BINDING & SUBSTITUTION

in

STRING DIAGRAMS

ROSS DUNCAN — UNIVERSITY OF STRATHCLYDE

PATTERN-MATCHING

~~BINDING~~ & SUBSTITUTION

in

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TOWARDS PATTERN-MATCHING

~~BINDING~~ & SUBSTITUTION

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1. OPERADs

- $A \xrightarrow{f} B \xrightarrow{g} C$ (arrows in a category)

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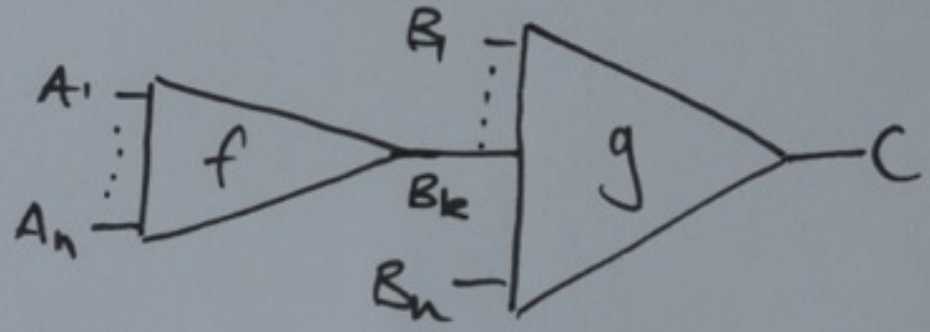

(arrows in an operad)

aka. multicategory.

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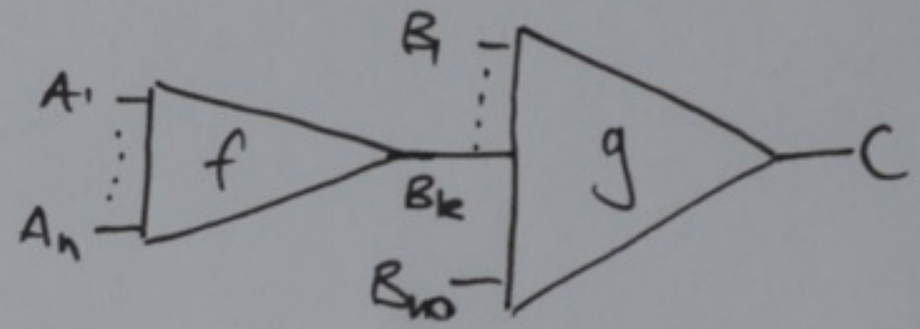

(arrows in an operad)

aka. multicategory.

$$x_1 : A_1, \dots, x_n : A_n \vdash f : B_k \qquad y_1 : B_1, \dots, y_m : B_m \vdash g : C$$

$$y_1 : B_1, \dots, y_{k-1} : B_{k-1}, x_1 : A_1, \dots, x_n : A_n, y_{k+1} : B_{k+1}, \dots, y_m : B_m \vdash g[f/y_k] : C$$

2. Making an Operad from a PRO

- Let (Σ, E) be a presentation of a PRO.
- Adjoin "enough" new generators $x : m \to n$ for every $m, n \in \mathbb{N}$. (variables.)
- Then $(\Sigma + Var, E)$ is again a PRO with (term) variables.

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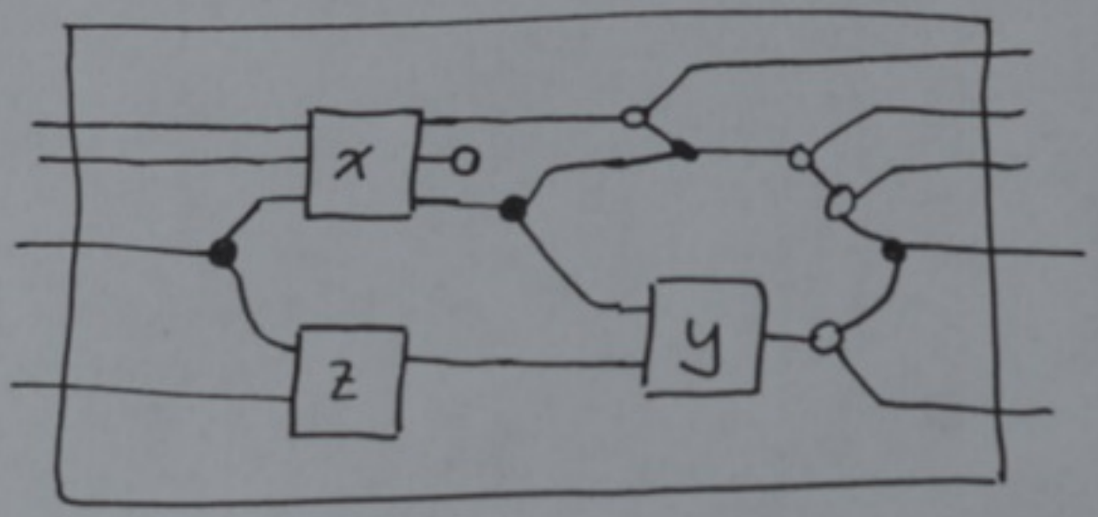
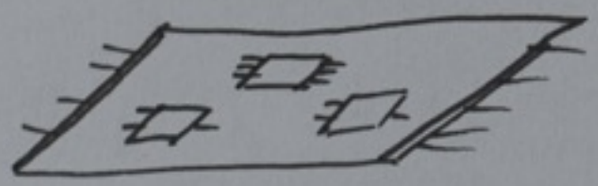


Assume variables only occur once (for now)

$x : (3,3), y : (2,1), z : (2,1) \vdash f : (4,5)$

2. OPERADS FROM PRO.S.



f

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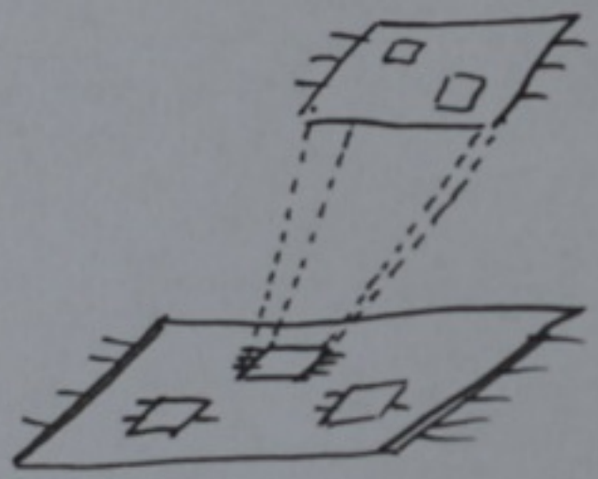

$f[g/x]$

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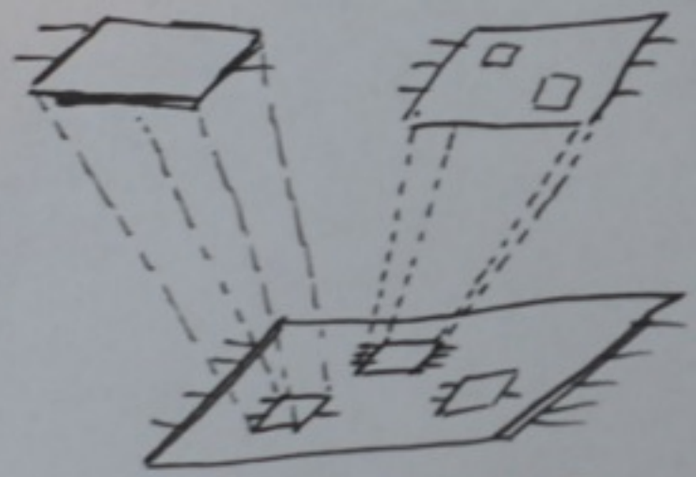

$f[g/x][h/z]$

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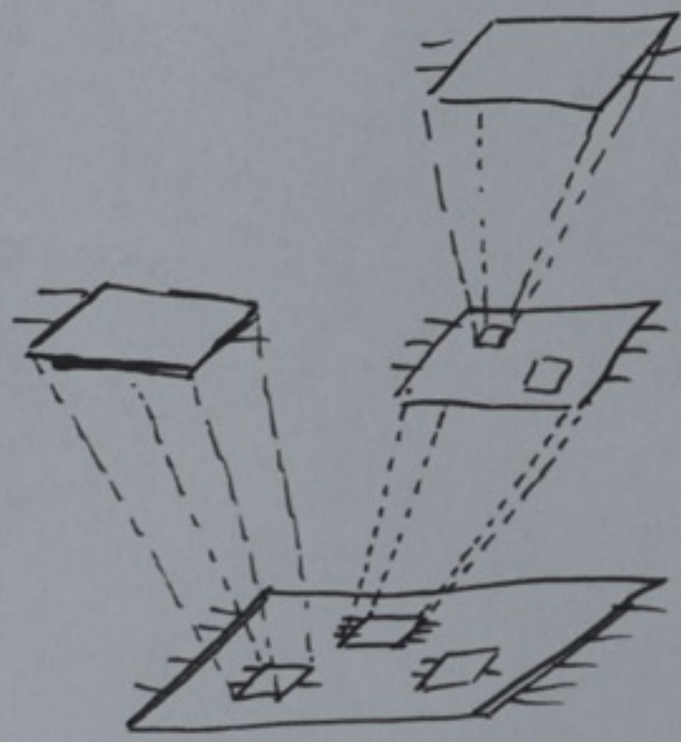

$f[g/x][h/z][k/\omega]$

etc.

