

Chapter 17.mw

```
> restart;  
> with(DEtools):  
> with(plots):
```

PART A: FISHERY MODEL EQUATIONS

```
> H(E,S)=e*E*S; Diff(S,t) =G(S)-H(E,S);  
H(E,S) = e E S
```

$$\frac{\partial}{\partial t} S = G(S) - H(E, S) \quad (1)$$

where

```
> G(S) := g*S*(1-(S/S[MAX]));
```

$$G(S) := g S \left(1 - \frac{S}{S_{MAX}} \right) \quad (2)$$

For the **open access fishery only** we have

```
> Diff(E,t)=delta*J*E;
```

$$\frac{\partial}{\partial t} E = \delta J E \quad (3)$$

where J = Profit per unit effort and so

```
> J=(P*H-w*E)/E;
```

$$J = \frac{P H - w E}{E} \quad (4)$$

The baseline parameter values used in this illustration are:

```
> g=0.15; S[MAX]=1; e=0.015; P=200; delta=0.4; w=0.6;
```

$$g = 0.15$$

$$S_{MAX} = 1$$

$$e = 0.015$$

$$P = 200$$

$$\delta = 0.4$$

$$w = 0.6$$

(5)

[If you wish to change parameter values from this baseline set, it will be necessary to edit the document below at various places; the spreadsheet is not currently set up to allow one set of parameter values to be input at the top of the sheet and to automatically update subsequently whenever the user changes those.]

PART B: OBTAINING THE OPEN ACCESS STEADY STATE SOLUTION

Defining the equations of the open access model:

$$\begin{aligned} > \text{eqn1} := H = e * E * S; \\ \text{eqn1} &:= H = e E S \end{aligned} \quad (6)$$

$$\begin{aligned} > \text{eqn2} := J = (P * H - w * E) / E; \\ \text{eqn2} &:= J = \frac{P H - w E}{E} \end{aligned} \quad (7)$$

$$\begin{aligned} > \text{eqn3} := \text{delta} * J = 0; \\ \text{eqn3} &:= \delta J = 0 \end{aligned} \quad (8)$$

$$\begin{aligned} > \text{eqn4} := H = g * S * (1 - (S / S[\text{MAX}])); \\ \text{eqn4} &:= H = g S \left(1 - \frac{S}{S_{\text{MAX}}} \right) \end{aligned} \quad (9)$$

THE OPEN ACCESS SOLUTION IN GENERAL PARAMETRIC FORM:

$$\begin{aligned} > \text{solve} (\{\text{eqn1}, \text{eqn2}, \text{eqn3}, \text{eqn4}\}, \{S, E, H, J\}); \\ \left\{ E = \frac{g (S_{\text{MAX}} P e - w)}{e^2 S_{\text{MAX}} P}, H = \frac{g (S_{\text{MAX}} P e - w) w}{e^2 S_{\text{MAX}} P^2}, J = 0, S = \frac{w}{P e} \right\} \end{aligned} \quad (10)$$

Note that the expressions just given for E, S and H are identical (although written slightly differently) to Equations 17.20, 17.21 and 17.22 in the text and in Table 17.2.

OPEN ACCESS STEADY STATE NUMERICAL SOLUTIONS

$$\begin{aligned} > \text{eq1} := \text{subs} (e = 0.015, \text{eqn1}); \\ \text{eq1} &:= H = 0.015 E S \end{aligned} \quad (11)$$

$$\begin{aligned} > \text{eq2} := \text{subs} (\{P = 200, w = 0.6\}, \text{eqn2}); \\ \text{eq2} &:= J = \frac{200 H - 0.6 E}{E} \end{aligned} \quad (12)$$

$$\begin{aligned} > \text{eq3} := \text{subs} (\text{delta} = 0.4, \text{eqn3}); \\ \text{eq3} &:= 0.4 J = 0 \end{aligned} \quad (13)$$

$$\begin{aligned} > \text{eq4} := \text{subs} (g = 0.15, S[\text{MAX}] = 1, \text{eqn4}); \\ \text{eq4} &:= H = 0.15 S (1 - S) \end{aligned} \quad (14)$$

$$\begin{aligned} > \text{solve} (\{\text{eq1}, \text{eq2}, \text{eq3}, \text{eq4}\}, \{S, E, H, J\}); \\ \{E = 8., H = 0.02400000000, J = 0., S = 0.2000000000\} \end{aligned} \quad (15)$$

(These are the solution values reported in Box 17.2.)

Part C: GENERATING & PLOTTING THE PHASE DIAGRAM FOR THE OPEN ACCESS SOLUTION

```
> restart;
> with(DEtools):
> with(plots):with(linalg):
```

The two differential equations are given below in 'diffsys' in parametric form, and then in 'eqs5' in numerical form. These consist of Equations 17.8 (with appropriate functions substituted in) and the open access entry rule Equation 17.13 (also given explicitly in Table 17.1) $dE/dt = \delta(PeS - wE)$

```
> diffsys:={diff(S(t),t)=g*S(t)*(1-(S(t)/S[MAX]))-e*E(t)*S(t),diff
(E(t),t)=delta*(P*e*S(t)-w)*E(t)};
```

$$\text{diffsys} := \left\{ \frac{d}{dt} E(t) = \delta (P e S(t) - w) E(t), \frac{d}{dt} S(t) = g S(t) \left(1 - \frac{S(t)}{S_{MAX}} \right) - e E(t) S(t) \right\} \quad (16)$$

```
> eqs5:=subs({S[MAX]=1, g=0.15, e=0.015, P=200, delta=0.4, w=0.6},
diffsys);
```

$$\text{eqs5} := \left\{ \frac{d}{dt} E(t) = 0.4 (3.000 S(t) - 0.6) E(t), \frac{d}{dt} S(t) = 0.15 S(t) (1 - S(t)) - 0.015 E(t) S(t) \right\} \quad (17)$$

Inputting initial conditions for the dynamic simulation on the phase diagram:

```
> init5:=[[S(0)=0.99,E(0)=0.01]];
init5 := [[S(0)=0.99,E(0)=0.01]] (18)
> curves51:=DEplot(eqs5,[E(t),S(t)],t=0..200,S=0..1, E=0..55,
init5,stepsize=.2,arrows=slim, linecolour=blue,thickness=1):
```

Generating the Equilibrium lines for S and E for the phase diagram

```
> equilibriumS:=0=g*S*(1-S/S[MAX])-e*E*S;
```

$$\text{equilibriumS} := 0 = g S \left(1 - \frac{S}{S_{MAX}} \right) - e E S \quad (19)$$

```
> equilibriumE:=0=delta*(P*e*E*S-w*E);
```

$$\text{equilibriumE} := 0 = \delta (P e E S - w E) \quad (20)$$

```
> oasol:=solve ({equilibriumS,equilibriumE}, {S,E});
```

$$oasol := \{E=0, S=0\}, \{E=0, S=S_{MAX}\}, \left\{ E = \frac{g (P e S_{MAX} - w)}{P e^2 S_{MAX}}, S = \frac{w}{P e} \right\} \quad (21)$$

```
> oasol[3];
```

$$\left\{ E = \frac{g (P e S_{MAX} - w)}{P e^2 S_{MAX}}, S = \frac{w}{P e} \right\} \quad (22)$$

```
> oasolution:=subs({g=0.15, S[MAX]=1, e=0.015, w=0.6, e=0.015, w=0.6, P=200}, oasol[3]);
```

$$oasolution := \{E = 8.000000000, S = 0.2000000000\} \quad (23)$$

```
> S(eqE) := solve (equilibriumE, S);
```

$$S(eqE) := \frac{w}{P e} \quad (24)$$

```
> S(eqS) := solve (equilibriumS, S);
```

$$S(eqS) := 0, -\frac{S_{MAX} (-g + e E)}{g} \quad (25)$$

```
> S(eqS)[1]; S(eqS)[2];
```

$$0$$

$$-\frac{S_{MAX} (-g + e E)}{g} \quad (26)$$

```
> S22:=subs({g=0.15, S[MAX]=1, e=0.015, w=0.6}, S(eqS)[2]);
```

