Effect of co-phasing diversity on CDMA downlink capacity

F. Tong, R.C. Atkinson, I. Glover, S.R. Pennock and P.R. Shepherd

The impact of distributed antennas using co-phasing transmission diversity on downlink CDMA channels is examined. An expression is derived that demonstrates that uniform downlink signal-to-interference ratio across users can be achieved by appropriately weighting the distribution of transmit powers across distributed antennas. The capacity of a downlink CDMA system employing distributed antennas with optimum power allocation is then found by simulation.

Introduction: This Letter examines the impact of applying co-phasing transmission techniques to distributed antennas. A distributed antenna is an irregular array of antennas with large separation connected to central processing via a range of technologies, e.g. radio over fibre [1]. It has previously been demonstrated that improved uplink signal-to-interference ratio (SIR) is realised by distributed antennas compared to a conventional (single) antenna [2]. This Letter considers the downlink.

In systems where multiple transmit antennas are used to transmit to a single receive antenna, the output of each antenna must be time shifted to compensate for the different delays inherent in the different paths between each antenna pair. However, this does not affect the path-induced phase shift of the received signal. Co-phasing implies pre-compensation for phase changes during transmission in order to maximise the likelihood of constructive interference and hence improve the overall SIR [3]. Maximum ratio transmission schemes have been studied before, but without SIR balancing between mobile terminals [4].

Signal power: In multi-user systems such as CDMA a distributed antenna can be used to transmit to a number of users at different geographical locations. The time and phase adjustment, therefore, differs for each user. In a system with K transmit antennas and M mobile users, a $K \times M$ channel matrix, G, can be defined such that the element g_{ij} represents the (power) gain between antenna i and user j. Throughout this Letter it is assumed that there is a one-to-one relationship between users and channels: user j is allocated downlink channel j. The terms inter-user interference and inter-channel interference are therefore synonymous.

The total downlink transmission power associated with a particular channel is distributed across the multiple antennas. The mapping of transmission power to antennas for a particular user is determined by a voltage weighting matrix, W. Weighting the transmit power in a particular antenna can be interpreted as modifiying the channel gain. The actual channel gain g_{ij} will be referred to the pre-weighted gain and the compensated channel gain will be referred to as the post-weighted gain. The voltage induced at the receive antenna of user j by a signal from antenna i is proportional to the product of the channel gain and the weighting factor.

In cases where the weighting is chosen to be the reciprocal of the channel gain, the received power level experienced by a particular user will be equal from all antennas. This Letter, however, considers the case in which weights are chosen proportional to channel gain. This approach is based on the notion that the greatest proportion of power should be transmitted by antennas that experience least attenuation (highest pre-weighted gain). Without loss of generality the constant of proportionality is chosen to be unity and thus equal to the pre-weighted power gain, i.e. $w_{ij} = g_{ij}$ and $W = G^T$.

The signals at antenna i are phase adjusted by angle θ_i before transmission such that they arrive in-phase at the receive antenna. The result is a summation of voltage magnitudes. The total voltage induced in the receive antenna of user j by all K transmit antennas is proportional to this sum.

The total signal power delivered to the receive antenna of user j is given by the product of the overall transmit power associated with user j, P_j, and the aggregated post-weighted power gain associated with that user, G_j, i.e.:

$$S_j = P_j G_j = P_j \left(\sum_{i=1}^{K} g_{ij} \right)^2 \tag{1}$$

Inter-channel interference power: The interfering power at a particular user, j, can be derived in a similar way to signal power. Total interference is the received power intended for every other user (i.e. all users except j) from each of the K transmit antennas. The total voltage induced in the receive antenna of user j from all K antennas by a signal intended for user m is proportional to the sum of the voltages induced by each transmit antenna individually. Therfore, for a system with M users (channels), the total inter-channel interference experienced by user j is given by the sum of the interference power from all other $M - 1$ users, i.e.:

$$I_j = \sum_{m=1, m \neq j}^{M} P_m \sum_{i=1}^{K} g_{im} g_{ij} \tag{2}$$

A channel loss factor ζ can be defined to reflect the degree to which lack of orthogonality exists in practical CDMA channels. (The lack of orthogonality could be due to multipath effects). This gives rise to a more realistic expression for inter-channel interference power:

$$I_j = \zeta \left(\sum_{m=1, m \neq j}^{M} P_m \sum_{i=1}^{K} g_{im} g_{ij} \right) \tag{3}$$

Since only relative performance across different numbers of antennas is of interest here, ζ has been set (arbitrarily) to unity for all channels.

Signal-to-interference ratio: In cases where there is a single transmit antenna and single receive antenna the path losses suffered by the signal S_j and the interference I_j are equal. The SIR on the downlink is then determined entirely by the transmit power values associated with each channel. Equal power allocation across channels will result in equal SIR across users.

Where there are multiple transmit antennas uniform SIR across receivers cannot be attained by equalising the transmit power to all users since each antenna–user pair will have a distinct channel gain and the overall SIR will be influenced by each of these. Uniform SIR is possible, however, if the transmit power for each user is distributed across antennas appropriately as described by the power weighting matrix W. The appropriate elements of W can be determined via eigenvalue decomposition.

A diagonal matrix of power loss values $\mathcal{G} = diag\,(G_j)$ is defined and the transmit powers are described by a $1 \times M$ column vector $\mathcal{P} = [P_j]$. Equation (1) can therefore be expressed as $1 \times M$ column vector $\mathcal{S} = \mathcal{P}\mathcal{G}$. A matrix expression for interference corresponding to (2) can be formulated by recognising that the interference received by a user, j, is the sum of all transmission powers from all antennas not meant for that particular user.