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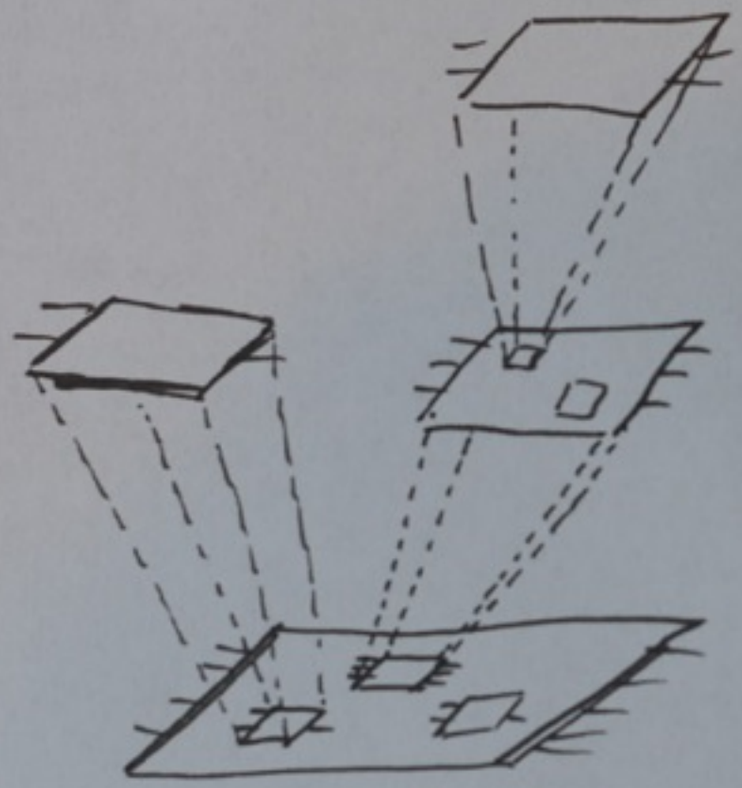

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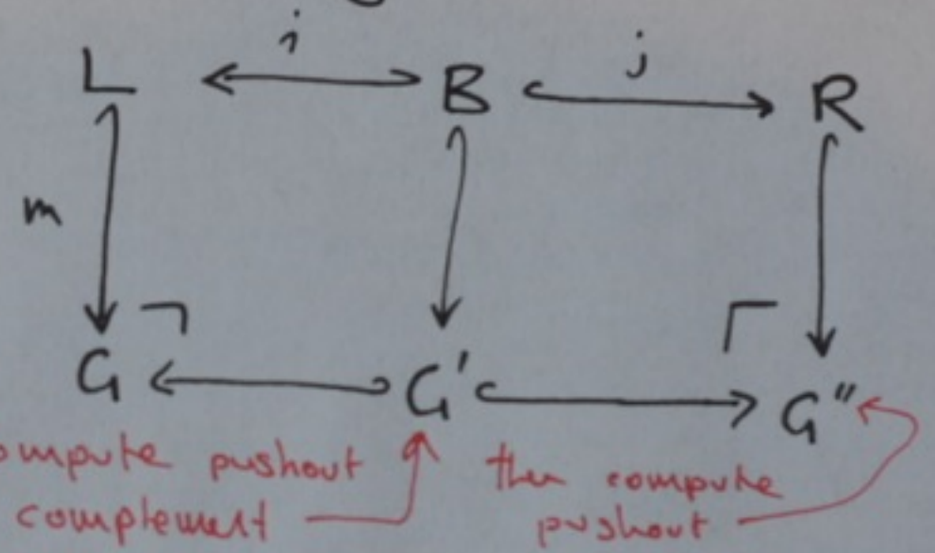
etc.

DPO Rewriting : $L \Rightarrow R$



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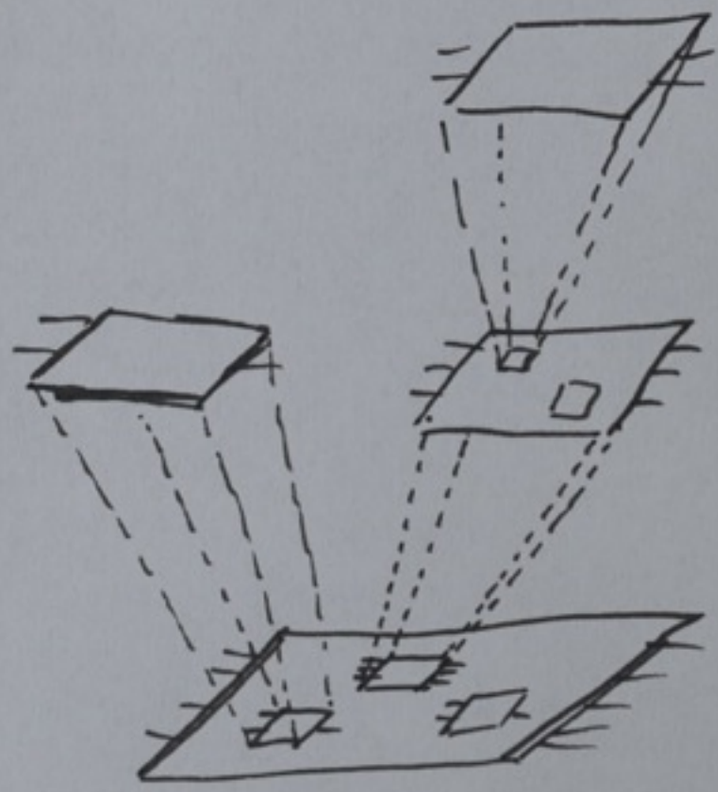

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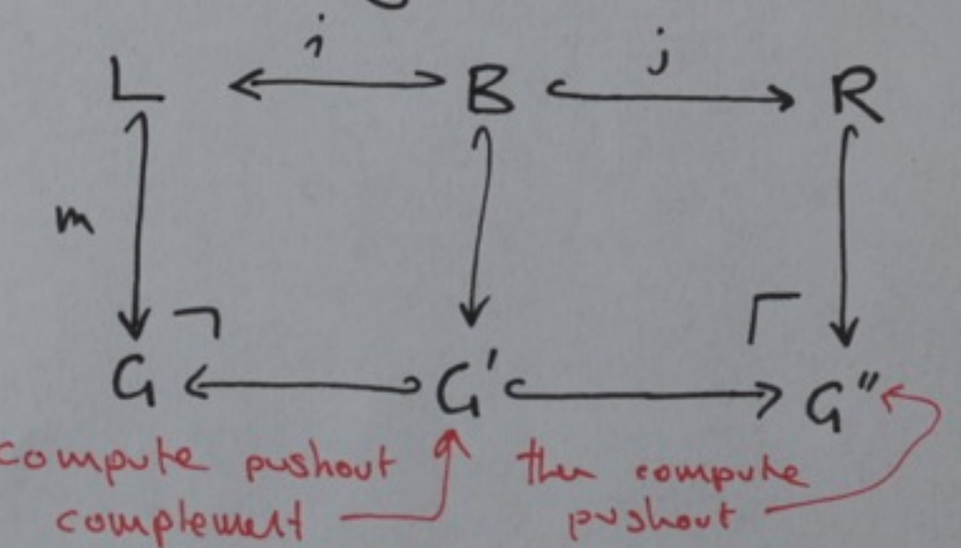

