CSE 507 Advanced Systems Optimization **Name (Print):** ____________
Spring 2018
Final Exam
30-APR-2016
Time Limit: 100 Minutes Signature ____________

This exam contains 10 pages (including this cover page and 1 blank pages) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may **NOT** use your books, notes, computers, smart phones, PDAs, tablets, calculator or any computing devices on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	16	
2	31	
3	33	
4	8	
5	12	
Total:	100	

1. Let (P) be a primal linear program and (D) be its dual problem. Can the following situation happen? (Yes or No, no explanation needed)
   (a) (2 points) Both (P) and (D) are infeasible.

   (b) (2 points) Both (P) and (D) are unbounded.

   (c) (2 points) Both (P) and (D) have a finite optimum.

   (d) (2 points) (P) is unbounded (D) has finite optimum.

   (e) (2 points) (D) has a unique optimal solution and (P) has no optimal solution.

   (f) (2 points) For any basic feasible solution in (P), there is a corresponding basic feasible solution in (D) with the same objective value.

   (g) (2 points) In simplex method, if the problem is degenerate, the Big-M method can be applied to avoid cycling.

   (h) (2 points) Revised simplex method usually requires less computations than the original simplex method.

2. Consider a primal linear programming problem (LP)

$$\begin{array}{ll} \text{minimize} & -4x_1 + 10x_2 - 20x_3 \\ \text{subject to} & 2x_1 - 2x_2 + 8x_3 + 2x_4 = 40 \\ & -x_1 + 6x_2 + 5x_3 + x_5 = 50 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{array}$$

(a) (9 points) Find its dual linear programming problem.

(b) (9 points) Use the graphic method to find the optimal solution and the optimal value of the dual problem. Mark the optimal point on the graph.

(c) (13 points) Use the information obtained from (b) and write down the complementary slackness conditions and compute the optimal point of the primal problem using complimentary slackness conditions.(Hint: Slackness conditions are: $w_i(\mathbf{A_i^T}\mathbf{x} - b_i) = 0 \quad \forall i$, and $(c_j - \mathbf{w}^T\mathbf{A}_j)x_j = 0 \quad \forall j$)

3. Consider the linear programming problem (LP) below,

$$\begin{array}{ll} \text{Minimize} & -2x_1 + x_2 - 3x_3 \\ \text{subject to} & x_1 \leq 1 \\ & \sqrt{7}x_2 \leq 2\sqrt{7} \\ & \pi x_3 \leq 3\pi \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

You are going to start the simplex method from a basic feasible solution with $x_1 = x_2 = x_3 = 0$. Recall that $\mathbf{r}_q = c_q - c_B^T \mathbf{B}^{-1}\mathbf{N}_q \geq \mathbf{0}$, $\mathbf{d}_q = \left(\frac{-\mathbf{B}^{-1}\mathbf{A}_q}{\mathbf{e}_q}\right)$, and $\alpha = \underset{j \in \tilde{\mathbf{B}}}{\text{minimize}} \left[-\frac{x_j}{d_j^q} | d_j^q < 0 \right]$.

(a) (4 points) Convert the LP into its standard form

(b) (4 points) What is your current coordinate in full dimension and what are the basic and the nonbasic variables?

(c) (10 points) List all directions to the adjacent extreme points. Show all work. (you can also use the fundamental matrix ($M^{-1} = \begin{bmatrix} \mathbf{B}^{-1} & -\mathbf{B}^{-1}\mathbf{N} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$))

(d) (5 points) Is the current solution optimal? Why? List all directions which will lead to better solutions

(e) (10 points) Pick any "good" direction listed above and determine the step length which ends at an adjacent extreme point. What is your new solution?

4. Starting simplex method.

   (a) (4 points) Consider the following linear programming problem,

$$\begin{array}{ll} \text{Minimize} & x_1 + x_2 - 2x_4 \\ \text{s.t.} & x_1 + 2x_2 + x_4 = 4 \\ & -x_2 + x_3 - x_4 = -1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

   Write down Phase-I problem to initiate 2-Phase method (do not solve it).

   (b) (4 points) Consider the following linear programming problem,

$$\begin{array}{ll} \text{Minimize} & x_1 + 2x_2 \\ \text{s.t.} & x_1 + x_2 \geq 2 \\ & -x_1 + 2x_2 \geq= 3 \\ & 2x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{array}$$

   Write down Big-M problem to initiate the problem (do not solve it).

5. (12 points) Consider the following (Primal) linear programming problem

$$\begin{array}{ll} \text{Maximize} & 3x_1 + 4x_2 + 9x_3 + 2x_4 + 5x_5 \\ \text{s.t.} & 4x_1 + 7x_2 + 10x_3 + 3x_4 + 7x_5 \le 20 \\ & x_j \ge 0; \text{for all } j \end{array}$$

Convert the problem into its dual and calculate the optimal dual value. Then, using complementary slackness conditions, calculate the optimal point $(x_1, x_2, x_3, x_4, x_5)$ and optimal objective value for the primal problem.