Research Statement

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1. Introduction

I am the author of 140+ publications including 100 peer-reviewed journal articles and 2 books published by Springer. In this statement, I will discuss some of the major aspects of my research. I will not discuss other substantial parts of my work, for example, those in combinatorics on words [1, 2, 5, 6, 13, 31, 32, 34, 36, 41, 72], in graph theory [1, 12, 16, 17, 22, 28, 51], in algebra [87], in formal languages theory [45], and in number theory [62].

Much of my research (65+ publications including a book) is dedicated to the subject of pattern avoidance in permutations and words. This very active area of research, introduced by Donald Knuth, has its roots in theoretical computer science (in the problem of sorting a permutation through a stack) and it has relevance to several areas of mathematics (e.g. algebraic combinatorics, bijective combinatorics, and combinatorics on words), theoretical physics and computational biology. Another major direction of my recent research is the theory of graph representations (20+ publications including a book), in particular, the theory of word-representable graphs I introduced in 2004, which has attracted the attention of many researchers around the globe.

Some of my achievements over the last ten years are:

- My book "Patterns in permutations and words" [44] (published by Springer in *EATCS monographs in Theoretical Computer Science book series* in 2011) is the only comprehensive survey to date on existing literature and modern research directions in a very active area of research with hundreds of researchers involved, and over a thousand papers published to date.
- In [8], in collaboration with other researchers, I enumerated interval orders, a classical object introduced by Fishburn, thus settling a 40+ years old problem. This paper has been very influential and it was for several years "the most cited paper of the preceding five years in *Journal of Combinatorial Theory, Series A*". This journal is widely regarded as the most prestigious journal in combinatorics.
- My second Springer book "Words and Graphs" [58], written in collaboration with Lozin in 2015 and published by Springer in *EATCS monographs in Theoretical Computer Science book series*, introduces the reader to the theory of word-representable graphs that I pioneered, a field enjoying ever greater attention by other researchers. These graphs do not only generalize several well-known classes of graphs and are interesting from the algorithmic point of view, but also can be linked to robot scheduling. I plan to further develop this theory and its generalizations.
- In collaboration with Seif [87] I solved the word problem for the *Perkins semigroup* in terms of certain digraphs. This semigroup has played a central role in semigroup theory since 1960, particularly as a source of examples and counterexamples.
- Tutte [94] introduced planar maps in the 1960s in connection with what later became the celebrated *Four Colour Theorem.* Even though self-dual graphs were well studied, nothing was known about self-dual maps, and in particular, their enumeration remained an open problem for 50+ years. In collaboration with de Meir and Noy [70] I not only solved the problem, but also enumerated 2-connected and 3-connected self-dual maps, showing that the latter are counted by the Fine numbers. The novel methods we introduced there are likely to be applicable, e.g. in the enumeration of 3-connected maps, still an open problem.

- In collaboration with Ehrenborg and Perry [27] I settled a conjecture of Warlimont on asymptotics for the number of permutations avoiding a consecutive pattern. In another paper [81], as a corollary to more general results, in collaboration with Remmel, I proved a conjecture of Mather, 3 conjectures of Hardin, and a conjecture of Baker.
- In collaboration with Remmel [80] I refined classic enumeration results of André on alternating permutations obtained in 1879.
- *Gray codes*, first applied in telegraphy in 1878, are an important listing method from theoretical and practical points of view. In [4], in collaboration with other researchers, I constructed Gray codes for exhaustive generation of three classes of planar maps. Planar maps are a natural model of discrete surfaces used, e.g. in 2D-quantum-gravity. The results would not be possible without our novel idea on applying description trees in the context.
- In [55], I proved a key result in the theory of graphs representable by words and patterns, establishing that any binary pattern of length at least 3 can be used to represent any graph. The significance of this rather unexpected existence is that there are only two interesting non-equivalent cases, one of which is the class of well-studied word-representable graphs.
- The study of *alternating permutations*, counted by Euler numbers and occurring in many places in mathematics, goes back to 1879, and still attracts much attention. However, no one managed to define the notion of an alternating word, a natural extension from permutations to words, until, in collaboration with Gao and Zhang [33], we not only closed that gap by defining and enumerating alternating words, but also enumerated classes of alternating words avoiding patterns of length 3 that gives links to the Narayana and Fibonacci numbers.