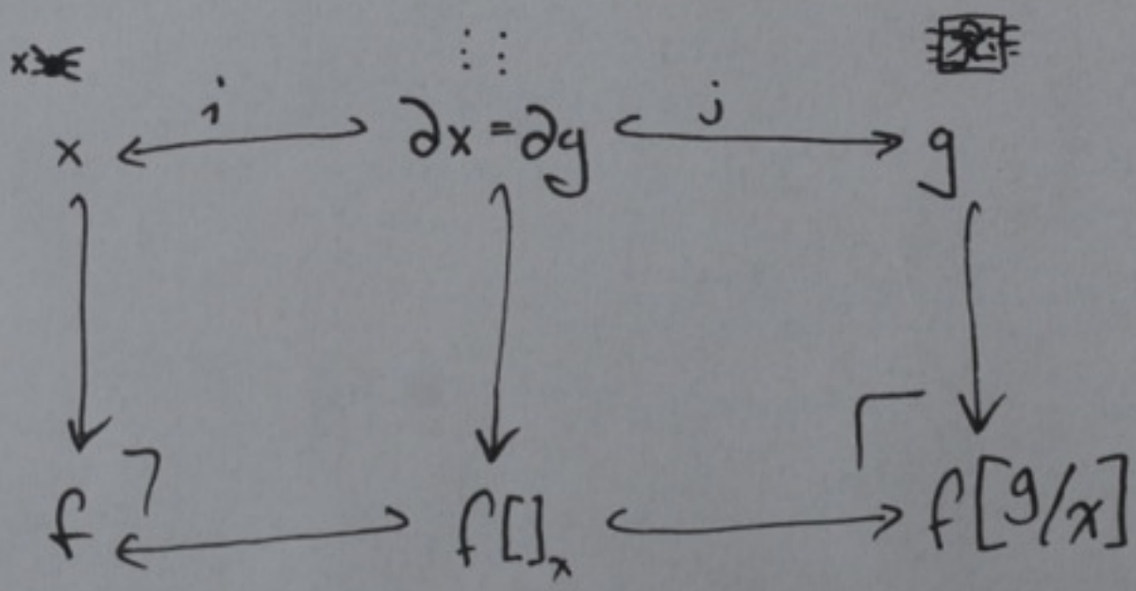

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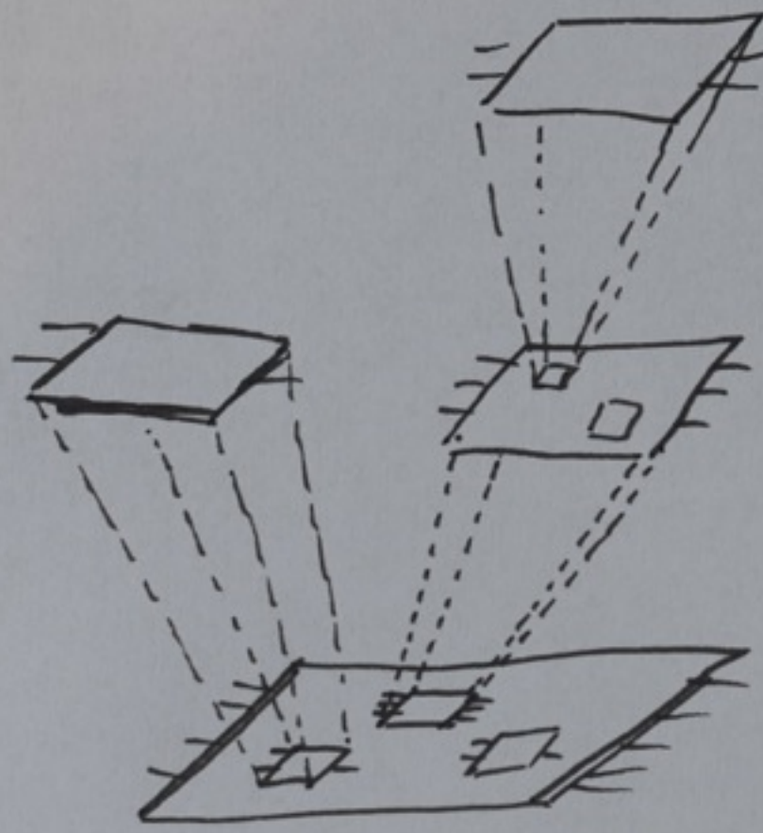

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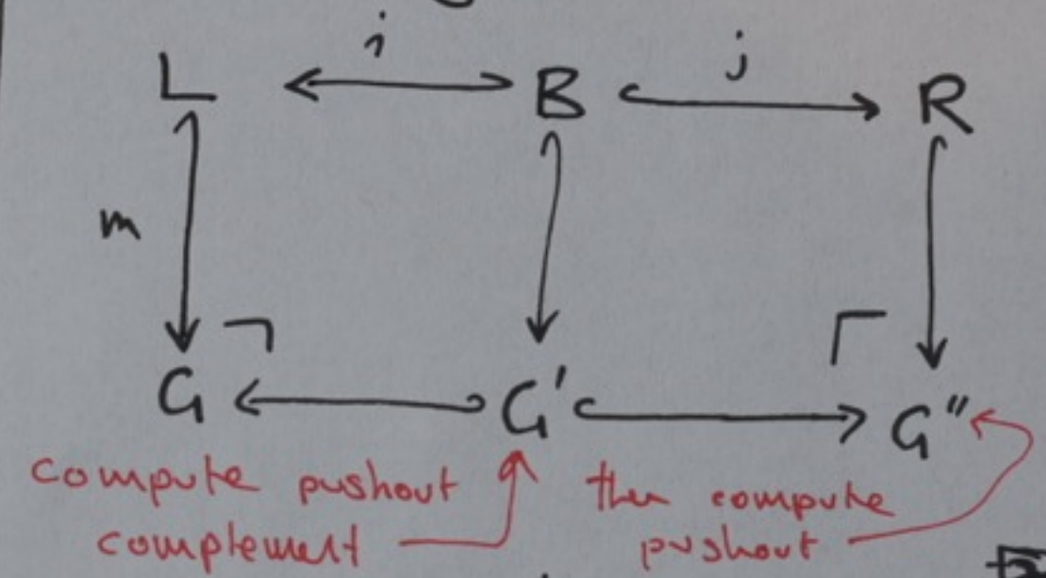
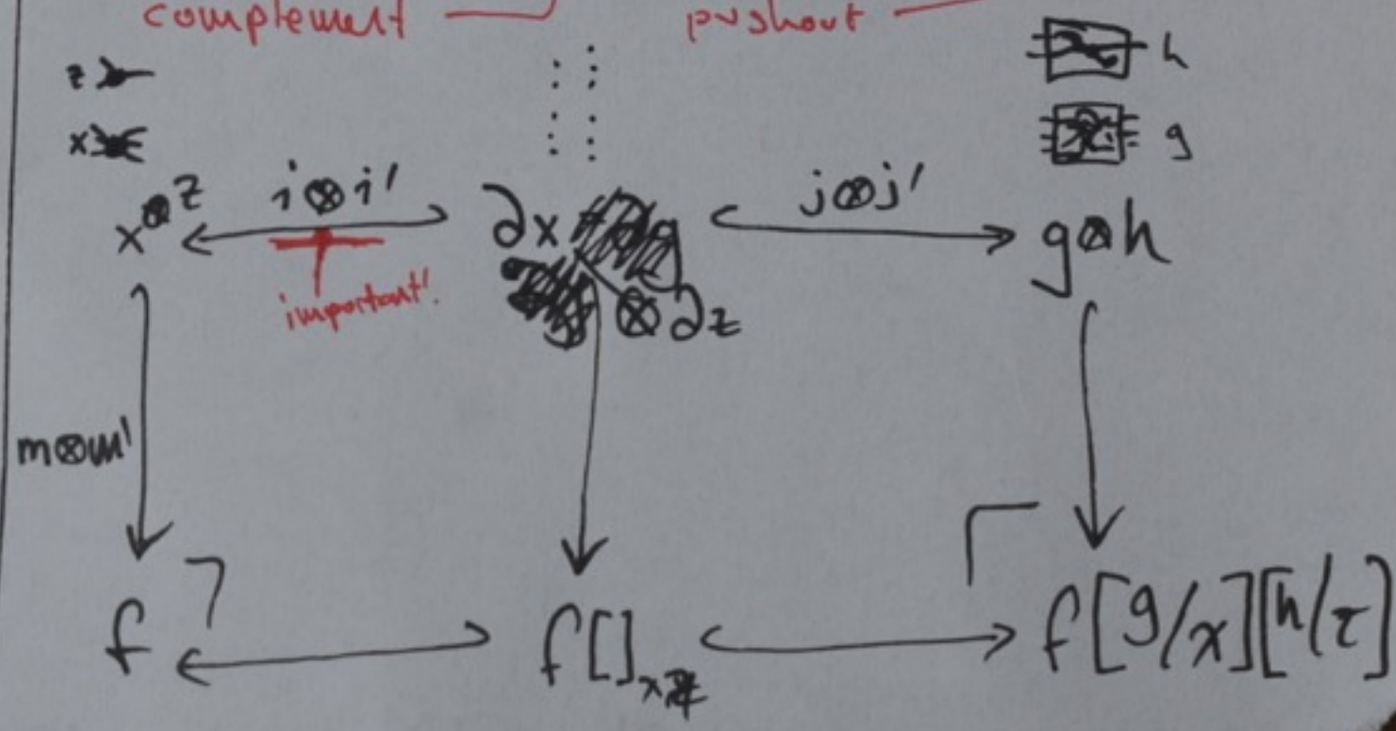
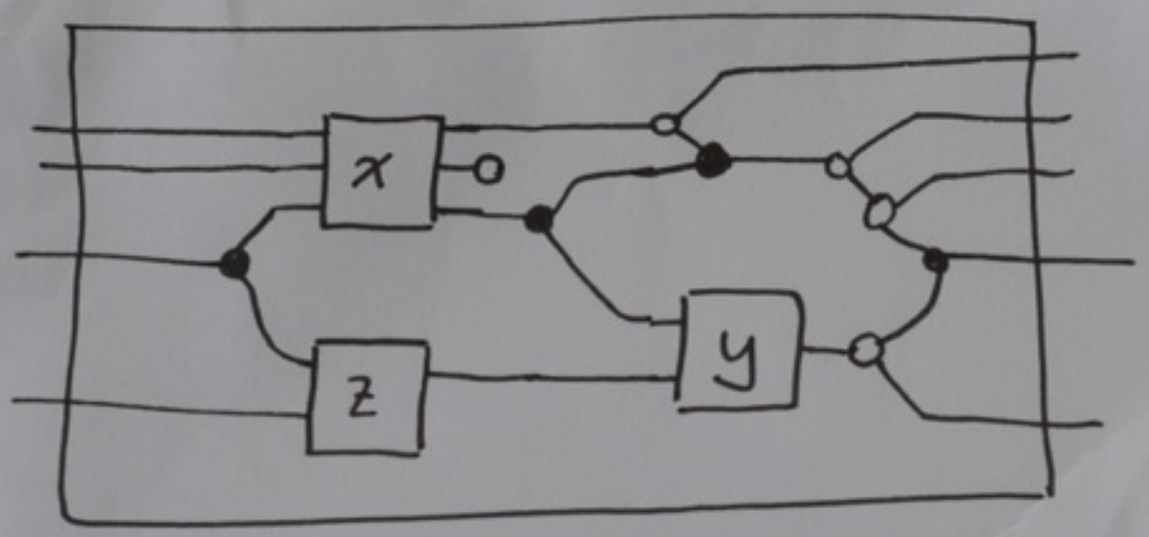




