A comment on “An arbitrage-free approach to quasi-option value” by Coggins and Ramezani

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Abstract

In their article "An Arbitrage-Free Approach to Quasi-Option Value" [J. Environm. Econom. Management 35, 103-125, 1998], Coggins and Ramezani interpreted the concept of quasi-option value introduced by Arrow and Fisher [Quart. J. Econom. 88, 1974, 312-319] as being identical to Dixit and Pindyck's real option value. This means their approach differs from the approach by Fisher and Hanemann [J. Environm. Econom. Management 14, 183-190, 1987] who formalized the concept of quasi-option value a decade before. By indirectly characterizing Dixit and Pindyck's real option value Coggins and Ramezani confirmed classic results in the field of real options theory.

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1 Introduction

In the context of irreversible decision making under uncertainty in a dynamic framework, concepts of option values have been developed independently in different strands of literature. Most prominently Dixit and Pindyck (e.g.[6]; for an overview see [14], section VI) contributed to the investment and financial literature by arguing that by making an irreversible investment that could have been postponed, the option to invest is exercised and therefore its value should be accounted for in the investment decision.[1] Arrow and Fisher [1] - analyzing a similar decision framework as Henry [9] - triggered a series of publications in the area of environmental and resource economics by describing the concept of quasi-option value. Their work lead to a formal definition of quasi-option value with respect to an irreversible investment under uncertainty by Fisher and Hanemann [7]. This option value, to be called OV^{FH} here, represents the value of the information that becomes available when uncertainty is resolved over time [7,8].

Attempts were made to unify both concepts. Lund [10] reconciled the Fisher-Hanemann decision framework with financial option valuation. Mensink and Requate [12] established the relation between the value of the option identified by Dixit & Pindyck and OV^{FH}. Finally, Coggins and Ramezani [4] interpreted quasi-option value as a financial option using an arbitrage-free ('contingent claims') valuation approach in the standard binomial model [5].

In this comment I argue that the option value that is the object of study of Coggins and Ramezani's article "An arbitrage-free approach to quasi-option value" [4], is equivalent to Dixit and Pindyck's real option value, and not equivalent to the quasi-option value OV^{FH} as defined by Fisher and Hanemann [7].

The paper is organized as follows. First, I present a decision framework to define Fisher and Hanemann's quasi-option value OV^{FH} (section 2). Then I show that the 'quasi-option value' Coggins and Ramezani [4] analyze is equivalent to Dixit and Pindyck's

[1] This option is often referred to as 'real option' because it is an option with respect to a 'real' investment, and not a financial derivative.

real option value and not Fisher and Hanemann's quasi-option value (section 3). After that, I discuss the consequences of this result and draw a conclusion (section 4).

2 The Model

The model presented here is identical to the one in Fisher and Hanemann [7] and Hanemann [8][2]: a firm has the opportunity to make an investment that can be made in either of two periods, $i = 1, 2$. By $d_i \in \{0, 1\}$ we denote the decision variable which indicates whether or not the investment is in place in period i. The investment is irreversible: this implies $d_2 \geq d_1$. Moreover, the investment can only be made once: $d_2 + d_1 \leq 1$. The investment generates a benefit $B_1(d_1)$ in period 1, and a stochastic benefit $B_2(d_1, d_2, \theta)$ in period 2, where θ is a random variable that represents uncertainty. Now consider two scenarios. In the first scenario, the firm knows the true value of θ in period 2 before it has to decide for the second time whether or not to invest. The expected benefit as a function of the investment decision in period 1 is given by

$$\hat{V}(d_1) = B_1(d_1) + E[\max_{d_2} B_2(d_1, d_2, \theta)] \tag{1}$$

In the second scenario the firm does not know the realization of the value of θ when it decides whether or not to invest in period 2. In this case the expected benefit as a function of the investment decision in the first period is determined by

$$V^*(d_1) = B_1(d_1) + \max_{d_2} E[B_2(d_1, d_2, \theta)] \tag{2}$$

The quasi-option value as defined by Fisher and Hanemann, OV^{FH}, is the difference between these two expected benefits if no investment is made in the first period [7,8]:

$$OV^{FH} = \hat{V}(0) - V^*(0). \tag{3}$$

[2] For an extensive presentation and analysis of this model the reader is referred to [7,8,12]

OV^{FH} measures the extra expected profit the firm can gain by taking into account information that will become available in the future. Formally speaking, OV^{FH} is the expected value of information with respect to θ conditional on non-investment in the first period [8, p.29]. It can also be interpreted as a shadow tax on investment that corrects myopic behavior. In other words, this shadow tax corrects the behavior of those investors that do not take into account information that will become available in the future [8, p.27].

