Stock dependent utility

$$\text{Max } \mathbf{SNPV} = \int_0^\infty \{B(H_t, S_t) - C(H_t, S_t)\} e^{-it} dt$$

subject to

$$\frac{dS}{dt} = G(S_t) - H_t$$

and initial stock level $S(0) = S_0$.

The current-value Hamiltonian, L, for this problem is

$$L(H, S)_t = B(H_t, S_t) - C(H_t, S_t) + p_t (G(S_t) - H_t) \qquad (17.28^{**})$$

The necessary conditions for a maximum (assuming an interior solution) include

$$\text{(i)} \frac{\partial L_t}{\partial H_t} = 0 = \frac{dB}{dH_t} - \frac{\partial C}{\partial H_t} - p_t \qquad (17.29)$$

$$\text{(ii)} \frac{dp_t}{dt} = ip_t - p_t \frac{dG}{dS_t} + \frac{\partial C}{\partial S_t} - \frac{\partial B}{\partial S_t} \qquad (17.24^{**})$$

and the resource net growth equation

$$\frac{dS}{dt} = G(S_t) - H_t$$

Notice the presence of an additional term in 17.24**.

We assume that the social benefits function is separable in H and S, and as before an inverse demand function for the resource given by $P_t = P(H_t)$. Then Equation 17.29 can be rewritten as

$$p_t = P(H_t) - \frac{\partial C}{\partial R_t} \qquad (17.23^*)$$

Equation 17.24** is the Hotelling efficient harvesting condition for a renewable resource in which costs depend upon the stock level. It is also sometimes called the asset-equilibrium condition. As described in the text, a decision about whether to defer some harvesting until the next period is made by comparing the marginal costs and benefits of adding additional units to the resource stock. The marginal cost is the foregone current return, the value of which is the net price of the resource, p. However, since we are considering a decision to *defer* this revenue by one period, the present value of this sacrificed return is ip.

The benefits obtained by the resource investment are now fourfold:

- The unit of stock may appreciate in value by the amount dp/dt.
- With an additional unit of stock, *total* harvesting costs will be reduced by the quantity $\partial C/\partial S$ (note that $\partial C/\partial S < 0$).
- The additional unit of stock will grow by the amount dG/dS, the value of which is this quantity multiplied by the net price of the resource, p.
- $\partial B/\partial S$, the marginal social benefit of having an incremental unit of the stock in existence.

Dividing both sides of Equation 17.24* by the net price p (and ignoring time subscripts from now on for simplicity) gives

$$\mathbf{i} = \frac{\left(\frac{\mathbf{dp}}{\mathbf{dt}}\right)}{\mathbf{p}} - \frac{\left(\frac{\partial \mathbf{C}}{\partial \mathbf{S}}\right)}{\mathbf{p}} + \frac{\left(\frac{\partial \mathbf{B}}{\partial \mathbf{S}}\right)}{\mathbf{p}} + \frac{\mathbf{dG}}{\mathbf{dS}} \qquad (17.30^{**})$$

This reformulation shows that it is Hotelling's rule of efficient resource use, albeit in a modified form. The left-hand side of Equation 17.30** is the rate of return that can be obtained by investing in assets elsewhere in the economy. The right-hand side is the rate of return that is obtained from the renewable resource. This is made up of four elements:

- the proportionate growth in net price
- the proportionate reduction in harvesting costs that arises from a marginal increase in the resource stock
- the proportionate increase in existence benefits
- the natural rate of growth in the stock from a marginal change in the stock size.