3. PATTERN-MATCHING.



$x:(3,3), y:(2,1), z:(2,1) \vdash f:(4,5)$

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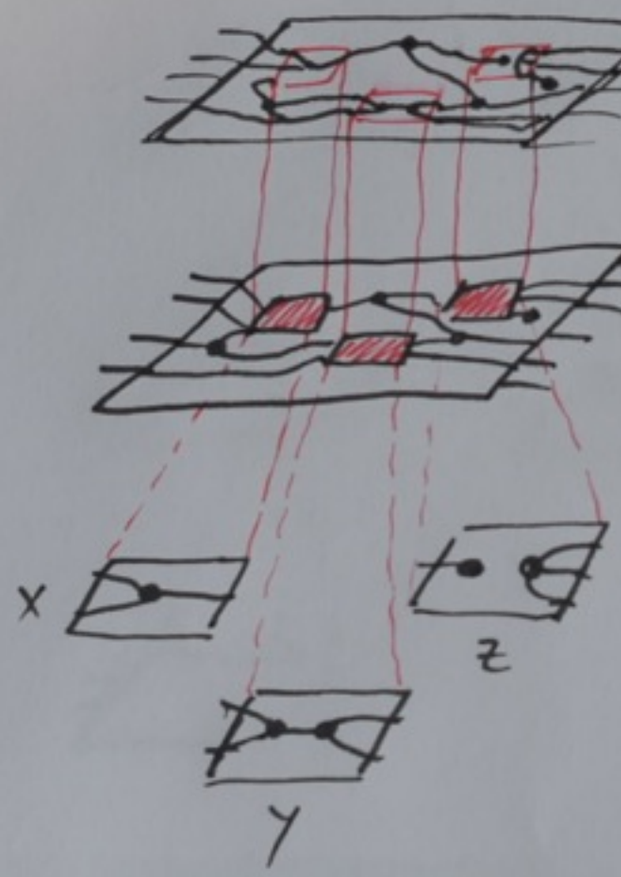

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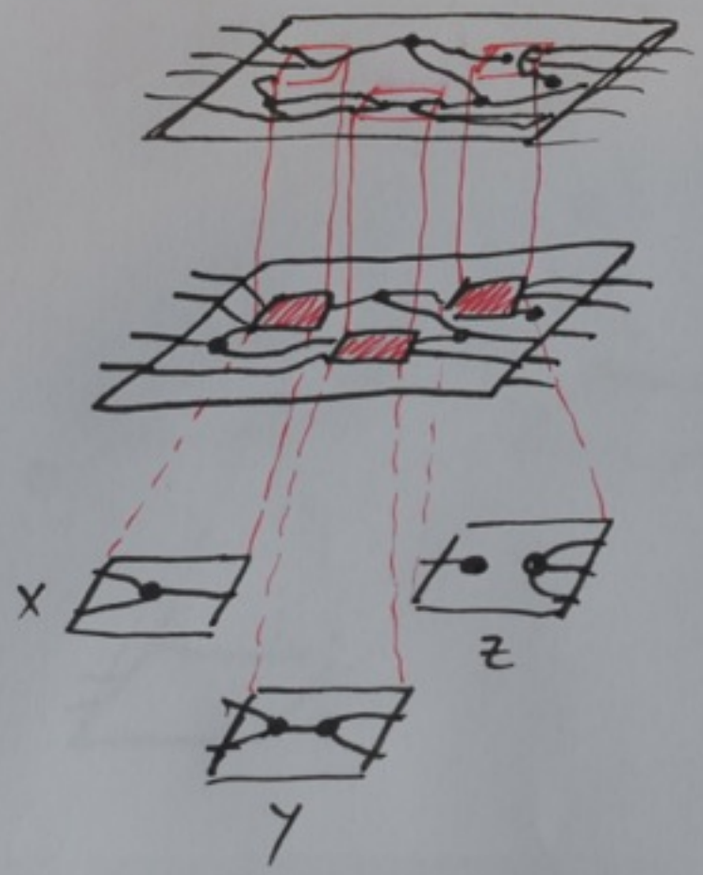
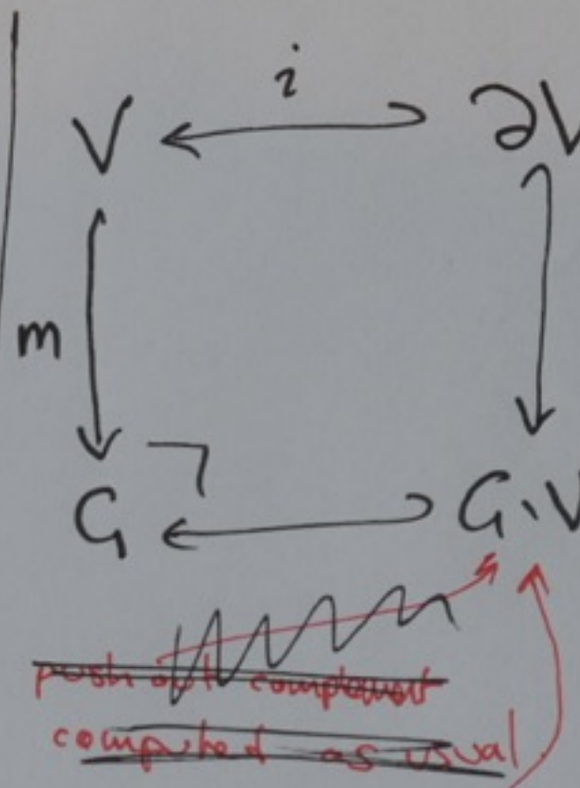
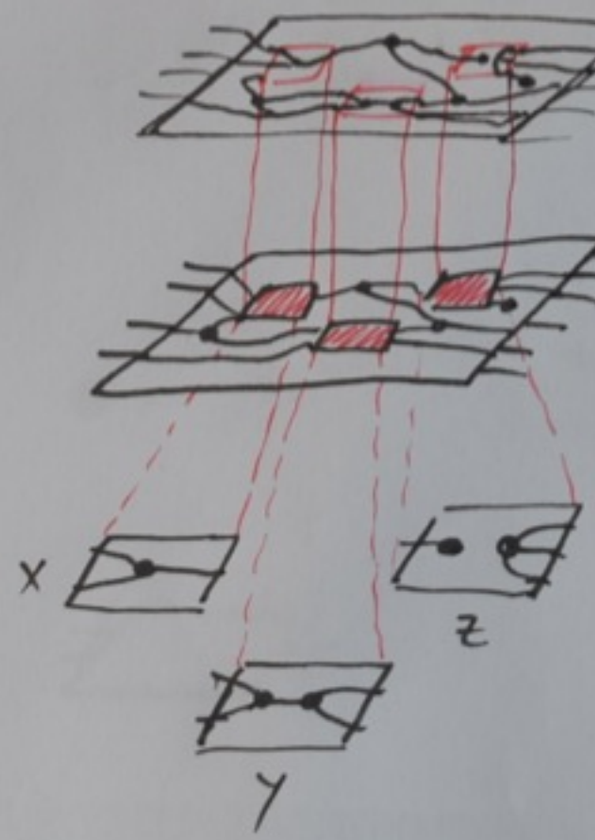