The 'real option value' with respect to an irreversible decision, as formulated by Dixit and Pindyck (see [6, p.96-97;12]) - expressed in terms of our model - has been defined as:

$$OV^{DP} = \max\{\hat{V}(0), \hat{V}(1)\} - NPV \tag{4}$$

with NPV being the maximum of the expected benefit investing in the first period and not investing at all:

$$NPV = \max\left\{V^*(1),\ B_1(0) + E[B_2(0,0,\theta)]\right\} \tag{5}$$

The real option value, OV^{DP}, is the value of the opportunity to postpone the investment decision. OV^{DP} is not equal to OV^{FH} in general, though it includes OV^{FH}, which is that part of the value of waiting that is related to information. For a detailed discussion see [12].

OV^{DP} is traditionally analyzed either using dynamic programming techniques or - if the return on investment can be replicated by a traded portfolio - by means of an arbitrage free argument in the tradition of Black, Scholes and Merton [6,13].

3 Quasi Option Value in Coggins and Ramezani [4]

To formally define the concept of quasi-option value as it was introduced by Arrow and Fisher [1], Coggins and Ramezani first describe the 'expected net present value rule'

(ENPV) in the context of a dynamic decision making problem as

> 'Replacing future stochastic variables with their expected values, discounting costs and returns back to the present, and investing whenever ENPV is positive [...]' [4,p.103-104].

In terms of the model in section 2, their ENPV decision rule says[3]:

$$
\begin{aligned}
d_1 &= 1,\ d_2 = 1 \quad \text{if} \ \ B_1(1) + B_2(1, 1, E[\theta]) \geq B_1(0) + B_2(0, 0, E[\theta]) && (6) \\
d_1 &= 0,\ d_2 = 0 \quad \text{otherwise.} && (7)
\end{aligned}
$$

Following [4], we define W_0 as the expected value of the investment if the decision maker would act according to this ENPV decision rule. The authors use this definition to claim quasi-option value in [1] is equal to:

> '[...] the extra value that can be gained if one eschews ENPV analysis in favor of the fully dynamic alternative.' [4, p.104].

This means that Coggins and Ramezani claim that the concept of quasi-option value decribed by Arrow and Fisher [1] is equal to $\hat{V}(\hat{d}_1) - W_0$, with $\hat{d}_1 = \arg\max \hat{V}(\hat{d}_1)$. At the bottom of page 107 they write that $O^* = \max\{0, W_0^* - W_0\} \equiv \max\{0, \hat{V}(0) - W_0\}$ is the definition of quasi-option value.

Neither of these expressions is equivalent to the quasi-option value OV^{FH} as defined and analyzed by Fisher and Hanemann [7,8] who stylized the framework of irreversible investment under uncertainty developed by Arrow and Fisher [1] and Henry [9]. Please recall that Fisher and Hanemann define OV^{FH} as follows:

[3] Although the definition of this rule leaves room for the decision-pair $d_1 = 0, d_2 = 1$ if $B_1(1) + B_2(1, 1, E[\theta]) < B_1(0) + B_2(0, 0, E[\theta])$ and $B_2(0, 1, E[\theta]) > B_2(0, 0, E[\theta])$, equation (1) in Coggins and Ramezani [4] makes clear that $d_1 = 0, d_2 = 1$ is not considered a possibility under this decision rule: (1) only takes $d_1 = 0, d_2 = 0$, and $d_1 = 1, d_2 = 1$ into account, though $d_1 = 0, d_2 = 1$ might be a rational choice when $A > (P_0 - c)$ and $u >> d$.