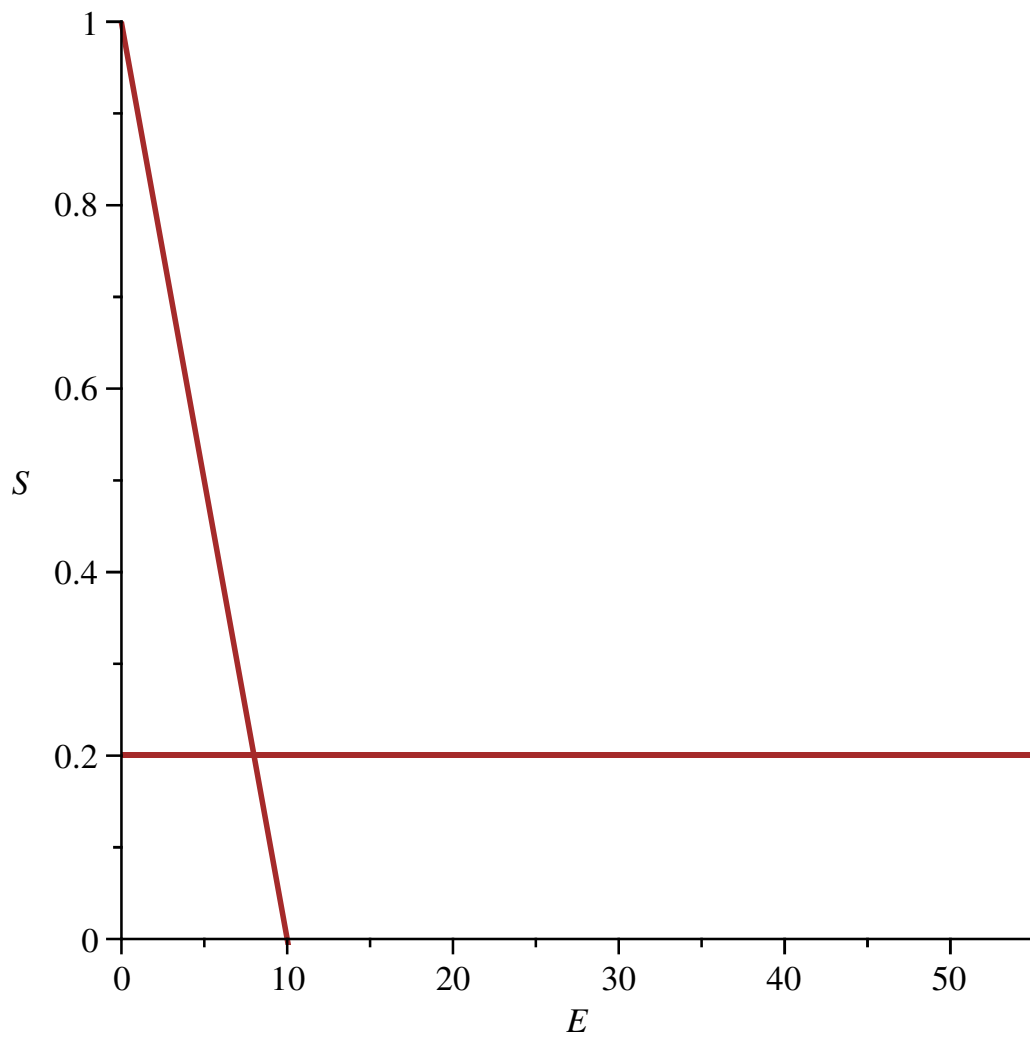
$$S22 := 1.000000000 - 0.1000000000 E \quad (27)$$

```
> S11:=subs({e=0.015, w=0.6, P=200}, S(eqE));
```

$$S11 := 0.2000000000 \quad (28)$$

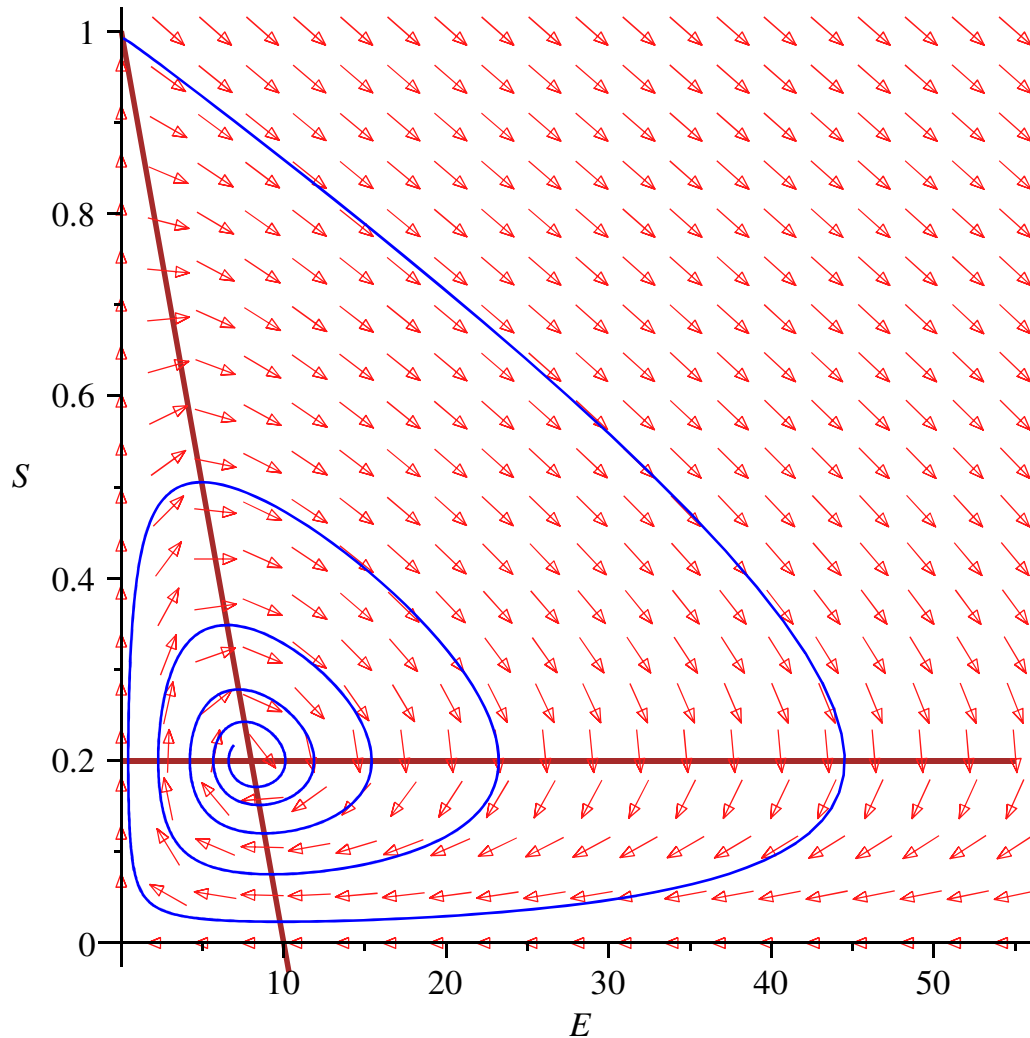
```
> phaselines :=plot({S22, S11}, E=0..55, S=0..1, colour=brown, thickness=2);
```

```
> display(phaselines);
```



We now generate Figure 17.6 in the text:

```
> display({curves51, phaselines});
```



Part D: Obtaining linear approximations for the non-linear system (open access model)

$$\begin{aligned} &> \text{taylor}((0.15*(1-s)-0.015*e)*s, s=S, 2); \\ &\quad (0.15 - 0.15 S - 0.015 e) S + (0.15 - 0.30 S - 0.015 e) (s - S) + O((s - S)^2) \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{taylor}((0.15*(1-s)-0.015*e)*s, e=E, 2); \\ &\quad (0.15 - 0.15 s - 0.015 E) s - 0.015 s (e - E) \end{aligned} \quad (30)$$

$$\begin{aligned} &> \text{taylor}((20*s-4)*e, s=S, 2); \\ &\quad (20 S - 4) e + 20 e (s - S) \end{aligned} \quad (31)$$

$$\begin{aligned} &> \text{taylor}((20*s-4)*e, e=E, 2); \\ &\quad (20 s - 4) E + (20 s - 4) (e - E) \end{aligned} \quad (32)$$

$$\begin{aligned} &> 0.15-0.30*0.2-0.015*8;-0.015*0.2;20*8;20*0.2-4; \\ &\quad -0.030 \\ &\quad -0.0030 \\ &\quad 160 \end{aligned} \quad (33)$$

$$0. \quad (33)$$

```
> alpha:=matrix([[ -0.03, -0.003], [160, 0]]);
```

$$\alpha := \begin{bmatrix} -0.03 & -0.003 \\ 160 & 0 \end{bmatrix} \quad (34)$$

```
> eigenvalues(alpha);
```

$$-0.01500000000 + 0.6926579242 I, -0.01500000000 - 0.6926579242 I \quad (35)$$

```
> z:=eigenvectors(alpha);
```

$$z := \left[-0.01500000000 + 0.6926579242 I, 1, \left[\begin{bmatrix} -0.004329112026 - 0.00009375000000 I & 0. + 1. I \end{bmatrix} \right], \left[-0.01500000000 - 0.6926579242 I, 1, \left[\begin{bmatrix} -0.004329112026 + 0.00009375000000 I & 0. - 1. I \end{bmatrix} \right] \right] \right] \quad (36)$$