An inter-channel interference gain matrix can be determined by multiplying the power weighting matrix, W, by the channel gain matrix, G, to give the post-weighted channel gain matrix, L (L is an $M \times M$ matrix in which l_{ij} represents the aggregated inter-channel interference gain from channel j to channel i). Removing the redundant components in the leading diagonal (since a channel does not interfere with itself) gives the inter-channel interference gain matrix $\mathcal{I}$ (i.e. $\mathcal{I} = L - \Lambda$, and $\Lambda = diag\,(l_{mm})$). The process of removing the elements of the main diagonal accounts for the case $m \neq j$ in (2). The expression for SIR therefore becomes:

$$\Gamma = [\gamma_j] = \frac{\mathcal{S}}{\mathcal{I}} = \frac{\mathcal{P}\mathcal{G}}{\mathcal{P}(WG - \Lambda)} = \frac{\mathcal{G}}{WG - \Lambda} \tag{4}$$

where

$$\gamma_j = \frac{S_j}{I_j} = \frac{P_j \left(\sum_{i=1}^{K} g_{ij} \right)^2}{\sum_{m=1, m \neq j}^{M} P_m \sum_{i=1}^{K} g_{im} g_{ij}} \tag{5}$$

Taking the reciprocal of (4) and letting $Z = 1/\Gamma$, the interference to signal ratio is given by $Z = (WG - \Lambda)\ \mathcal{G}^{-1}$. The appropriate transmission powers which give rise to uniform SIR values across users can be determined by solving:

$$PZ = \lambda P \tag{6}$$

The best solution is given be the smallest eigenvalue, λ_{min}, and its associated eigenvector. The SIR that will be experienced by all users is then given by $\gamma = 1/\lambda_{min}$.

Results: Simulations have been conducted to determine the overall impact on SIR of a distributed antenna system employing co-phasing diversity. A specimen case is shown in Fig. 1 which depicts the cumulative distribution function (CDF) of the SIR for eight users with up to five transmit antennas. The Figure shows that, when power is allocated to channels in accordance with the solution to (6), significant SIR gain is realised as antennas are added.

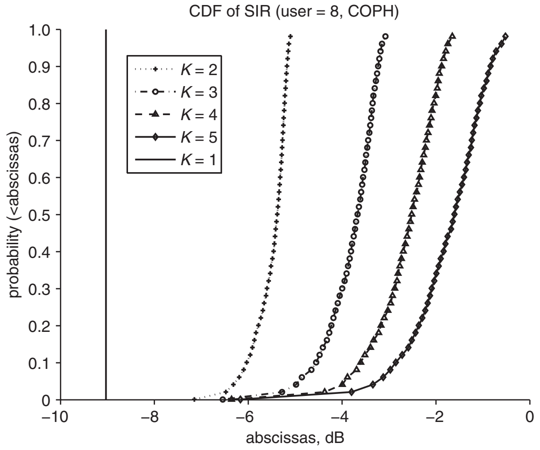

Fig. 1 *Eight users with co-phasing*

Although the power control algorithm equalises SIR for all users, the SIR changes with the spatial distribution of users. The gradient of the CDF is related to the number of antenna units. The greater the number of antenna units, the smaller the slope. This can be explained by the observation that, as more antenna units are deployed, the number of ways that mobile users can cluster around these units increases. When the number of users is small, one user's signal power as a fraction of the total received power is large. This will result in a more significant influence on SIR when changing mobile user position. The CDF gradient for eight users, therefore, might be expected to be smaller than the slope for 16 users. This has been confirmed by simulation results not included here.

Capacity gain has been estimated by obtaining the CDF for different numbers of users. For each user capacity we obtain a 90% SIR exceedance. Fig. 2 shows this exceedance value against number of users. The parameter, K, is the number of antenna units. As the number of mobile users increases the SIR achieved decreases. For the same number of users, an increased number of antenna units results in better SIR. This improvement arises as a consequence of co-phase combining.

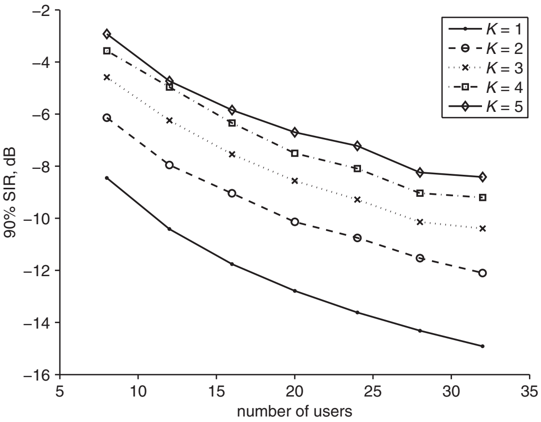

Fig. 2 *Downlink*

Conclusions: A power allocation scheme for distributed antenna co-phasing transmission diversity assigns power to antenna units in proportion to the elements of the channel gain vector. This scheme results in unbalanced SIR between mobile users. To achieve balanced SIR over all users, a user power allocation scheme is required. Such a scheme is derived by solving an eigenvalue problem based on the channel gain matrix. The downlink SIR for one cell using combined co-phasing transmission diversity and power control has been simulated. The resulting SIR CDFs show that this can improve downlink SIR. The SIR obtained depends on the spatial distribution of users. 90% exceedance values of SIR have been found as a function of the number of mobile users. With the same SIR threshold, a distributed antenna can accommodate more users than a single antenna.

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F. Tong (*Motorola Ltd, Swindon, United Kingdom*)

R.C. Atkinson and I. Glover (*Mobile Communications Group, University of Strathclyde, Glasgow, United Kingdom*)

E-mail: r.atkinson@eee.strath.ac.uk

S.R. Pennock and P.R. Shepherd (*Department Electronic & Electrical Engineering, University of Bath, Bath, United Kingdom*)

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