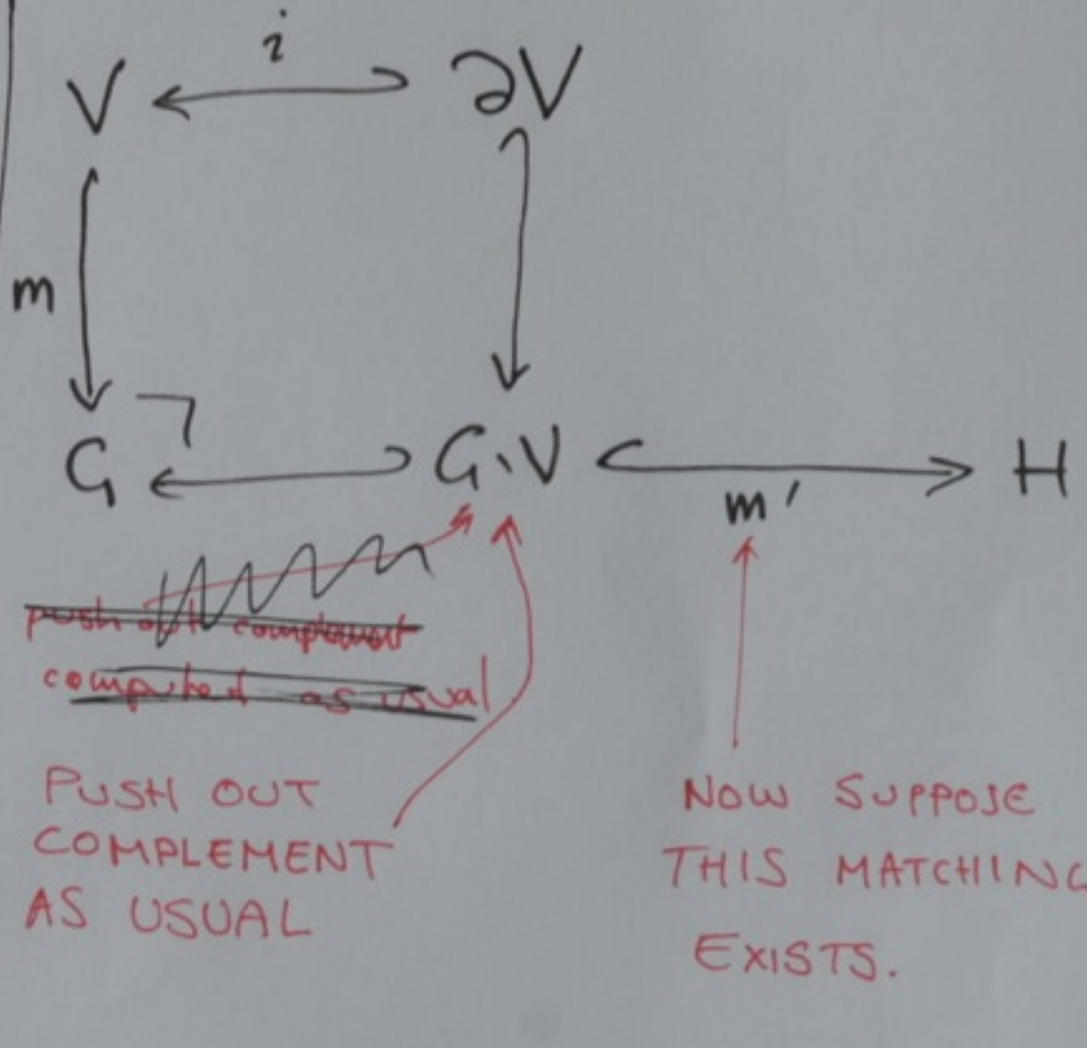


~~push out complement~~
~~computed as usual~~

PUSH OUT
COMPLEMENT
AS USUAL

3. PATTERN-MATCHING.





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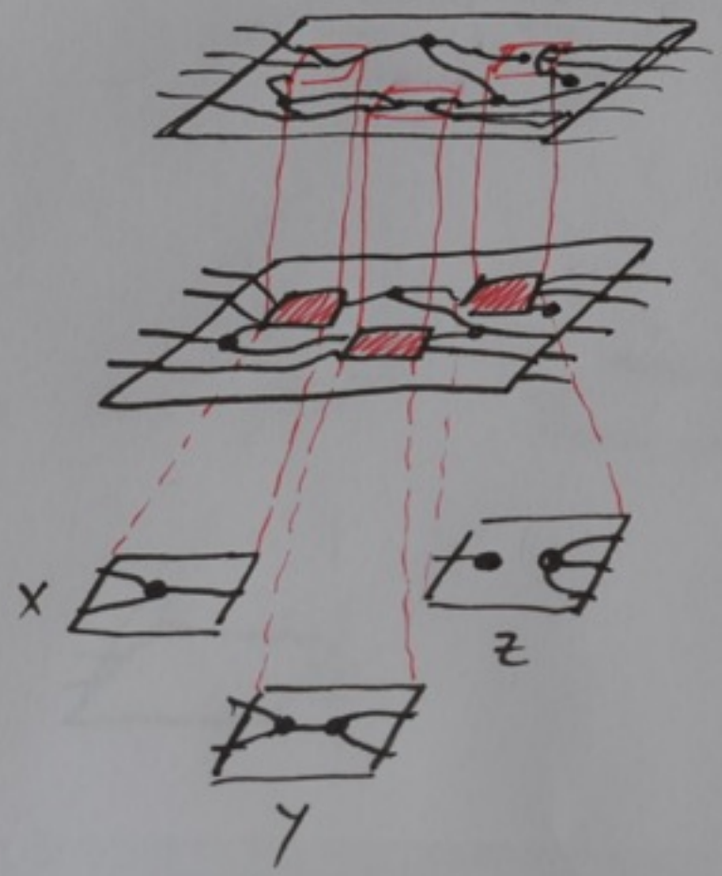