In the rest of the statement, I will discuss some of my research in the theory of patterns in combinatorial structures (Section 2), in the theory of word-representable graphs (Section 3), and in the theory of planar maps (Section 4).

2. Patterns in permutations, words and matrices

We write permutations as words $\pi = a_1 \cdots a_n$, whose letters are distinct and usually consist of the integers $1, \ldots, n$. An occurrence of a "classical" *pattern* τ in a permutation π is a subsequence in π (of the same length as τ) whose letters are in the same relative order as those in τ. We denote by $\mathcal{S}_n(\tau)$ the set of all permutations in $\mathcal{S}_n$ which *avoid* τ, that is, have no occurrences of τ. If $R = \{\tau_1, \ldots, \tau_m\}$, we let $\mathcal{S}_n(R) = \bigcap_{1 \le i \le m} \mathcal{S}_n(\tau_i)$. Fundamental questions are to determine $|\mathcal{S}_n(R)|$ viewed as a function of n, and if $|\mathcal{S}_n(R)| = |\mathcal{S}_n(R')|$ to find an explicit bijection between $\mathcal{S}_n(R)$ and $\mathcal{S}_n(R')$. It is also interesting to find relations between $\mathcal{S}_n(R)$ and other combinatorial structures. By determining $|\mathcal{S}_n(R)|$ we mean finding an explicit formula, or *ordinary* or *exponential generating functions* (*GF* and *EGF* respectively). A more general question is to find the *distribution* of a given pattern τ, that is, to find out how many permutations of length n have exactly k occurrences of τ for any n and k.

The origin of the modern day study of permutation patterns can be traced back to papers by Rotem, Rogers, and Knuth in the 1970s and early 1980s. The first systematic study of these patterns was not undertaken until [93] by Simion and Schmidt in 1985.

In [7] Babson and Steingrímsson introduced *generalized permutation patterns*, now known as *vincular patterns*, that allow the requirement that two adjacent letters in a pattern must be adjacent in the permutation. If we write, say 2-31, then we mean that if this pattern occurs in a permutation π, then the letters in π that correspond to 3 and 1 are adjacent. E.g., $\pi = 516423$ has only one occurrence of the pattern 2-31, namely the subword 564.

The motivation for introducing these patterns in [7] was the study of *Mahonian statistics*. Many interesting results on vincular patterns appear in the literature (see [44, Chapter 7]). In particular, [19] provides relations of vincular patterns to several well studied combinatorial structures, such as set partitions, *Dyck paths*, *Motzkin paths* and involutions.

2.1. **My research in permutation patterns.** It is a classical result, first established by Knuth [88] in 1969, that the number of 3-2-1-avoiding permutations is equal to that of 2-3-1-avoiding permutations and both are given by *Catalan numbers*. In the literature one can find many subsequent bijective proofs confirming this fact. In [20] we classify the existing bijections showing which of them are equivalent modulo trivial bijections and how many permutation statistics each of the bijections preserves. Moreover, in [20] we introduce a recursive description of the algorithmic bijection given by Richards in 1988 (combined with a bijection by Knuth from 1969). This bijection respects 11 statistics (the largest number of statistics any of the bijections respects).

In [23] we give a new bijection, preserving 7 statistics, between $\beta(1,0)$-trees (introduced by Jacquard and Schaeffer in [24]) and permutations avoiding 3-1-4-2 and 2-41-3 which are the reverse of *nonseparable permutations* equinumerous with *nonseparable planar maps*. As a corollary to this bijection, we give a new bijection between nonseparable permutations and rooted nonseparable planar maps. In connection with this we give a nontrivial involution on the $\beta(1,0)$-trees, which specializes to an involution on unlabeled rooted plane trees, where it yields interesting results.