$$OV^{FH} = \hat{V}(0) - V^*(0). \quad (8)$$

It is easy to see that (8) is neither equal to $\hat{V}(\hat{d}_1) - W_0$, nor to O^*: the difference is most obvious when $W_0 < V^*(0)$. This is the case when even without knowing the true value of θ, investing in the second period is still more attractive than either investing in the first period or not investing at all. The crucial difference between OV^{FH} and both $\hat{V}(\hat{d}_1) - W_0$ and O^*, lies in the fact that, in their ENPV rule, Coggins and Ramezani do not account for the opportunity to invest in the second period in the absence of extra information.[4]

In fact, $O^* = \hat{V}(0) - W_0$ is strongly related to OV^{DP} - and for Coggins and Ramezani's model they are equivalent as we will show below. Therefore it should not come as a surprise that Coggins and Ramezani show that O^* is equal to the value of the right to delay the investment decision, which after all is the verbal definition of OV^{DP}. Please recall that

$$W_0 = \max\{B_1(0) + \ B_2(0, 0, E[\theta]), V^*(1)\} \text{ (eq.1 in Coggins and Ramezani, 1998)} \quad (9)$$

We know that[5]

[4] This is confirmed by two mathematical errors in what I interpreted as their attempt to show equivalence between the Fisher and Hanemann [7] and Hanemann [8] definition of quasi-option value and O^* (footnote 10): neither $W_1(q)/R = W_0(q)$ nor $V^*(0) = V^* = W_1(q)/R$ hold in general. In stead $V^*(0) = V^* = A + (W_1(q)/R)$. $W_1(q)/R = W_0(q)$ can not hold in general because the term on the right contains P_0 and the term on the left side does not.

[5] Please note that equation (14) holds because by definition $NPV \geq V^*(1) = \hat{V}(1)$ and if $NPV > \hat{V}(1)$ then $\hat{V}(0) - NPV \geq 0$.

$$
\begin{aligned}
O^* &= \max\{0, W_0^* - W_0\} && (10)\\
&= \max\{0, \hat{V}(0) - \max\{B_1(0) + B_2(0,0,E[\theta]), V^*(1)\}\} && (11)\\
&\quad \text{(and if } B_2 \text{ is linear in } \theta\text{, as in Coggins and Ramezani (1998))} \\
&= \max\{0, \hat{V}(0) - \underbrace{\max\{B_1(0) + E[B_2(0,0,\theta)], V^*(1)\}}_{NPV}\} && (12)\\
&= \max\{0, \hat{V}(0) - NPV\} && (13)\\
&= \max\{\hat{V}(1) - NPV, \hat{V}(0) - NPV\} && (14)\\
&= \max\{\hat{V}(0), \hat{V}(1)\} - NPV && (15)\\
&= OV^{DP} && (16)
\end{aligned}
$$

The fact that O^* is equal to OV^{DP} confirms that O^* is not equivalent to OV^{FH}, because OV^{FH} and OV^{DP} are not equal in general [12].[6]

4 Concluding remarks

This comment showed that when in sections I and II of "An Arbitrage-Free Approach to Quasi-Option Value" [4] Coggins and Ramezani define 'quasi-option value' they define a concept that differs from the 'quasi-option value' as it was defined and analyzed by Fisher and Hanemann [7,8]. This means they re-interpreted the concept of quasi-option value that was introduced by Arrow and Fisher [1]. What Coggins and Ramezani define as 'quasi-option value' was shown here to be equivalent to the 'real option value' as formulated and analyzed by Dixit and Pindyck [6,13].

This means that in section III, Coggins and Ramezani express Dixit and Pindyck's real option value in terms of a risk free portfolio. This was done before by Brennan & Schwartz and Pindyck [3; 13, section A]. This also means that in the same section Coggins and Ramezani analyse the effect of increasing uncertainty on Dixit and Pindyck's

[6]Bosetti and Messina [2] based their model on Coggins and Ramezani's and therefore seem to adopt the same definition of quasi-option value as Coggins and Ramezani did.

real option value, and show it is non-decreasing in the level of uncertainty. Thereby confirming a result by McDonald and Siegel [11].

Conclusion: after re-interpreting the concept of quasi-option value as being equal to Dixit and Pindyck's real option value, Coggins and Ramezani [4] confirmed classic results in the field of financial economics.

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