PART E: STATIC PRIVATE PROPERTY STEADY STATE SOLUTION

Note that the equation $P \cdot e \cdot S_{MAX} - 2 \cdot P \cdot (e^2) \cdot E \cdot (S_{MAX}/g) = w$ in the expression below is the marginal condition for profit maximisation, given by Equation 17.25 in the textbook.

```
> restart;
```

```
> with(DEtools):
```

```
> with(plots):with(linalg):
```

```
> eqn1:= H=e*E*S;
```

$$eqn1 := H = e E S \quad (37)$$

```
> eqn2:= J=(P*H-w*E)/E;
```

$$eqn2 := J = \frac{P H - w E}{E} \quad (38)$$

```
> zz:=solve ({eqn1, eqn2, (P*e*S[MAX]-2*P*(e^2)*E*(S[MAX]/g))=w,
H=g*S*(1-(S/S[MAX]))}, {S, E, H, J});
```

$$zz := \left\{ E = \frac{1}{2} \frac{g (P e S_{MAX} - w)}{P e^2 S_{MAX}}, H = 0, J = -w, S = 0 \right\}, \left\{ E = \frac{1}{2} \frac{g (P e S_{MAX} - w)}{P e^2 S_{MAX}}, H = \frac{1}{4} \frac{g (P e S_{MAX} - w) (P e S_{MAX} + w)}{e^2 P^2 S_{MAX}}, J = \frac{1}{2} P e S_{MAX} - \frac{1}{2} w, S = \frac{1}{2} \frac{P e S_{MAX} + w}{P e} \right\} \quad (39)$$

Only the second set of these solutions is non-trivial. Notice that these correspond exactly to the formulae for S*PP, E*PP and H*PP in Table 17.4 in the textbook. We now copy this second set, substitute numerical parameter values, and solve for numerical solution:

```
> eqH := H = 1/4/e^2*g*(P*e*S[MAX]-w)/P^2/S[MAX]*(P*e*S[MAX]+w);
eqJ := J=1/2*P*e*S[MAX]-1/2*w; eqS := S=1/2*(P*e*S[MAX]+w)/P/e;
eqE := E=1/2*g*(P*e*S[MAX]-w)/P/e^2/S[MAX];
```

$$eqH := H = \frac{1}{4} \frac{g (P e S_{MAX} - w) (P e S_{MAX} + w)}{e^2 P^2 S_{MAX}}$$

$$eqJ := J = \frac{1}{2} P e S_{MAX} - \frac{1}{2} w$$

$$eqS := S = \frac{1}{2} \frac{P e S_{MAX} + w}{P e}$$

$$eqE := E = \frac{1}{2} \frac{g (P e S_{MAX} - w)}{P e^2 S_{MAX}} \quad (40)$$

```
> eqHH :=subs({g=0.15,S[MAX]=1,w=0.6,P=200,e=0.015},eqH);eqJJ :=
subs({g=0.15,S[MAX]=1,w=0.6,P=200,e=0.015},eqJ);eqSS :=subs({g=
0.15,S[MAX]=1,w=0.6,P=200,e=0.015},eqS);eqEE :=subs({g=0.15,S
[MAX]=1,w=0.6,P=200,e=0.015},eqE);
```

$$eqHH := H = 0.03600000000$$

$$eqJJ := J = 1.200000000$$

$$eqSS := S = 0.6000000000$$

$$eqEE := E = 4.000000000 \quad (41)$$

These solution values correspond exactly to those given in Table 17.4 and Table 17.6.

PART F: NPV MAXIMISING FISHERY MODEL SOLUTION

CASE 1: For downward sloping linear demand curve (this is not dealt with explicitly in the textbook)

```
> restart;
> with(DEtools):
> with(plots):
```

The second part of eqng that follows (the bit starting diff(H(t)) =) is Equation 17.68 in the appendices to chapter 17 for the text:

```
> eqng :={diff(S(t),t)=g*S(t)*(1-S(t))-H(t),diff(H(t),t)=-1/b}
```



```
* ((i-g+(2*g*S(t)))*(a-b*H(t)-(w/(e*S(t))))-(w*g/(e*S(t)))*(1-
(S(t)))));
```

$$eqng := \left\{ \frac{d}{dt} H(t) = - \frac{(i-g+2gS(t)) \left(a - bH(t) - \frac{w}{eS(t)} \right) - \frac{wg(1-S(t))}{eS(t)}}{b}, \right. \quad (42)$$

$$\left. \frac{d}{dt} S(t) = gS(t)(1-S(t)) - H(t) \right\}$$

```
> init51 := [[S(0)=0.6, H(0)=.06], [S(0)=0.35, H(0)=0.02], [S(0)=
0.32, H(0)=0.005], [S(0)=0.55, H(0)=0.045]];
```

```
init51 := [[S(0) = 0.6, H(0) = 0.06], [S(0) = 0.35, H(0) = 0.02], [S(0) = 0.32, H(0)
= 0.005], [S(0) = 0.55, H(0) = 0.045]] \quad (43)
```

```
> eqns := subs({g=0.15, S[MAX]=1, w=0.6, a=250, b=2000, i=0.1, e=
0.015}, eqng);
```

$$eqns := \left\{ \frac{d}{dt} H(t) = - \frac{1}{2000} (-0.05 + 0.30 S(t)) \left(250 - 2000 H(t) - \frac{40.00000000}{S(t)} \right) \right. \quad (44)$$

$$\left. + \frac{0.003000000000 (1 - S(t))}{S(t)}, \frac{d}{dt} S(t) = 0.15 S(t) (1 - S(t)) - H(t) \right\}$$

To generate the two curves for the phaselines, need to insert two equations in {(),()} to the right of "plot" below. First of these is obtained by setting dS/dt = 0 giving H = G and then writing the expression

gS - sS^2 below (with g = 0.15 in this case)

Second equation is obtained by finding from the equation dh/dt = 0 an equation from which H is expressed as a function of S, and then inserting the function in the place below). To do this, cut the relevant expression from eqns above, and solve for H.

```
> solve(0=-1/2000*(-.5e-1+.30*S)*(250-2000*H-40.0000000/S)
+.300000000e-2/S*(1-S), H);
```

$$\frac{0.005000000000 (-37. S - 8. + 150. S^2)}{S (-1. + 6. S)} \quad (45)$$

```
> eqq11:= 0 = -1/2000*((0.1-0.15*(1-2*S))*(250-2000*H-0.6/(0.015*
S))-(0.6/(0.015*S^2))*0.15*(S-S^2));
```

$$eqq11 := 0 = - \frac{1}{2000} (-0.05 + 0.30 S) \left(250 - 2000 H - \frac{40.00000000}{S} \right) \quad (46)$$

$$+ \frac{0.003000000000 (S - S^2)}{S^2}$$

This expression is now ready for cutting and pasting in the PLOT command below. But first we obtain

the steady state solutions:

```
> exp2:= H = 0.15*(S*(1-S));
```

$$\text{exp2} := H = 0.15 S (1 - S) \quad (47)$$

```
> exp3 := P = 250 -2000*H;
```

$$\text{exp3} := P = 250 - 2000 H \quad (48)$$

```
> solve ({eqq11, exp2, exp3}, {H, S,P});
```