COMPUTE THIS P.O. COMPLEMENT

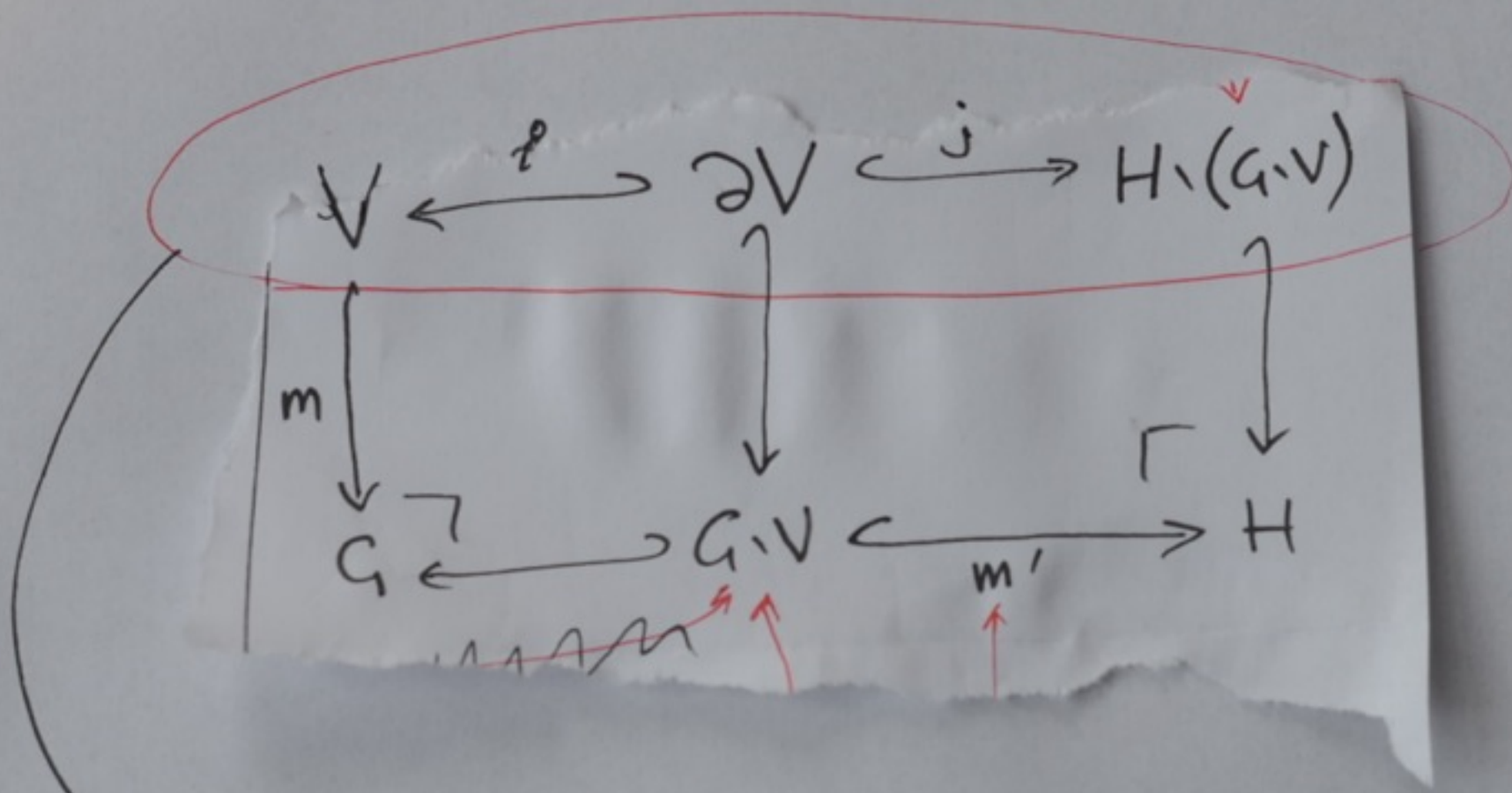
$$\begin{array}{ccccc} V & \xhookrightarrow{i} & \partial V & \xhookrightarrow{j} & H\setminus(G\cdot V) \\ {\scriptstyle m}\downarrow & & \downarrow & & \downarrow \\ G & \xhookrightarrow{} & G\cdot V & \xhookrightarrow{m'} & H \end{array}$$

~~push out complement~~
~~computed as usual~~

PUSH OUT COMPLEMENT AS USUAL

NOW SUPPOSE THIS MATCHING EXISTS.

3. PATTERN MATCHING.

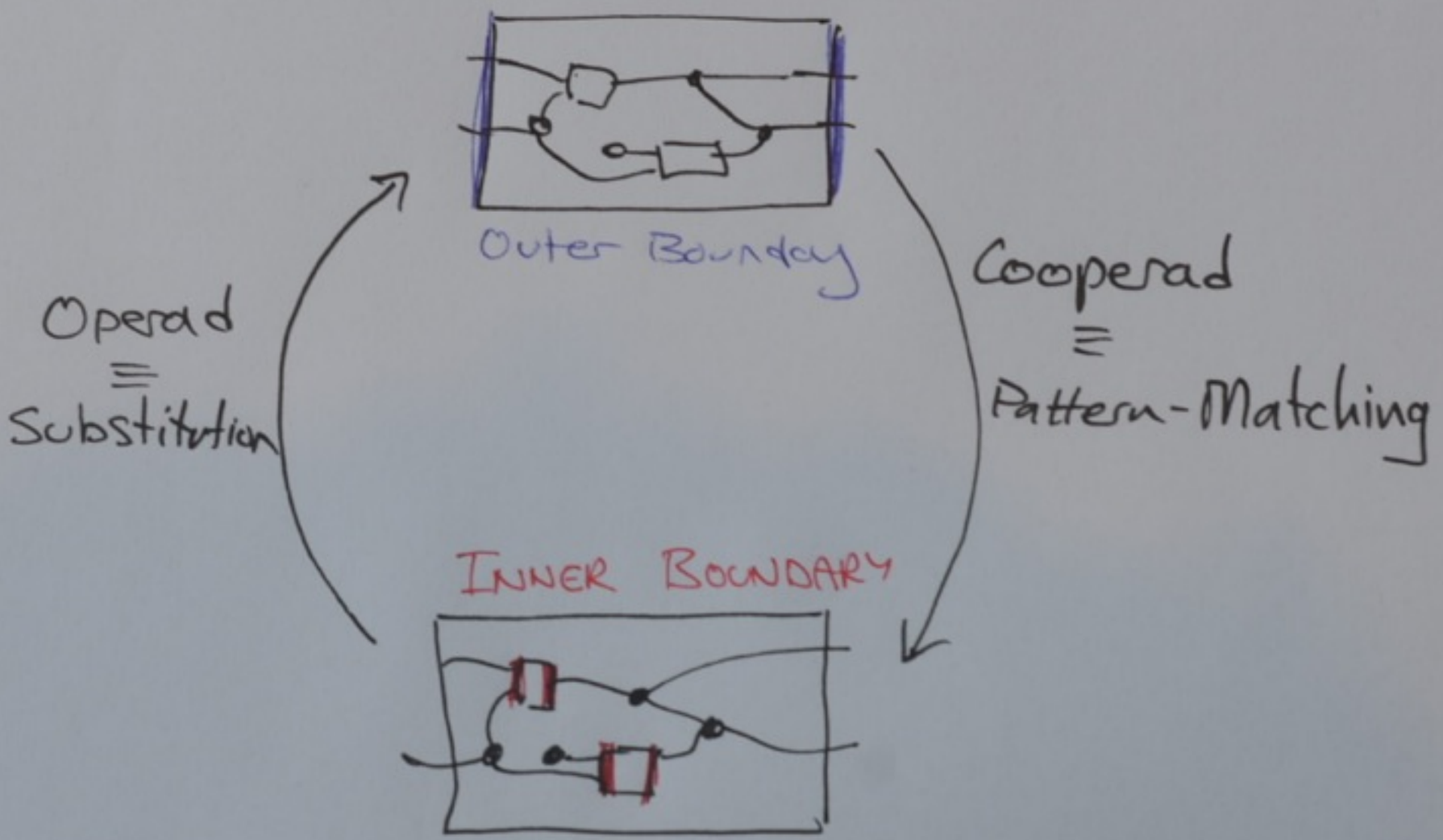


→ This span determines a cover of

$\{H \cdot (G \cdot V)$ by V.

