrix IX includes residual wastes generated by households and other final buyers (CO₂, solid wastes, scrap, and so on).

More complex versions can be constructed, and alternative systems of accounting can be utilised, but the above schema captures the essential features of the environmental input–output system, from which a model can be constructed. Like the basic input–output model described in the chapter, the version presented below is an open, comparative-static model in which final demands are exogenous (determined outside the model). There are no explicit capacity constraints on outputs, or limits to the supply of factors of production, which is equivalent to treating factor supplies as completely elastic at prevailing factor prices.¹¹

¹¹ Other than natural resources, inputs of factors of production (labour and capital) are not shown in Figure A8.1, but the system could readily be extended to include them, in a manner similar to that used for natural resource flows.

To simplify the algebra, we assume in what follows that submatrices V and VIII are empty. For the n 'conventional' input–output sectors the balance equations are

$$X_{i} - \sum_{j=1}^{n} a_{ij} X_{j} - \sum_{q=n+1}^{n+m} a_{iq} Z_{q} = F_{i}$$
(8.27)

or

$$X - A_1 X - A_2 Z = F \tag{8.28}$$

where **X** is the output vector for the conventional industries, **Z** is the output vector for the abatement industries (to be discussed below) and **F** is a vector of deliveries to final buyers. (For convenience we assume here that **F** is a vector.) The coefficients $a_{ij} \in \mathbf{A_1}$ and $a_{iq} \in \Box \mathbf{A_2}$ are derived from the system of accounts in Figure A8.1 as

$$a_{ij} = X_{ij} / X_j \tag{8.29}$$

where X_{ij} is purchases of commodity *i* to produce output X_j , and

$$a_{iq} = X_{iq} / Z_q \tag{8.30}$$

where Z_q is the output of abatement sector q (or volume of residual q eliminated).

These assumptions of constant proportional input coefficients mirror those of the basic input–output model of the chapter text, and reflect the properties of the Leontief production function, notably constant returns to scale and zero substitution between inputs, in contrast to the more usual neoclassical function used elsewhere in this book.

For the residuals submatrices (III, VI and IX), the production or generation of residuals can be written as

$$P_{g} = \sum_{j=1}^{n} w_{gj} X_{j} + \sum_{q=n+1}^{n+m} w_{gq} Z_{q} + w_{gF}$$
(8.31)

where P_g is the amount of residual g generated by production, by abatement activities and by final demand.

In matrix form

$$\mathbf{P} = \mathbf{W}_1 \mathbf{X} + \mathbf{W}_2 \mathbf{Z} + \mathbf{W}_F \tag{8.32}$$

Equation 8.32 measures gross production of residuals. The net production is gross production less the volume treated or eliminated, which is the measured output of the abatement sector. How is this

determined? For residual g, the net production (the volume of the residual returned to the environment) can be written

$$D_{g} = Z_{g} - P_{g} = Z_{g} - \sum w_{gj} Xj - \sum w_{gq} Z_{q} - w_{gF}$$
(8.33)

where Z_g is the volume of residual g eliminated (the output of abatement sector g) and D_g is the net production of g (the volume not eliminated). Unless there is complete elimination, D_g will typically be negative, but its level may be amenable to control, and in ideal circumstances may be taken as a measure of the permitted level of net emission, waste or damage, where marginal damage and abatement costs are equal. By specifying this level as a negative final demand for the residual concerned, we have an equilibrium condition which enables us to determine the output of the abatement activity for that residual.

In matrix form,

$$\mathbf{Z} - \mathbf{W}_1 \mathbf{X} - \mathbf{W}_2 \mathbf{Z} = \mathbf{D} + \mathbf{W}_F \tag{8.34}$$

We now write the complete model in the following partitioned form:

$$\begin{bmatrix} \mathbf{X} \\ \mathbf{Z} \end{bmatrix} - \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{W}_1 & \mathbf{W}_2 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{Z} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{D} + \mathbf{W}_{\mathbf{F}} \end{bmatrix}$$
(8.35)

F and **D** are the vectors of independent variables: **F** is final demand for the standard commodities, and **D** is tolerated or permitted emission, waste or damage levels. Once **F** and **D** are specified, we can solve for **X** (industry output levels) and **Z** (abatement levels):

$$\begin{bmatrix} \mathbf{X} \\ \mathbf{Z} \end{bmatrix} - \begin{bmatrix} \mathbf{I} - \mathbf{A}_1 & -\mathbf{A}_2 \\ -\mathbf{W}\mathbf{1} & \mathbf{I} - \mathbf{W}_2 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F} \\ \mathbf{D} + \mathbf{W}_F \end{bmatrix}$$
(8.36)

Given the solution vector [X Z], the level of natural resource consumption can be calculated as

 $\mathbf{N} = \mathbf{R}[\mathbf{X} \ \mathbf{Z}] \tag{8.37}$

where \mathbf{R} is a matrix of natural resource input coefficients.

Although there have been numerous applications of environmental input–output models, none has attained the degree of detail and comprehensiveness of the model structure outlined above. Data problems have been severe, particularly in relation to the cost and production structures of abatement activities, but also in the definition and measurement of certain types of environmental degradation.

For this extended environmental input–output system, cost–price calculations can be introduced in a manner similar to that outlined in the third section of the chapter. In practice a range of approaches

have been adopted, governed partly by data availability and partly by the particular form of model. For instance, cost–price equations for the abatement sectors (submatrices IV, V and VI of Figure A8.1) could be formulated so that the price of a unit of abatement is determined by its cost of production. In practice, abatement or elimination activity may be undertaken by and within the industry or industries which generate the pollution, and it may be difficult to identify the costs of the abatement activity.

If adequate data on abatement costs are available or can be collected, price equations can be formulated for the abatement sectors as

$$P_g = \sum a_{ij} P_i + v_g \tag{8.38}$$

where P_g is the price or cost of eliminating one unit of pollutant g. How these equations are used in the extended model depends on the mechanism adopted for paying for abatement or elimination. If legislation obliges the polluter to pay, then polluting industries will buy abatement services from the abatement sectors, and the cost of these services will be included in the polluting industries' prices.

The output price for industry *j* now becomes

$$P_j = \sum a_{ij} P_i + \sum a_{gj} P_g + v_j \tag{8.39}$$

where a_{gj} is the quantity of abatement service g per unit of output which industry j is required to purchase, and P_g is the unit cost of abatement service g. The general solution to the extended price model then becomes

$$P = (I - A')^{-1}v$$
(8.40)

where **P**, **v** and **A** now include the abatement sectors. Alternatively, abatement/treatment may be financed through general taxation. In this case, polluting industry prices are unaffected (at least directly). Abatement services are provided by or purchased by government (central or local) and delivered to consumers as a public service (for example, river purification, household waste collection, nuclear waste disposal).