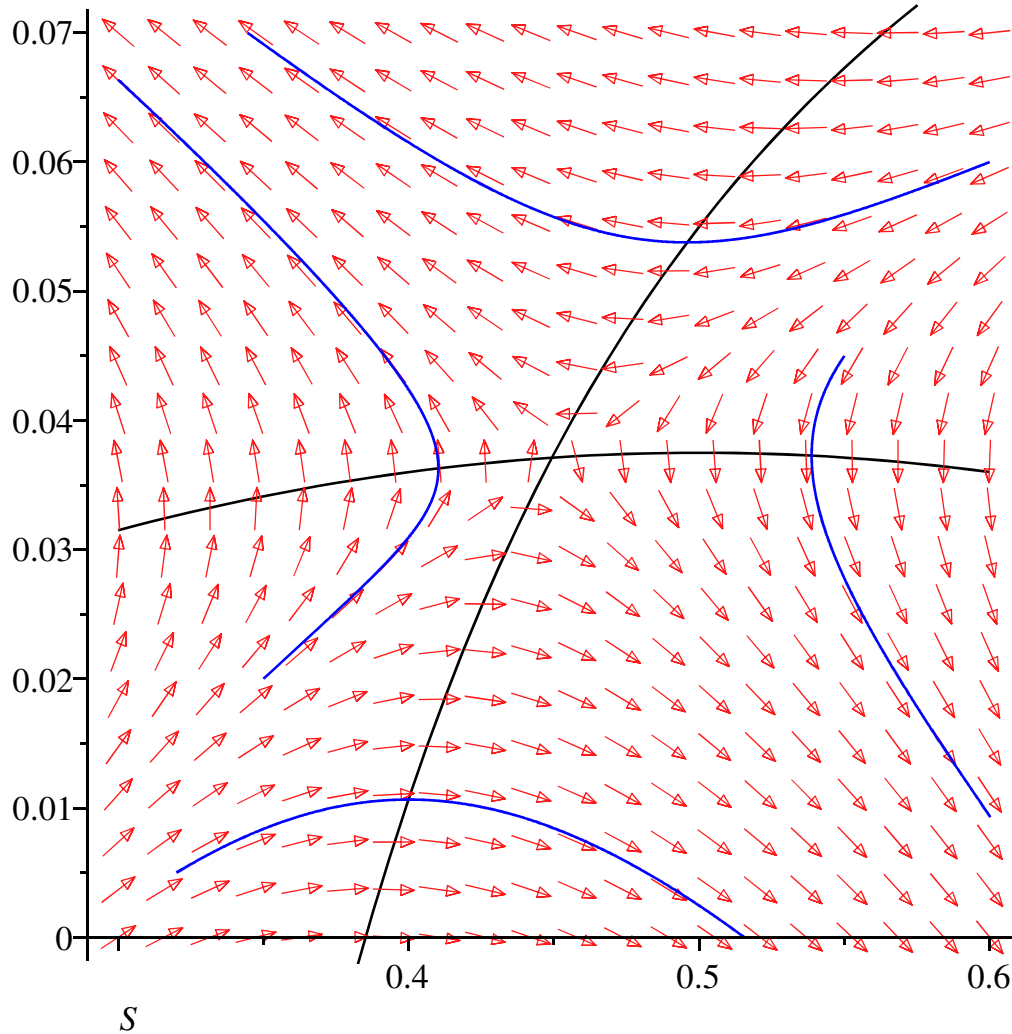
$$\{H=0.03711423127, P=175.7715375, S=0.4492871659\}, \{H=0.1275151340 + 0.01827324506 I, P=-5.030267978 - 36.54649012 I, S=0.4217689893 + 0.7786019421 I\}, \{H=-0.02131116591, P=292.6223318, S=-0.1261584779\}, \{H=0.1275151340 - 0.01827324506 I, P=-5.030267978 + 36.54649012 I, S=0.4217689893 - 0.7786019421 I\} \quad (49)$$

Note that only one of these sets is the appropriate solution (the one in which all variables are real valued and positive).

```
> curves51:=DEplot(eqns, [S(t), H(t)], t=0..200, S=0.3..0.6, H=0.07, init51, stepsize=0.02, linecolour=blue, thickness=1, arrows=SLIM);
```

```
> phaselines :=plot({(0.15*S-0.15*S^2), (.5000000000e-2*(-37.*S(t)-8.+150.*S(t)^2)/S(t)/(-1.+6.*S(t)))}, S=0.3..0.6, colour=black);
```

```
> display({curves51, phaselines});
```



Case (b) CONSTANT PRICE CASE

For perfectly elastic linear demand curve (P is constant)

```
> restart;
> with(DEtools):
> with(plots):
> solve ({(i*p) = p*(g - 2*g*S/S[MAX]) + (w*H)/(e*S^2)}, {H});
```

$$\left\{ H = \frac{p (i S_{MAX} - g S_{MAX} + 2 g S) e S^2}{S_{MAX} w} \right\} \quad (50)$$

```
> eqna := H = p*(i*S[MAX]-g*S[MAX]+2*g*S)/S[MAX]/w*e*S^2;
```

$$eqna := H = \frac{p (i S_{MAX} - g S_{MAX} + 2 g S) e S^2}{S_{MAX} w} \quad (51)$$

```
> eqnb :=subs({p=P-w/(e*S)},eqna);
```

$$eqnb := H = \frac{\left(P - \frac{w}{eS}\right) (iS_{MAX} - gS_{MAX} + 2gS) eS^2}{S_{MAX} w} \quad (52)$$

> eqnc := subs({H = g*(1-S/S[**MAX**)]*S}, eqnb);

$$eqnc := g \left(1 - \frac{S}{S_{MAX}}\right) S = \frac{\left(P - \frac{w}{eS}\right) (iS_{MAX} - gS_{MAX} + 2gS) eS^2}{S_{MAX} w} \quad (53)$$

> solve ({g*(1-S/S[**MAX**)]*S = (P-w/e/S)*(i*S[**MAX**]-g*S[**MAX**]+2*g*S)/S[**MAX**]/w*e*S^2}, {S});

$$\left\{S = \frac{1}{4} \frac{1}{Peg} \left(-PeiS_{MAX} + PegS_{MAX} + gw\right. \quad (54)$$

+

$$\left(P^2 e^2 i^2 S_{MAX}^2 - 2P^2 e^2 i S_{MAX}^2 g + 6PegwiS_{MAX} + P^2 e^2 g^2 S_{MAX}^2\right.$$

$$\left. + 2Peg^2 S_{MAX} w + g^2 w^2\right)^{1/2}\bigg\}, \left\{S = -\frac{1}{4} \frac{1}{Peg} \left(PeiS_{MAX} - PegS_{MAX} - gw\right.$$

+

$$\left(P^2 e^2 i^2 S_{MAX}^2 - 2P^2 e^2 i S_{MAX}^2 g + 6PegwiS_{MAX} + P^2 e^2 g^2 S_{MAX}^2\right.$$

$$\left. + 2Peg^2 S_{MAX} w + g^2 w^2\right)^{1/2}\bigg\}$$

> eqnd := S = 1/4/P/e/g*(-P*e*i*S[**MAX**]+P*e*g*S[**MAX**]+g*w+sqrt(P^2*e^2*i^2*S[**MAX**]^2-2*P^2*e^2*i*S[**MAX**]^2*g+6*P*e*g*w*i*S[**MAX**]+P^2*e^2*g^2*S[**MAX**]^2+2*P*e*g^2*S[**MAX**] *w+g^2*w^2));

$$eqnd := S = \frac{1}{4} \frac{1}{Peg} \left(-PeiS_{MAX} + PegS_{MAX} + gw\right. \quad (55)$$

+

$$\left(P^2 e^2 i^2 S_{MAX}^2 - 2P^2 e^2 i S_{MAX}^2 g + 6PegwiS_{MAX} + P^2 e^2 g^2 S_{MAX}^2\right.$$

$$\left. + 2Peg^2 S_{MAX} w + g^2 w^2\right)^{1/2}\bigg\}$$

> eqne := subs({g=0.15, S[**MAX**]=1, w=0.6, P=200, i=0, e=0.015}, eqnd);
eqne := S = 0.6000000000

(56)

Note that the solution value for this case ($S^* = 0.6$) is equal to that given in Table 17.6 for the PV maximising fishery for the case where $i = 0$.