2.2. **Results on pattern avoidance.** As [93] deals with multi-avoidance of "classical" patterns in [46] we give either an explicit formula or a recursive formula for many cases of simultaneous avoidance of more than one consecutive 3-patterns (such patterns correspond to contiguous subwords anywhere in permutation). The remaining cases were solved in [59, 60]. In [27] we suggest an analytic approach using the spectral theory of integral operators on $L^2([0,1]^m)$ (where $m+1$ is the length of prohibited patterns) to study asymptotics for consecutive patterns. Our methods give detailed asymptotic expansions and allow for explicit computation of leading terms in many cases. We obtain in a different way, and improve, some of the results from [29]. Moreover, as a corollary to our results, we settle a conjecture of Warlimont on asymptotics for the number of permutations avoiding a consecutive pattern. In general, there are several approaches to study occurrences of consecutive patterns in permutations. In [3] we propose yet another approach which is based on considering the *graph of patterns overlaps*, which is a certain subgraph of the *de Bruijn graph*.

In [48, 59, 60] we consider avoidance of vincular 3-patterns with additional restrictions. The restrictions consist of demanding that permutations in question must begin and/or end with certain patterns. One motivation for considering such additional restrictions is their connection to some classes of trees. Also, in [48] we get relations to partitions of special type, and obtain a new simple identity involving the *Bell numbers* and the *Stirling numbers* of the second kind.

2.3. **An extension of vincular patterns.** We define the following class of permutations:

$$\mathcal{R}_n = \{\pi_1 \dots \pi_n \in \mathcal{S}_n \mid \text{if } \pi_i\pi_j\pi_k \text{ forms 2-3-1 then } j \neq i+1 \text{ or } \pi_i \neq \pi_k + 1\}.$$

Essentially, it is a class of permutations that avoid a particular pattern of length three. Such a pattern is an instance of *bivincular patterns* introduced by us in [8], and in addition to the properties of vincular patterns, one can also control consecutive values. We note that *mesh patterns* introduced in [9] by Brändén and Claesson arose from bivincular patterns.

In [8] we present bijections between four classes of combinatorial objects. Two of them, the class of *unlabeled* $(2+2)$*-free posets* and a certain class of *chord diagrams* (or *involutions*), already appear in the literature. The third one is $\mathcal{R}_n$. The fourth class is formed by certain integer sequences, called *ascent sequences*. Our bijections preserve numerous statistics. Moreover, in [8] we determine the generating function of these classes of objects, thus recovering a series obtained by Zagier for chord diagrams. The fact that this series counts (2+2)-free posets is new. In any case, in [78] we extend the enumerative result by finding the generating function for (2+2)-free posets when four statistics are taken into account, one of which is the number of minimal elements in a poset. Also, in [8] we characterize ascent sequences that correspond to permutations avoiding the barred pattern 3-$\bar{1}$-5-2-$\bar{4}$, and enumerate those permutations, thus settling a conjecture of Lara Pudwell. Finally, in [26] we not only give enumeration of (2+2)-free posets by *indistinguishable elements*, but also settle and generalize a conjecture of Jovovic on the number of certain upper-triangular matrices.

2.4. **Partially ordered patterns in permutations and words.** In [47, 53] I introduced a further generalization of vincular patterns (VPs), namely *partially ordered patterns* (*POPs*). A POP is a VP some of whose letters are incomparable. The notion of a POP allows us to collect under one roof (to provide a uniform notation for) several combinatorial structures studied in the literature such as *peaks*, *valleys*, *modified maxima* and *modified minima* in permutations, *Horse permutations*, *p-descents* and others.

In any case, the motivation for introducing POPs in [47, 53] is that they allow us to find the EGF for the *entire distribution* of the maximum number of non-overlapping occurrences of a pattern τ with no dashes, if we only know the EGF for the number of permutations that avoid τ (we do know the EGF for many VPs with no dashes due to [29]).