PUTTING IT TOGETHER

STRING DIAGRAMS W/ VARIABLES



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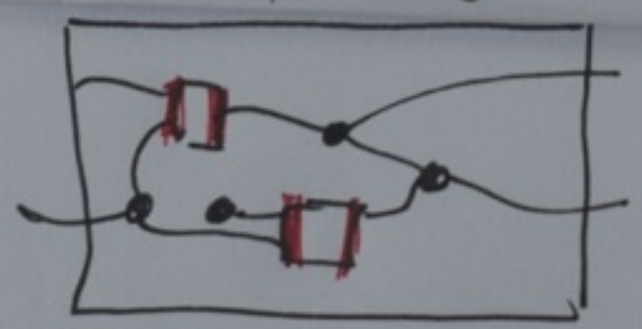
But obviously:

is totally legit



PUTTING IT TOGETHER

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But obviously:

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$\Rightarrow$ COMPUTAD } OF STRING DIAGRAMS
POLYCATEGORY } W/ VARIABLES

4. DITCHING LINEARITY

What about:

$$\frac{\Delta \vdash f : A}{x : B, \Delta \vdash f : A}$$ Weaken.

$$\frac{x : A, y : A, \Delta \vdash f : B}{z : A, \Delta \vdash f[z/x, z/y]}$$ Contraction

??

4. Ditching Linearity

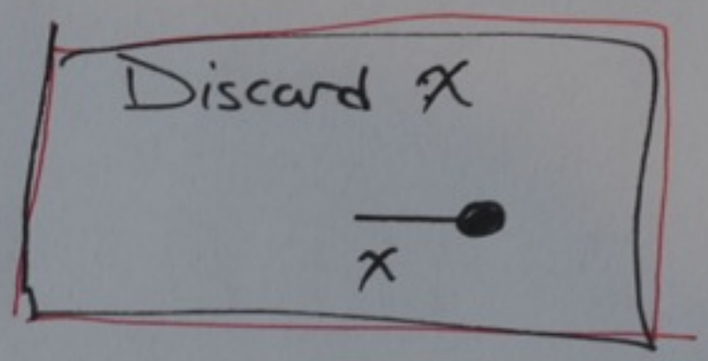
What about:

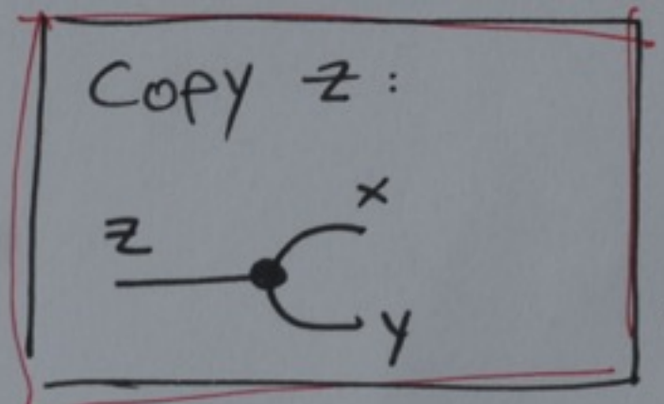
$$\frac{\Delta \vdash f : A}{x : B, \Delta \vdash f : A}\ \text{Weaken.}$$

$$\frac{x : A, y : A, \Delta \vdash f : B}{z : A, \Delta \vdash f[z/x, z/y]}\ \text{Contraction}$$

??

Relax matchings to relations:





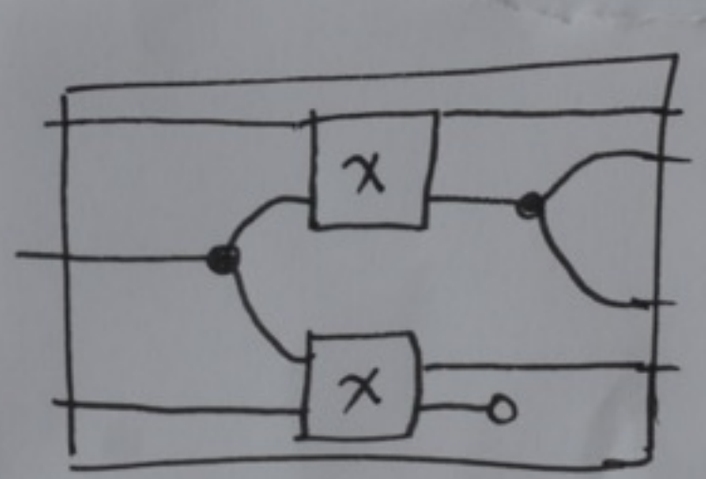
Cocommutative Comonoid.

4. Ditching Linearity

- $$\frac{\Delta \vdash t:A,\ t':A}{\Delta \vdash\ t'':A}\ \text{Contraction}$$

with $t = t' = t''$

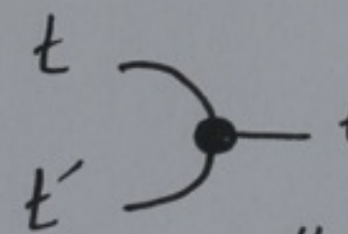
$$\frac{\Delta \vdash t:A}{\Delta \vdash t:A,\ t'':B}\ \text{Weakening}$$



4. Ditching Linearity

$$\frac{\Delta \vdash t:A,\ t':A}{\Delta \vdash\ t'':A}\ \text{Contraction}$$

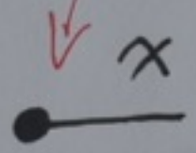
with ~~$t \neq t' \neq t''$~~



$t'' = \text{M.C.U.}(t, t')$ *

↑ Commutative Monoid!

$$\frac{\Delta \vdash t:A}{\Delta \vdash t:A,\ t'':B}\ \text{Weakening}$$

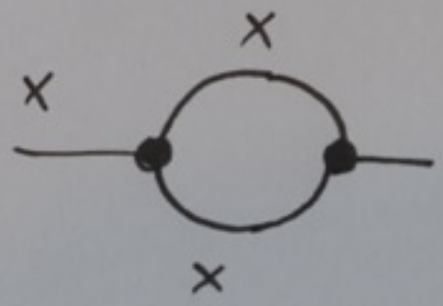


where x is a fresh variable.

* Check Assoc!

4. Ditching Linearity.

SPECIAL!



MGU(x, x) = x

= _____

4. Ditching Linearity.

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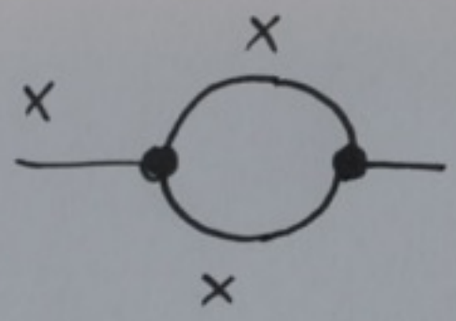

mgu(x,x) = x

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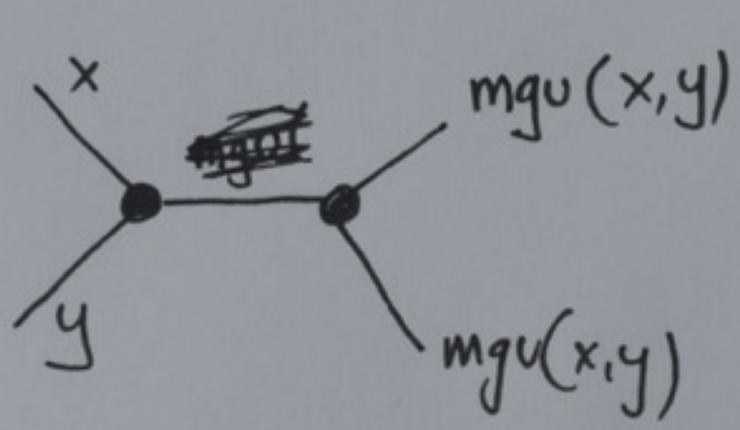