Part G: DYNAMIC PRESENT VALUE MAXIMISING

FISHERY MODEL: OBTAINING THE STEADY STATE SOLUTION

```
> restart;with(DEtools):
> with(plots):with(linalg):
```

Obtain the derivative dG/dS:

```
> GGS :=g*(1-S/S[MAX])*S;
```

$$GGS := g \left(1 - \frac{S}{S_{MAX}} \right) S \quad (57)$$

```
> diff(GGS,S);
```

$$-\frac{g S}{S_{MAX}} + g \left(1 - \frac{S}{S_{MAX}} \right) \quad (58)$$

We can obtain this derivative in a slightly simpler form by expanding our expression for G(S) before differentiating it:

```
> EqG :=g*S - (g*S^2)/S[MAX];
```

$$EqG := g S - \frac{g S^2}{S_{MAX}} \quad (59)$$

```
> dGdS :=diff(EqG,S);
```

$$dGdS := g - \frac{2 g S}{S_{MAX}} \quad (60)$$

Note that this can also be written as

```
> dGdS2 := g(1-2*S/S[MAX]);
```

$$dGdS2 := g \left(1 - \frac{2 S}{S_{MAX}} \right) \quad (61)$$

We next define the fishery production function, $H = H(E,S)$ and express this in terms of effort, E:

```
> PF :=H=e*E*S;
```

$$PF := H = e E S \quad (62)$$

```
> E1 :=solve(PF,E);
```

$$E1 := \frac{H}{e S} \quad (63)$$

The cost function is given by:

```
> C1 :=C(H,S)=w*E;
```

$$C1 := C(H, S) = w E \quad (64)$$

which can be further expressed explicitly in terms of H and S as:

```
> C2 :=subs(E=E1, C1);
```

$$C2 := C(H, S) = \frac{wH}{eS} \quad (65)$$

We next obtain the partial derivatives dC/dH and dC/dS:

> eq1 :=rhs(C2);

$$eq1 := \frac{wH}{eS} \quad (66)$$

> dCdH :=diff(eq1,H);

$$dCdH := \frac{w}{eS} \quad (67)$$

> dCdS :=diff(eq1,S);

$$dCdS := -\frac{wH}{eS^2} \quad (68)$$

First order condition 17.29 is given by

> FOC1 :=p=P-diff(C(H),H);

$$FOC1 := p = P - \left(\frac{d}{dH} C(H) \right) \quad (69)$$

which after substitution of dC/dH from above gives:

> FOC11 := p=P-dCdH;

$$FOC11 := p = P - \frac{w}{eS} \quad (70)$$

> FOC111 :=rhs(FOC11);

$$FOC111 := P - \frac{w}{eS} \quad (71)$$

First order condition (17.32) is given by:

> FOC2 :=i*p=p*diff(G(S),S)-diff(C(S),S);

$$FOC2 := ip = p \left(\frac{d}{dS} G(S) \right) - \left(\frac{d}{dS} C(S) \right) \quad (72)$$

Substituting previously obtained expressions for the derivatives dG/dS and dC/dS into this first order condition gives:

> FOC21 :=i*p=p*dGdS-dCdS;

$$FOC21 := ip = p \left(g - \frac{2gS}{S_{MAX}} \right) + \frac{wH}{eS^2} \quad (73)$$

Solve the first order condition to obtain an expression for H as a function of p (net price) and S:

> qH :=H =solve(FOC21,H);

$$qH := H = \frac{p (iS_{MAX} - gS_{MAX} + 2gS) eS^2}{S_{MAX} w} \quad (74)$$

Substitute the first order condition $p = P - dC/dH$ for the unknown **variable** p to give H as a function of S and the gross price **parameter**, P :

> **qH1 := subs (p=FOC111, qH) ;**

$$qH1 := H = \frac{\left(P - \frac{w}{eS}\right) (iS_{MAX} - gS_{MAX} + 2gS) eS^2}{S_{MAX} w} \quad (75)$$

Given that $G(S) = H$ in steady state, substitute the biological growth function, $G(S)$ into the previous equation to get one equation expressing the steady state stock in terms of the model parameters alone:

> **qH2 :=subs (H=EqGS, qH1) ;**

$$qH2 := EqGS = \frac{\left(P - \frac{w}{eS}\right) (iS_{MAX} - gS_{MAX} + 2gS) eS^2}{S_{MAX} w} \quad (76)$$

The next few lines put expression $qH2$ into a more manageable form:

> **qH3 :=rhs (qH2) ;**

$$qH3 := \frac{\left(P - \frac{w}{eS}\right) (iS_{MAX} - gS_{MAX} + 2gS) eS^2}{S_{MAX} w} \quad (77)$$

> **qH4 :=simplify(%);**

$$qH4 := \frac{(PeS - w) (iS_{MAX} - gS_{MAX} + 2gS) S}{S_{MAX} w} \quad (78)$$

and so we have:

> **g*(1-S/S[MAX])*S = (P*e*S-w)*(i*S[MAX]-g*S[MAX]+2*g*S)*S/(S[MAX]*w);**

$$g \left(1 - \frac{S}{S_{MAX}}\right) S = \frac{(PeS - w) (iS_{MAX} - gS_{MAX} + 2gS) S}{S_{MAX} w} \quad (79)$$

Re-expressing the term in parentheses on the left-hand side and then dividing both sides of this equation by S gives

> **qH5 :=g((S[MAX]-S)/S[MAX]) = (P*e*S-w)*(i*S[MAX]-g*S[MAX]+2*g*S)/(S[MAX]*w);**

$$qH5 := g \left(\frac{S_{MAX} - S}{S_{MAX}}\right) = \frac{(PeS - w) (iS_{MAX} - gS_{MAX} + 2gS)}{S_{MAX} w} \quad (80)$$

Multiplying both sides by S_{MAX}/g :

> **qH7 :=(S[MAX]-S) = (P*e*S-w)*(i*S[MAX]-g*S[MAX]+2*g*S)/(w*g);**

$$qH7 := S_{MAX} - S = \frac{(PeS - w)(iS_{MAX} - gS_{MAX} + 2gS)}{wg} \quad (81)$$

and so

$$\begin{aligned} > qH8 := S = (P*e*S-w)*(-i*S[MAX]+g*S[MAX]-2*g*S)/(w*g)+S[MAX]; \\ qH8 := S = \frac{(PeS - w)(-iS_{MAX} + gS_{MAX} - 2gS)}{wg} + S_{MAX} \end{aligned} \quad (82)$$

We are now in a position to obtain an explicit solution for S* in terms of model parameters alone (note that we will obtain two solutions as this is a quadratic in terms of S):

$$\begin{aligned} > qS1 := \text{solve}(qH8, S); \\ qS1 := \frac{1}{4} \frac{1}{Peg} \left(-PeiS_{MAX} + PegS_{MAX} + wg \right. \\ &+ \\ &\left. (P^2 e^2 i^2 S_{MAX}^2 - 2P^2 e^2 i S_{MAX}^2 g + 6PegwiS_{MAX} + P^2 e^2 g^2 S_{MAX}^2 \right. \\ &\left. + 2Peg^2 S_{MAX} w + w^2 g^2)^{1/2} \right), -\frac{1}{4} \frac{1}{Peg} \left(PeiS_{MAX} - PegS_{MAX} - wg \right. \\ &+ \\ &\left. (P^2 e^2 i^2 S_{MAX}^2 - 2P^2 e^2 i S_{MAX}^2 g + 6PegwiS_{MAX} + P^2 e^2 g^2 S_{MAX}^2 \right. \\ &\left. + 2Peg^2 S_{MAX} w + w^2 g^2)^{1/2} \right) \end{aligned} \quad (83)$$

We ignore the negative solution, and take the positive solution for S:

$$\begin{aligned} > qS2 := 1/4/P/e/g*(-P*e*i*S[MAX]+P*e*g*S[MAX]+w*g+sqrt(P^2*e^2*i^2*S[MAX]^2-2*P^2*e^2*i*S[MAX]^2*g+6*P*e*g*w*i*S[MAX]+P^2*e^2*g^2*S[MAX]^2+2*P*e*g^2*S[MAX]*w+w^2*g^2)); \\ qS2 := \frac{1}{4} \frac{1}{Peg} \left(-PeiS_{MAX} + PegS_{MAX} + wg \right. \\ &+ \\ &\left. (P^2 e^2 i^2 S_{MAX}^2 - 2P^2 e^2 i S_{MAX}^2 g + 6PegwiS_{MAX} + P^2 e^2 g^2 S_{MAX}^2 \right. \\ &\left. + 2Peg^2 S_{MAX} w + w^2 g^2)^{1/2} \right) \end{aligned} \quad (84)$$

To obtain a numerical solution, we can plug the assumed parameter values into this expression:

$$\begin{aligned} > SolS := \text{subs}(\{S[MAX]=1, g=0.15, P=200, e=0.015, i=0.1, w=0.6\}, \\ &qS2); \\ SolS := 0.4239265962 \end{aligned} \quad (85)$$

Note that the $S^* = 0.4239$ solution here is that given in Table 17.6 for the PV maximising fishery where $i = 0.1$.