Theorem 2.1 ([47, 53])**.** *Let τ be a consecutive pattern. Let $A(x)$ be the EGF for the number of permutations that avoid τ. Then,*

$$\sum_{\pi} y^{\tau\text{-}nlap(\pi)} \frac{x^{|\pi|}}{|\pi|!} = \frac{A(x)}{1-y((x-1)A(x)+1)}$$

where τ-nlap(π) is the maximum number of non-overlapping occurrences of τ in π.

An alternative, more complicated proof of Theorem 2.1, as well as that of Theorem 2.3 below, appears in [91, 92]. Theorem 2.1 is a starting point in [91, 92] where it is proved by exploiting the relationship between the elementary and homogeneous symmetric functions. Also, the class of permutation patterns for which Theorem 2.1 holds is enlarged in [91, 92]. Moreover, in [53, 91, 92] a q-analogue for Theorem 2.1 is found.

Also, in [47] we give alternative proofs, using inclusion-exclusion, of some of the results in [29]. Our proofs result in explicit formulas for the coefficients of the EGF whereas in [29] the authors obtained differential equations for these EGF. One more result in [47] is worth mentioning (it is cited in [89] by Knuth, who actually introduced the concept of a permutation pattern in [88]):

Theorem 2.2 ([47])**.** *Let $\tau = \sigma$-k, where σ is an arbitrary consecutive pattern on the elements $1, \dots, k-1$. Let $A(x)$ (resp. $B(x)$) be the EGF for the number of permutations that avoid σ (resp. τ). Then $B(x) = e^{F(x,A(y))}$, where $F(x, A(y)) = \int_0^x A(y)\, dy$.*

As a corollary to a much more general theorem on certain POPs, in [53] I found a bivariate generating function for the distribution of peaks in permutations (x is responsible for length of permutations and y for the number of occurrences of peaks):

$$1 - \frac{1}{y} + \frac{1}{y}\sqrt{y-1} \cdot \tan\left(x\sqrt{y-1} + \arctan\left(\frac{1}{\sqrt{y-1}}\right)\right).$$

Even though the "peaks in permutations" is a classical combinatorial object, the generating function for the peaks' distribution was only known in terms of a continued fraction while we provided an exact formula for it.

Further study of POPs in permutations is conducted in [52], where we not only prove a result from [46] in a much simpler way, but also establish a connection between POPs with no dashes and walks on lattice points starting from the origin and remaining in the positive quadrant. Also, in [52] we find the EGF for the number of permutations avoiding consecutive POPs of length 4 in few cases.

In [73] we study a special type of POPs, called V- and Λ-patterns, which generalize valleys and peaks, as well as *increasing* and *decreasing runs*, in permutations. A complete classification of permutations (multi)-avoiding V- and Λ-patterns of length 4 is given. We also establish a connection between restricted permutations and matchings in the *coronas of complete graphs*.

In [11] we study the encoding of several combinatorial objects by POPs. That is, given a class of objects, say, a class T_n of certain graphs on n nodes, we find a set of POPs to avoid in n-permutations which gives the same cardinality as the cardinality of T_n. This idea was used in [68] to enumerate occurrences of consecutive patterns in *compositions* (a composition, after removing all "+" signs, can be viewed as a word over the alphabet of natural numbers).

In [62] we extend the concept of POPs in permutations to that in words. We give analogies, extend and generalize several known results, and get some new results. In particular, we give the GF for the entire distribution of the maximum number of non-overlapping occurrences of a pattern τ with no dashes (and possibly with repeated letters) in k-ary words, provided we know the GF for the number of k-ary words that avoid τ (we do know the GF for many of such patterns). The next theorem can be compared with Theorem 2.1.