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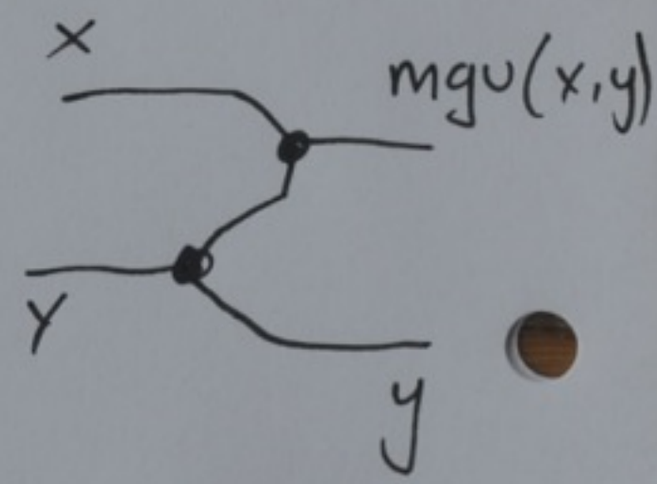
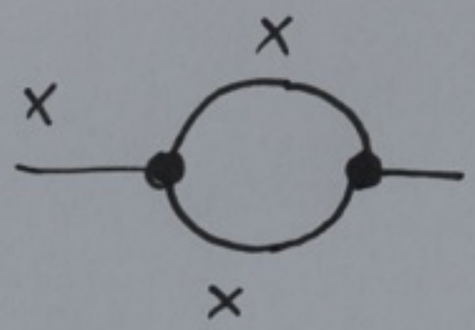


NOT FROB ☹

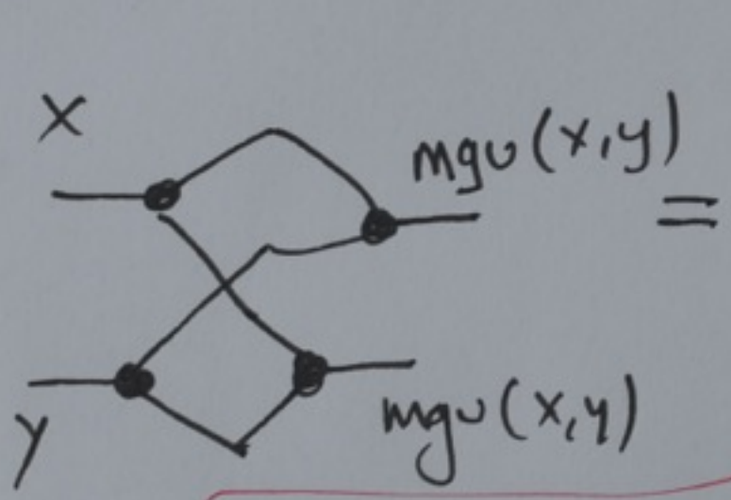
4. DITCHING LINEARITY.

SPECIAL!



$mgu(x,x) = x$

$=$ _____



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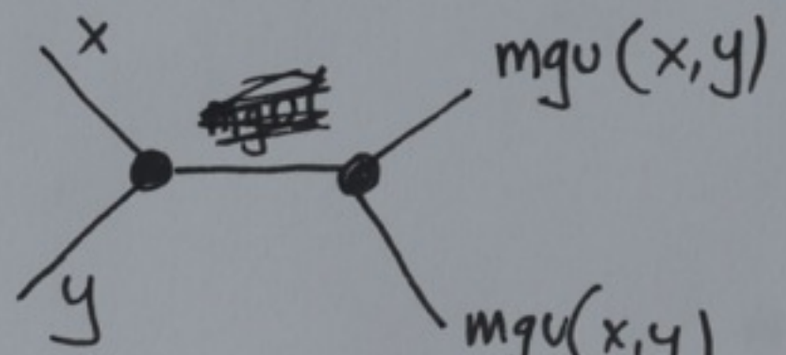
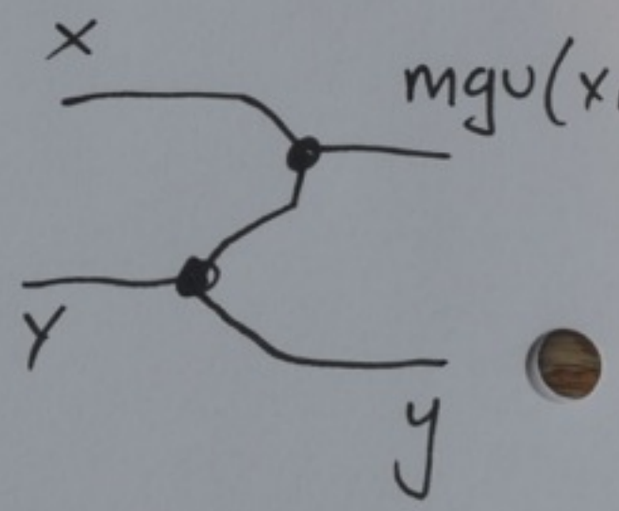
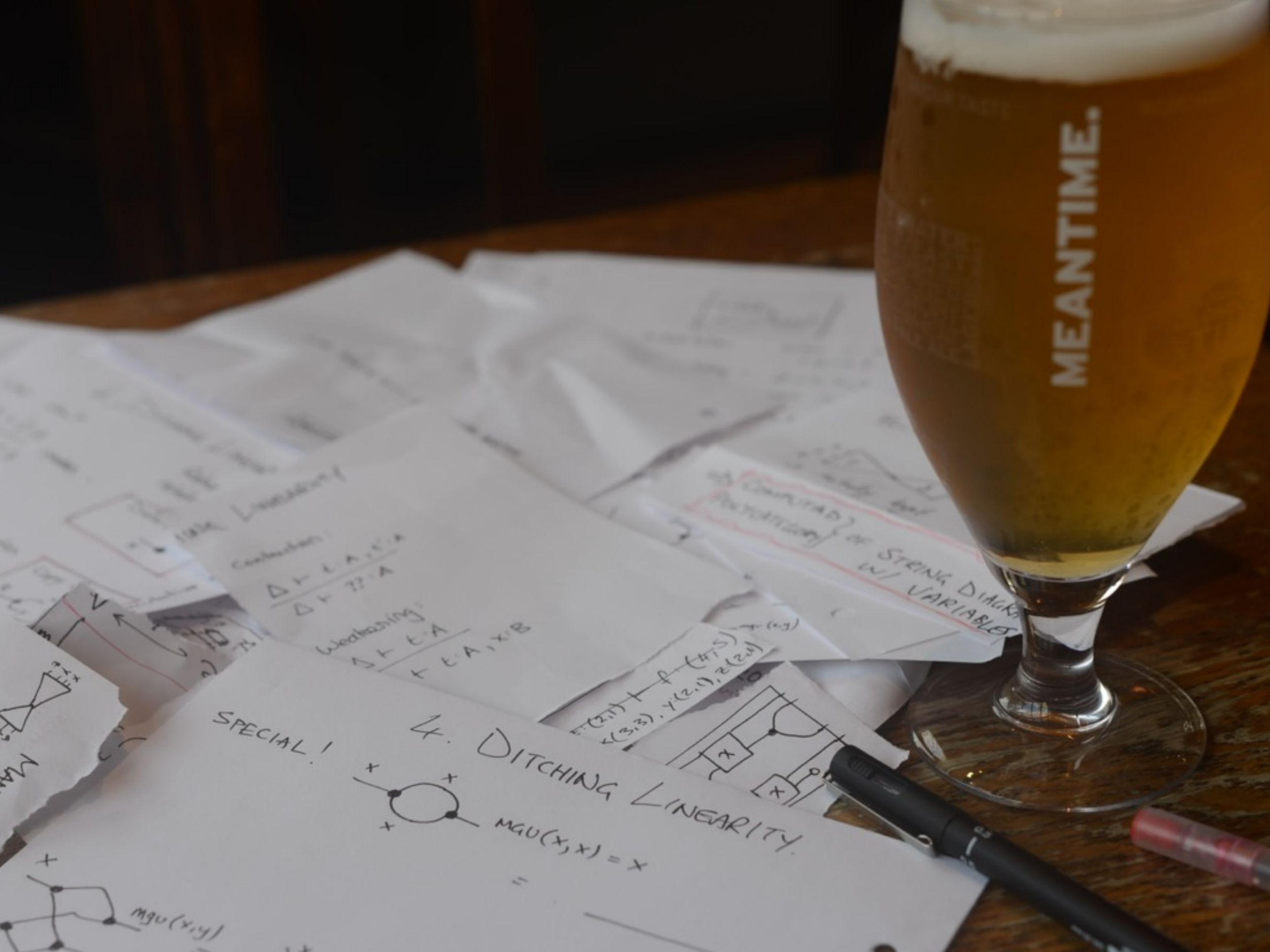


$\neq$



IS BIALGEBRA! ☺

NOT FROB ☹


MEANTIME.
Weakening:
4. DITCHING LINEARITY.
SPECIAL!
mgu(x,x) = x
mgu(x,y)