Expression qS2 is typical of the cumbersome expressions which can emerge from Maple calculations. It is the expression which we have used to (rather laboriously) programme our Excel worksheets. It would benefit from some work to put it into a simpler form. One such simpler form which emerges after some manipulation is:

$$\begin{aligned}
 &> \text{SSTAR} := (S[\text{MAX}]/4) * ((w/(P*e*S[\text{MAX}])) + 1 - i/g) + \text{sqrt}((w/(P*e*S[\text{MAX}])) \\
 &\quad + 1 - i/g)^2 + (8*w*i)/(P*e*S[\text{MAX}]*g)); \\
 \text{SSTAR} &:= \frac{1}{4} S_{\text{MAX}} \left(\frac{w}{P e S_{\text{MAX}}} + 1 - \frac{i}{g} + \sqrt{\left(\frac{w}{P e S_{\text{MAX}}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{P e g S_{\text{MAX}}}} \right) \quad (86)
 \end{aligned}$$

which we can verify is equivalent to expression qS2 by evaluation at the assumed parameter values:

$$\begin{aligned}
 &> \text{SolS2} := \text{subs}(\{S[\text{MAX}]=1, g=0.15, P=200, e=0.015, i=0.1, w=0.6\}, \\
 &\quad \text{SSTAR}); \\
 \text{SolS2} &:= 0.4239265960 \quad (87)
 \end{aligned}$$

$$\begin{aligned}
 &> \text{H2STAR} := \text{subs}(S=qS2, \text{EqG}); \\
 \text{H2STAR} &:= \frac{1}{4} \frac{1}{P e} \left(-P e i S_{\text{MAX}} + P e g S_{\text{MAX}} + w g \right. \\
 &\quad + \\
 &\quad \left(P^2 e^2 i^2 S_{\text{MAX}}^2 - 2 P^2 e^2 i S_{\text{MAX}}^2 g + 6 P e g w i S_{\text{MAX}} + P^2 e^2 g^2 S_{\text{MAX}}^2 \right. \\
 &\quad \left. \left. + 2 P e g^2 S_{\text{MAX}} w + w^2 g^2 \right)^{1/2} \right) - \frac{1}{16} \left(-P e i S_{\text{MAX}} + P e g S_{\text{MAX}} + w g \right. \\
 &\quad + \\
 &\quad \left. \left(P^2 e^2 i^2 S_{\text{MAX}}^2 - 2 P^2 e^2 i S_{\text{MAX}}^2 g + 6 P e g w i S_{\text{MAX}} + P^2 e^2 g^2 S_{\text{MAX}}^2 \right. \right. \\
 &\quad \left. \left. + 2 P e g^2 S_{\text{MAX}} w + w^2 g^2 \right)^{1/2} \right)^2 \\
 &\quad \left(g P^2 e^2 \left(\frac{1}{4} \frac{1}{P e g} \left(-P e i S_{\text{MAX}} + P e g S_{\text{MAX}} + w g \right. \right. \right. \\
 &\quad + \\
 &\quad \left. \left. \left(P^2 e^2 i^2 S_{\text{MAX}}^2 - 2 P^2 e^2 i S_{\text{MAX}}^2 g + 6 P e g w i S_{\text{MAX}} + P^2 e^2 g^2 S_{\text{MAX}}^2 \right. \right. \right. \\
 &\quad \left. \left. \left. + 2 P e g^2 S_{\text{MAX}} w + w^2 g^2 \right)^{1/2} \right) \right) \right)_{\text{MAX}}
 \end{aligned} \quad (88)$$

Substitute S* into G(S) to get H*:

> **simplify(%);**

$$-\frac{1}{16} \left((-P e i S_{MAX} + P e g S_{MAX} + w g \right. \tag{89}$$

+

$$(P^2 e^2 i^2 S_{MAX}^2 - 2 P^2 e^2 i S_{MAX}^2 g + 6 P e g w i S_{MAX} + P^2 e^2 g^2 S_{MAX}^2$$

$$+ 2 P e g^2 S_{MAX} w + w^2 g^2)^{1/2} \left($$

$$-4 g P e \left(\frac{1}{4} \frac{1}{P e g} (-P e i S_{MAX} + P e g S_{MAX} + w g$$

+

$$(P^2 e^2 i^2 S_{MAX}^2 - 2 P^2 e^2 i S_{MAX}^2 g + 6 P e g w i S_{MAX} + P^2 e^2 g^2 S_{MAX}^2$$

$$+ 2 P e g^2 S_{MAX} w + w^2 g^2)^{1/2} \Big)_{MAX} - P e i S_{MAX} + P e g S_{MAX} + w g$$

+

$$(P^2 e^2 i^2 S_{MAX}^2 - 2 P^2 e^2 i S_{MAX}^2 g + 6 P e g w i S_{MAX} + P^2 e^2 g^2 S_{MAX}^2$$

$$+ 2 P e g^2 S_{MAX} w + w^2 g^2)^{1/2} \Big) \Big/$$

$$\left(g P^2 e^2 \left(\frac{1}{4} \frac{1}{P e g} (-P e i S_{MAX} + P e g S_{MAX} + w g$$

+

$$(P^2 e^2 i^2 S_{MAX}^2 - 2 P^2 e^2 i S_{MAX}^2 g + 6 P e g w i S_{MAX} + P^2 e^2 g^2 S_{MAX}^2$$

$$+ 2 P e g^2 S_{MAX} w + w^2 g^2)^{1/2} \Big) \Big)_{MAX}$$

> **H1STAR :=subs (S=SSTAR, EqG);**

(90)

$$\begin{aligned}
H1STAR &:= \frac{1}{4} g S_{MAX} \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} + \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right) \\
&- \frac{1}{16} \left(g S_{MAX}^2 \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right. \right. \\
&+ \left. \left. \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right)^2 \right) / \left(\frac{1}{4} S_{MAX} \left(\frac{w}{Pe S_{MAX}} + 1 \right. \right. \\
&- \left. \left. \frac{i}{g} + \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right) \right)_{MAX}
\end{aligned} \tag{90}$$

> H1STAR := 1/4*g*(3/4-1/4*w/P/e/S[MAX]+1/4*i/g-1/4*sqrt((w/P/e/S[MAX]+1-i/g)^2+8*w*i/P/e/g/S[MAX]))*S[MAX]*(w/P/e/S[MAX]+1-i/g+sqrt((w/P/e/S[MAX]+1-i/g)^2+8*w*i/P/e/g/S[MAX]));

$$\begin{aligned}
H1STAR &:= \frac{1}{4} g \left(\frac{3}{4} - \frac{1}{4} \frac{w}{Pe S_{MAX}} + \frac{1}{4} \frac{i}{g} \right. \\
&- \left. \frac{1}{4} \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right) S_{MAX} \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right. \\
&+ \left. \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right)
\end{aligned} \tag{91}$$

> HSTAR := H1STAR;

$$\begin{aligned}
HSTAR &:= \frac{1}{4} g \left(\frac{3}{4} - \frac{1}{4} \frac{w}{Pe S_{MAX}} + \frac{1}{4} \frac{i}{g} \right. \\
&- \left. \frac{1}{4} \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right) S_{MAX} \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right. \\
&+ \left. \sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}} \right)
\end{aligned} \tag{92}$$

Evaluate steady state harvest at assumed parameter values:

> HPPSTAR :=subs({S[MAX]=1, g=0.15, P=200, e=0.015, i=0.1, w=0.6}, H1STAR);

$$HPPSTAR := 0.03663192558 \tag{93}$$

This confirms the entry in Table 17.6.

Obtain expression for steady-state effort, E*, by using E* = H*/(eS*):

> ESTAR :=HSTAR/(e*SSTAR);

$$ESTAR := \tag{94}$$

$$g \left(\frac{3}{4} - \frac{1}{4} \frac{w}{P e S_{MAX}} + \frac{1}{4} \frac{i}{g} - \frac{1}{4} \sqrt{\left(\frac{w}{P e S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{P e g S_{MAX}}} \right)$$

Evaluate steady state effort, E^* , at assumed parameter values:

$$\begin{aligned} > \text{E6STAR} := \text{subs}(\{S[\text{MAX}] = 1, g = 0.15, P = 200, e = 0.015, i = 0.1, w = 0.6\}, \\ & \text{ESTAR}); \\ & \text{E6STAR} := 5.760734039 \end{aligned} \quad (95)$$

$$\begin{aligned} > \text{ALL} := \text{subs}(\{S[\text{MAX}] = 1, g = 0.15, P = 200, e = 0.015, i = 0.1, w = 0.6\}, \\ & \{\text{ESTAR}, \text{HSTAR}, \text{SSTAR}\}); \\ & \text{ALL} := \{0.03663192558, 0.4239265960, 5.760734039\} \end{aligned} \quad (96)$$

These confirm the entries in Table 17.6.

