CSE 507 Advanced Systems Optimization **Name (Print):** ____________________
Spring 2016
Midterm Exam 1
21-FEB-2016
Time Limit: 110 Minutes Signature ____________________

This exam contains 9 pages (including this cover page and 2 blank pages) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may **NOT** use your books, notes, computers, smart phones, PDAs, tablets, calculator or any computing devices on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **True False** questions require justification. If you think it is TRUE, you need to prove it. If you think it is FALSE, you need to show a counterexample.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	15	
2	20	
3	10	
4	55	
Total:	100	

Do not write in the table to the right.

1. For each of the following statements, if you think it can be **TRUE**, please provide a reasonable proof. Otherwise, if it is **FALSE** please provide a counterexample.

   (a) (3 points) The difference of two linear functions is always a linear function.

   (b) (3 points) Let $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ be a function. If $f(x, \bar{y})$ is linear in x for any given $\bar{y} \in \mathbb{R}$ and $f(\bar{x}, y)$ is linear in y, for any given $\bar{x} \in \mathbb{R}$, then $f(x, y)$ is a linear function.

   (c) (3 points) If set C is a nonempty polytope in $\mathbb{R}^2$, then C is also affine.

   (d) (3 points) It is impossible that every basic feasible solution of a linear program is degenerate.

   (e) (3 points) The intersection of two polyhedral sets in $\mathbb{R}^n$ is a polyhedral set.

2. Consider a linear programming problem (LP) in standard form

$$\begin{aligned} \text{minimize} \quad & \mathbf{c^T x} \\ \text{subject to} \quad & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

Where $\mathbf{A}$ is an $m \times n$ matrix, $\mathbf{c}$ an $n \times 1$ real vector, $\mathbf{b}$ is an $m \times 1$ real vector, m and n are positive integers with $m \leq n$. We define $P = \{\mathbf{x} | \ \mathbf{Ax} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$, $z = \mathbf{c^T x}$, $P^* = \{\mathbf{x}^* | \ \mathbf{x}^* \text{is an optimal solution to LP}\}$ and $z^* = \mathbf{c^T x}^*$ for $\mathbf{x}^* \in P^*$. **Select all correct answers for the following questions. Your answer has to be completely correct to earn full credit. There will be a penalty of -1.5 points for each wrong choice.** No proofs needed.

(a) (4 points) P may have exactly
A. 0 B. 1 C. 2 D. 3 E. $+\infty$ elements.

(b) (4 points) P may have exactly
A. 0 B. 1 C. 2 D. 3 E. $+\infty$ vertices.

(c) (4 points) P may be:
A. a cone B. a convex set C. an affine set D. a half space
E. an unbounded set.

(d) (4 points) P^* may be
A. a cone B. a convex set C. an affine set D. a half space
E. an unbounded set.

(e) (4 points) P^* may contain
A. no element of P B. exactly 1 vertex of P
C. exactly 1 edge (but not a vertex) of P D. exactly 1 interior point of P
E. all elements of P.

3. Important theorems of LP.

   (a) (5 points) State the "Resolution Theorem" for convex polyhedrons (no proof needed).

   (b) (5 points) State the "Fundamental Theorem" of linear programming (no proof needed).

4. Consider the linear programming problem (LP) below,

$$\begin{array}{ll} \text{maximize} & x_1 + 2x_2 \\ \text{subject to} & 5x_1 + x_2 \leq 10 \\ & -x_1 + x_2 \leq 4 \\ & x_2 \leq 4 \\ & x_1 \leq 2 \\ & x_1, x_2 \geq 0 \end{array}$$

(a) (8 points) Convert the LP into its standard form

(b) (15 points) Draw a 2-dimensional graph of its feasible domain P. Be sure to mark the coordinates of each extreme point of P. Apply the graphic method to find its optimal solution **and** optimal objective value.

(c) (18 points) How many basic solutions are there? Generate all basic solutions of the standard for linear program and match them with the points in the graph. Show all work.

(d) (8 points) How many basic feasible solutions are there? Point out the corresponding extreme point of each basic feasible solution on the graph of the feasible domain P.

(e) (6 points) Which points on the graph are degenerate basic feasible solution?