HBKU
CSE 507 - Spring 2018
Homework # 5 Solution

Part I
4.2

a.

The Primal:

Minimize $9x_1 + 6x_2$

Subject to $3x_1 + 8x_2 \geq 4$

$5x_1 + 2x_2 \geq 7$

$x_1, x_2 \geq 0$

The Dual:

Maximize $4\pi_1 + 7\pi_2$

Subject to $3\pi_1 + 5\pi_2 \leq 9$

$8\pi_1 + 2\pi_2 \leq 6$

$\pi_1, \pi_2 \geq 0$

b.

The Primal:

Maximize $4x_1 + 7x_2$

Subject to $3x_1 + 5x_2 \leq 9$

$8x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

The Dual:

Minimize $9\pi_1 + 6\pi_2$

Subject to $3\pi_1 + 8\pi_2 \geq 4$

$5\pi_1 + 2\pi_2 \geq 7$

$\pi_1, \pi_2 \geq 0$

Comparing the results of (a) and (b) above, we conclude that the dual of a dual is the original primal problem.

4.4

The Primal:

Minimize $9x_1 + 6x_2 - 4x_3 + 100$

Subject to $3x_1 + 8x_2 - 5x_3 \geq 14$

$5x_1 - 2x_2 + 6x_3 = 17$

$2x_1 + 4x_2 \leq 19$

$x_1 \leq 0, x_2 \geq 0, x_3$ unrestricted

The Dual:

Maximize $14\pi_1 + 17\pi_2 + 19\pi_3 + 100$

Subject to $3\pi_1 + 5\pi_2 + 2\pi_3 \geq 9$

$8\pi_1 - 2\pi_2 + 4\pi_3 \leq 6$

$-5\pi_1 + 6\pi_2 = -4$

$\pi_1 \geq 0, \pi_2$ unrestricted, $\pi_3 \leq 0$

Part II

Minimize $15x_1 + 64x_2 - 6x_3 - 6x_4 - 8x_5$

Subject to $x_1 + 8x_2 - x_3 - 2x_4 - x_5 = 0$

$x_1 - x_2 - 3x_3 - x_4 = -1$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

a. Derive the dual LP problem:

Maximize $-w_2$

Subject to $w_1 + w_2 \leq 15$

$8w_1 - w_2 \leq 64$

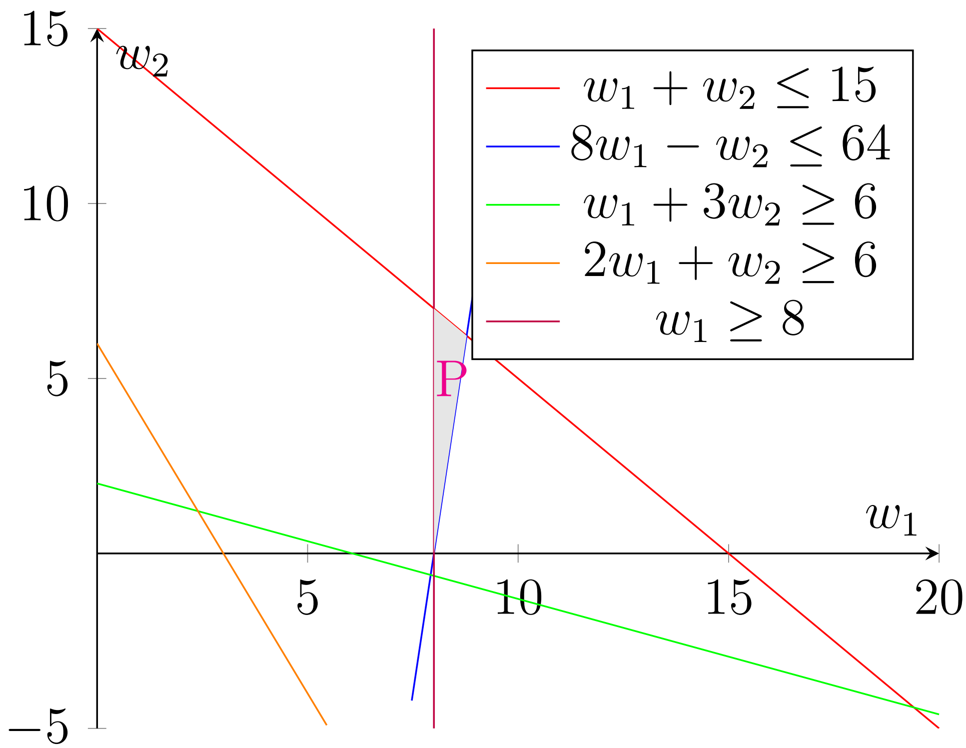
$-w_1 - 3w_2 \leq -6$

$-2w_1 - w_2 \leq -6$

$-w_1 \leq -8$

w_1, w_2 unrestricted

b. Solve the dual problem using the graphic method:



Our cost vector is $\mathbf{c} = \begin{bmatrix} 0 & -1 \end{bmatrix}$. The optimal point is $\begin{bmatrix} 8 & 0 \end{bmatrix}$ and the optimal objective value $z^* = 0$.

c. Write down complementary slackness conditions for this LP problem:

Either $r_j = (\mathbf{c} - \mathbf{A}^T\mathbf{w})_j = 0 \quad$ or $x_j = 0 \quad \forall j$

Either $s_i = (\mathbf{A}\mathbf{x} - \mathbf{b})_i = 0 \quad$ or $w_i = 0 \quad \forall i$

$r_1 = (15 - w_1 - w_2) = 0 \quad$ or $x_1 = 0$
$r_2 = (64 - 8w_1 + w_2) = 0 \quad$ or $x_2 = 0$
$r_3 = (-6 + w_1 + 3w_2) = 0 \quad$ or $x_3 = 0$
$r_4 = (-6 + 2w_1 + w_2) = 0 \quad$ or $x_4 = 0$
$r_5 = (-8 + w_1) = 0 \quad$ or $x_5 = 0$

$s_1 = (x_1 + 8x_2 - x_3 - 2x_4 - x_5) = 0 \quad$ or $w_1 = 0$
$s_2 = (x_1 - x_2 - 3x_3 - x_4 + 1) = 0 \quad$ or $w_2 = 0$

d. Find an optimal solution to the primal problem:

An optimal solution can be obtained iff $\mathbf{r}^T\mathbf{x} = 0$. Hence,

$(15 - w_1 - w_2)x_1 = 0 \rightarrow x_1 = 0$
$(64 - 8w_1 + w_2)x_2 = 0 \rightarrow x_2 \neq 0$
$(-6 + w_1 + 3w_2)x_3 = 0 \rightarrow x_3 = 0$
$(-6 + 2w_1 + w_2)x_4 = 0 \rightarrow x_4 = 0$
$(-8 + w_1)x_5 = 0 \rightarrow x5 \neq 0$

Substituting for x_1, x_3 and x_4 back into the constraints, we get $x_2 = 1$ and $x_5 = 8$. Hence, the primal optimal solution is $\begin{bmatrix} 0 & 1 & 0 & 0 & 8 \end{bmatrix}$ and the primal optimal objective value z^* is 0.

END OF HOMEWORK