Part H: DEMONSTRATING THAT, FOR THIS MODEL, THE PROFIT MAX RULE WHICH SAYS INCREASE EFFORT IF MARGINAL PROFIT (OF EFFORT) IS POSITIVE LEADS TO CORRECT SOLUTION.

$$\begin{aligned} > \text{eqn11} := \text{diff}(S(t), t) = g * S(t) * (1 - S(t) / S[\text{MAX}]) - e * E(t) * S(t); \\ & \text{eqn11} := \frac{d}{dt} S(t) = g S(t) \left(1 - \frac{S(t)}{S_{MAX}} \right) - e E(t) S(t) \end{aligned} \quad (97)$$

$$\begin{aligned} > \text{eqn1} := \text{subs}(v = 1, S[\text{MAX}] = 1, g = 0.15, P = 200, e = 0.015, i = 0.1, w = 0.6, \\ & \text{eqn11}); \\ & \text{eqn1} := \frac{d}{dt} S(t) = 0.15 S(t) (1 - S(t)) - 0.015 E(t) S(t) \end{aligned} \quad (98)$$

$$\begin{aligned} > \text{eqn22} := \text{diff}(E(t), t) = v * (P * e * S[\text{MAX}] - (2 * P * e^2 * E(t) * S[\text{MAX}]) / g - w); \\ & \text{eqn2} := \text{subs}(v = 1, S[\text{MAX}] = 1, g = 0.15, P = 200, e = 0.015, i = 0.1, w = 0.6, \\ & \text{eqn22}); \\ & \text{eqn22} := \frac{d}{dt} E(t) = v \left(P e S_{MAX} - \frac{2 P e^2 E(t) S_{MAX}}{g} - w \right) \\ & \text{eqn2} := \frac{d}{dt} E(t) = 2.400 - 0.5999999998 E(t) \end{aligned} \quad (99)$$

$$\begin{aligned} > \text{sys} := (\text{eqn1}, \text{eqn2}); \\ \text{sys} := \frac{d}{dt} S(t) = 0.15 S(t) (1 - S(t)) - 0.015 E(t) S(t), \frac{d}{dt} E(t) = 2.400 \\ - 0.5999999998 E(t) \end{aligned} \quad (100)$$

$$\begin{aligned} > \text{initcon} := \{S(0) = 1, E(0) = 1\}; \\ & \text{initcon} := \{E(0) = 1, S(0) = 1\} \end{aligned} \quad (101)$$

The following dsolve commands needs to be edited: at present the syntax is not correct.

```
> dsolve([sys_ode, ics]);
```

Error, (in dsolve) required an indication of the dependent variables in the given system

```
> soln :=dsolve({eqn1, eqn2, S(0)=1, E(0)=1}, {S(t), E(t)});
```

$$\text{soln} := \left\{ E(t) = \frac{12000000000}{2999999999} - \frac{9000000001}{2999999999} e^{-\frac{2999999999}{5000000000} t}, S(t) \right. \quad (102)$$

$$= e^{\frac{5399999997}{59999999980} t - \frac{675000000075000000}{8999999994000000001} e^{-\frac{2999999999}{5000000000} t}}$$

$$\left. \int_0^t \frac{3}{20} e^{\frac{5399999997}{59999999980} -zI} - \frac{675000000075000000}{8999999994000000001} e^{-\frac{2999999999}{5000000000} -zI} d_zI + \frac{1}{e^{\frac{675000000075000000}{8999999994000000001}}} \right\}$$

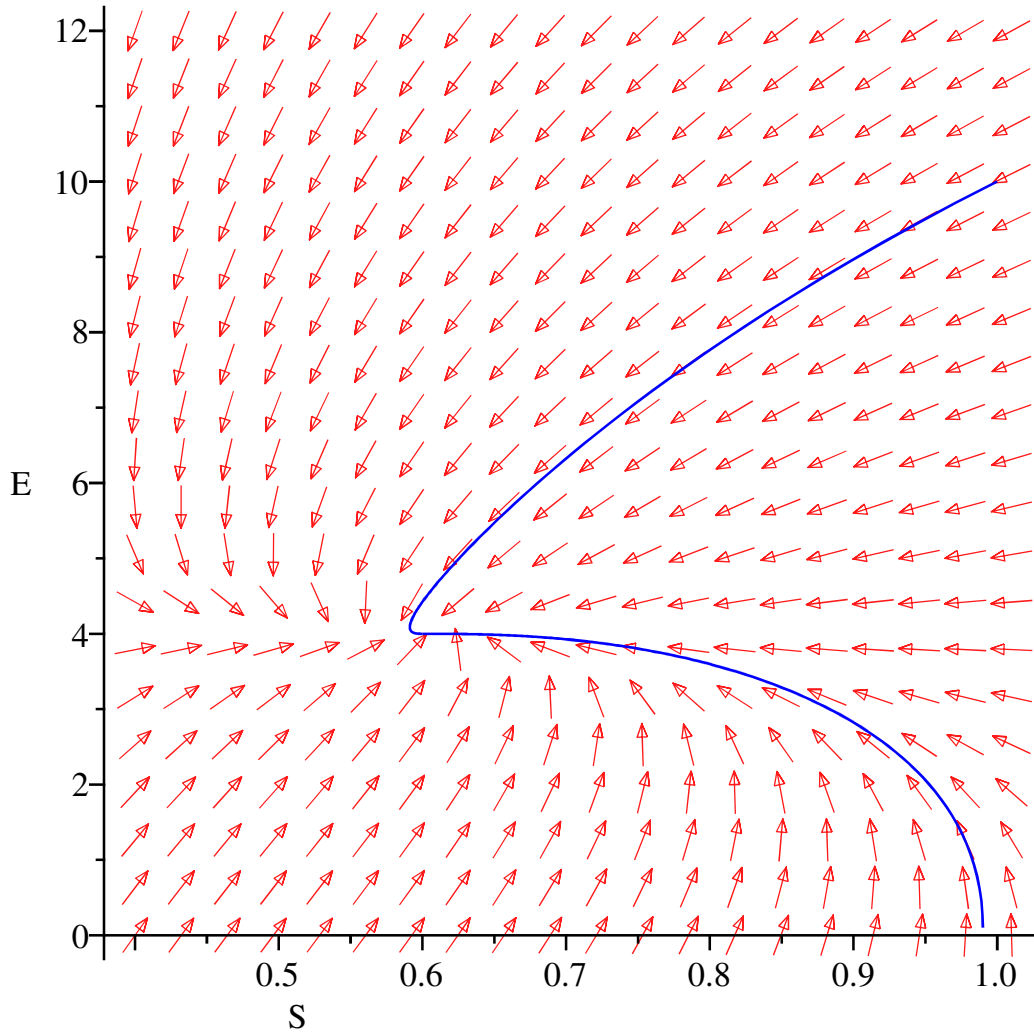
```
> eqns :={diff(S(t),t)=0.15*S(t)*(1-S(t)/1)-0.015*E(t)*S(t), diff(E(t),t)=0.4*(200*0.015*1-(2*200*0.015^2*E(t)*1)/0.15 -0.6)};
```

$$\text{eqns} := \left\{ \frac{d}{dt} E(t) = 0.9600 - 0.2400000000 E(t), \frac{d}{dt} S(t) = 0.15 S(t) (1 - S(t)) - 0.015 E(t) S(t) \right\} \quad (103)$$

```
> init51 :=[[S(0)=0.99, E(0)=0.1], [S(0)=0.3, E(0)=6], [S(0)=1, E(0)=10]];
```

```
init51 := [[S(0) = 0.99, E(0) = 0.1], [S(0) = 0.3, E(0) = 6], [S(0) = 1, E(0) = 10]] \quad (104)
```

```
> DEplot(eqns, [S(t), E(t)], t=0..100, S=0.4..1.01, E=0..12, init51, stepsize=.2, linecolour=blue, thickness=1, arrows=SLIM);
```



Part I: OPEN ACCESS MODEL: COMPARATIVE STATICS:

- > SOA := w / (P * e) ;
- > dS/dw = diff(SOA, w) ; dS/dP = diff(SOA, P) ; dS/de = diff(SOA, e) ;

$$SOA := \frac{w}{P e}$$

$$\frac{dS}{dw} = \frac{1}{P e}$$

$$\frac{dS}{dP} = -\frac{w}{P^2 e}$$

$$\frac{dS}{de} = -\frac{w}{P e^2}$$

(105)

- > EOA := (g/e) * (1 - (w / (P * e * S [MAX]))) ;

(106)

$$EOA := \frac{g \left(1 - \frac{w}{P e S_{MAX}} \right)}{e} \quad (106)$$

> $dE/dw = \text{diff}(EOA, w)$; $dE/dP = \text{diff}(EOA, P)$; $dE/de = \text{diff}(EOA, e)$; $dE/dg = \text{diff}(EOA, g)$;

$$\begin{aligned} \frac{dE}{dw} &= - \frac{g}{e^2 P S_{MAX}} \\ \frac{dE}{dP} &= \frac{g w}{e^2 P^2 S_{MAX}} \\ \frac{dE}{de} &= - \frac{g \left(1 - \frac{w}{P e S_{MAX}} \right)}{e^2} + \frac{g w}{e^3 P S_{MAX}} \\ \frac{dE}{dg} &= \frac{1 - \frac{w}{P e S_{MAX}}}{e} \end{aligned} \quad (107)$$

Note that

(1) The expression for dE/de cannot be unambiguously signed.

(2) In equilibrium, $S = w/Pe$. As $S[MAX] > S$ (except when there is no harvesting), $S[MAX] > w/Pe$, and so $w < PeS[MAX]$. Hence dE/dg is positive.

> $HOA := (g*w) / (P*e) * (1 - (w / (P*e*S[MAX])))$;

$$HOA := \frac{g w \left(1 - \frac{w}{P e S_{MAX}} \right)}{P e} \quad (108)$$

> $dH/dw = \text{diff}(HOA, w)$; $dH/dP = \text{diff}(HOA, P)$; $dH/de = \text{diff}(HOA, e)$; $dH/dg = \text{diff}(HOA, g)$;

$$\begin{aligned} \frac{dH}{dw} &= \frac{g \left(1 - \frac{w}{P e S_{MAX}} \right)}{P e} - \frac{g w}{e^2 P^2 S_{MAX}} \\ \frac{dH}{dP} &= - \frac{g w \left(1 - \frac{w}{P e S_{MAX}} \right)}{P^2 e} + \frac{g w^2}{P^3 e^2 S_{MAX}} \\ \frac{dH}{de} &= - \frac{g w \left(1 - \frac{w}{P e S_{MAX}} \right)}{P e^2} + \frac{g w^2}{P^2 e^3 S_{MAX}} \end{aligned}$$

$$\frac{dH}{dg} = \frac{w \left(1 - \frac{w}{P e S_{MAX}} \right)}{P e} \quad (109)$$

Part J: COMPARATIVE STATICS: STATIC PRIVATE PROPERTY MODEL

- > $SPP := 1/2 * (P * e * S_{MAX} + w) / e / P$; $EPP := 1/2 * g * (P * e * S_{MAX} - w) / P / e^2 / S_{MAX}$; $HPP := 1/4 / e^2 * g * (P * e * S_{MAX} - w) / P^2 / S_{MAX} * (P * e * S_{MAX} + w)$;
- > $dS/dw = \text{diff}(SPP, w)$; $dS/dP = \text{diff}(SPP, P)$; $dS/de = \text{diff}(SPP, e)$; $dS/dg = \text{diff}(SPP, g)$;

$$SPP := \frac{1}{2} \frac{P e S_{MAX} + w}{e P}$$

$$EPP := \frac{1}{2} \frac{g (P e S_{MAX} - w)}{P e^2 S_{MAX}}$$

$$HPP := \frac{1}{4} \frac{g (P e S_{MAX} - w) (P e S_{MAX} + w)}{e^2 P^2 S_{MAX}}$$

$$\frac{dS}{dw} = \frac{1}{2 P e}$$

$$\frac{dS}{dP} = \frac{1}{2} \frac{S_{MAX}}{P} - \frac{1}{2} \frac{P e S_{MAX} + w}{e P^2}$$

$$\frac{dS}{de} = \frac{1}{2} \frac{S_{MAX}}{e} - \frac{1}{2} \frac{P e S_{MAX} + w}{e^2 P}$$

$$\frac{dS}{dg} = 0 \quad (110)$$

- > $dE/dw = \text{diff}(EPP, w)$; $dE/dp = \text{diff}(EPP, P)$; $dE/de = \text{diff}(EPP, e)$; $dE/dg = \text{diff}(EPP, g)$;

$$\frac{dE}{dw} = -\frac{1}{2} \frac{g}{e^2 P S_{MAX}}$$

$$\frac{dE}{dp} = \frac{1}{2} \frac{g}{e P} - \frac{1}{2} \frac{g (P e S_{MAX} - w)}{P^2 e^2 S_{MAX}}$$

$$\frac{dE}{de} = \frac{1}{2} \frac{g}{e^2} - \frac{g (P e S_{MAX} - w)}{P e^3 S_{MAX}}$$

$$\frac{dE}{dg} = \frac{1}{2} \frac{P e S_{MAX} - w}{P e^2 S_{MAX}} \quad (111)$$

- > $dH/dw = \text{diff}(HPP, w)$; $dH/dP = \text{diff}(HPP, P)$; $dH/de = \text{diff}(HPP, e)$; $dH/dg = \text{diff}(HPP, g)$;

(HPP, g) ;

$$\frac{dH}{dw} = -\frac{1}{4} \frac{g (P e S_{MAX} + w)}{e^2 P^2 S_{MAX}} + \frac{1}{4} \frac{g (P e S_{MAX} - w)}{P^2 e^2 S_{MAX}}$$

$$\frac{dH}{dP} = \frac{1}{4} \frac{g (P e S_{MAX} + w)}{e P^2} - \frac{1}{2} \frac{g (P e S_{MAX} - w) (P e S_{MAX} + w)}{e^2 P^3 S_{MAX}}$$

$$+ \frac{1}{4} \frac{g (P e S_{MAX} - w)}{e P^2}$$

$$\frac{dH}{de} = -\frac{1}{2} \frac{g (P e S_{MAX} - w) (P e S_{MAX} + w)}{e^3 P^2 S_{MAX}} + \frac{1}{4} \frac{g (P e S_{MAX} + w)}{e^2 P}$$

$$+ \frac{1}{4} \frac{g (P e S_{MAX} - w)}{e^2 P}$$

$$\frac{dH}{dg} = \frac{1}{4} \frac{(P e S_{MAX} - w) (P e S_{MAX} + w)}{e^2 P^2 S_{MAX}}$$

(112)

COMPARATIVE STATICS: PV MAXIMISING MODEL

> dS/dw=diff (SSTAR, w) ;

$$\frac{dS}{dw} = \frac{1}{4} S_{MAX} \left(\frac{1}{P e S_{MAX}} + \frac{1}{2} \frac{\frac{2 \left(\frac{w}{P e S_{MAX}} + 1 - \frac{i}{g} \right)}{P e S_{MAX}} + \frac{8 i}{P e g S_{MAX}}}{\sqrt{\left(\frac{w}{P e S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{P e g S_{MAX}}}} \right)$$

(113)

> pdSP :=diff (SSTAR, P) ;

$$pdSP := \frac{1}{4} S_{MAX} \left(-\frac{w}{P^2 e S_{MAX}} + \frac{1}{2} \frac{-\frac{2 \left(\frac{w}{P e S_{MAX}} + 1 - \frac{i}{g} \right) w}{P^2 e S_{MAX}} - \frac{8 w i}{P^2 e g S_{MAX}}}{\sqrt{\left(\frac{w}{P e S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{P e g S_{MAX}}}} \right)$$

(114)

> pdSe :=diff (SSTAR, e) ;

(115)

$$pdSe := \frac{1}{4} S_{MAX} \left(-\frac{w}{Pe^2 S_{MAX}} + \frac{1}{2} \frac{-\frac{2 \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right) w}{Pe^2 S_{MAX}} - \frac{8 w i}{Pe^2 g S_{MAX}}}{\sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}}} \right) \quad (115)$$

> pdEw :=diff(ESTAR,w);

$$pdEw := \frac{g \left(-\frac{1}{4 Pe S_{MAX}} - \frac{1}{8} \frac{\frac{2 \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)}{Pe S_{MAX}} + \frac{8 i}{Pe g S_{MAX}}}{\sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}}} \right)}{e} \quad (116)$$

> pdEP :=diff(ESTAR,P);

$$pdEP := \frac{g \left(\frac{1}{4} \frac{w}{P^2 e S_{MAX}} - \frac{1}{8} \frac{-\frac{2 \left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right) w}{P^2 e S_{MAX}} - \frac{8 w i}{P^2 e g S_{MAX}}}{\sqrt{\left(\frac{w}{Pe S_{MAX}} + 1 - \frac{i}{g} \right)^2 + \frac{8 w i}{Pe g S_{MAX}}}} \right)}{e} \quad (117)$$

>