Theorem 2.3 ([62]). *Let τ be a consecutive pattern and let $A_\tau(x;k) = \sum_{n\geq 0} a_\tau(n;k)x^n$ be the GF for the numbers $a_\tau(n;k)$ of words in $[k]^n$ avoiding the pattern τ. Then, for all $k \geq 1$,*

$$\sum_{n\geq 0}\sum_{\sigma\in[k]^n} y^{\tau\text{-}nlap(\sigma)}x^n = \frac{A_\tau(x;k)}{1-y((kx-1)A_\tau(x;k)+1)}$$

where τ-$nlap(\sigma)$ is the maximum number of non-overlapping occurrences of τ in σ.

A q-analogue to Theorem 2.3, based on considering patterns in compositions, is found in [42].

2.5. **Other notions of a "pattern".** In [50, 67, 83] we generalize the concept of pattern occurrence in permutations and words first to that in matrices and then to pattern occurrences in n-dimensional objects, which are basically sets of $(n+1)$-tuples. For certain patterns or sets of patterns related to our objects, we get an unexpected connection to *Ramsey Theory*, which makes our research interesting from a graph theoretic point of view. For example, occurrences of particular matrix patterns in the adjacency matrix of a graph pose certain restrictions on the set of edges of the graph and on its cycles. The *bipartite Ramsey numbers* and the concept of "*vanishing borders*" arise in our study. In some cases, we employ direct combinatorial considerations to obtain either explicit closed formulas or generating functions; in other cases, we use the transfer matrix method to derive an algorithm which gives closed formulas.

One more way to proceed with patterns is to pay attention to parity of elements. In [75] we refine the well-known permutation statistic "descent" by fixing parity of (exactly) one of the descent's numbers. We provide explicit formulas for the distribution of these (four) new statistics. We use certain differential operators to obtain the formulas. Moreover, we discuss connections of our new statistics to the *Genocchi numbers*, the study of which goes back to Euler. We also provide bijective proofs of some of our results from this paper. In [76] we have generalized the results of [75] to classify descents according to equivalence mod k for $k \geq 3$. The papers [75] and [76] can be viewed as a first step in a more general program which is to study pattern avoiding conditions on permutations where generalized parity considerations are taken into account. The work in this direction is continued in [39, 77, 90]. However, the most general framework in modern permutation pattern theory is suggested in [77].

2.6. **More on my research in patterns.** In [49, 65, 66] we count the occurrences of certain patterns in certain words. These words were chosen to be the set of all finite approximations of certain sequences. In [66] we start a general study of counting the number of occurrences of patterns in words generated by *morphisms*. In [65] we treat the sequence obtained from the *Peano curve*. The Peano curve was studied by Peano in 1890 as an example of a continuous space filling curve. Finally, in [49] we study the *sigma-sequence*, which was used by Evdokimov [30] to construct chains of maximal length in the n-dimensional unit cube. The sigma-sequence is interesting, e.g., in connection with the well-known *Dragon curve*, discovered by physicist John E. Heighway.

3. Word-representable graphs and their generalizations

A graph $G = (V, E)$ is *word-representable* if there exists a word w over the alphabet V such that letters x and y alternate in w if and only if $xy \in E$ for each $x \neq y$. For example, the 4-cycle labeled by 1, 2, 3 and 4 in a clockwise direction, can be represented by the word 13243142. Word-representable graphs generalize several important classes of graphs such as 3-*colorable graphs*, *comparability graphs* and *circle graphs*. A comprehensive introduction to, and the state of the art of the theory of word-representable graphs is given in the book [58] coauthored by me, and also in my paper [56]. A long line of research on word-representable graphs includes, but is not limited to [14, 15, 35, 37, 38, 74, 85, 86].

Word-representable graphs appear in real-life situations as follows. Consider a scenario with n recurring tasks and requirements on alternation of certain pairs of tasks, which captures typical situations in periodic scheduling, where there are recurring *precedence* requirements. For example, the following five tasks may be involved in the operation of a given machine: 1) Initialize controller, 2) Drain excess fluid, 3) Obtain permission from supervisor, 4) Ignite motor, 5) Check oil level. Tasks 1 & 2, 2 & 3, 3 & 4, 4 & 5, and 5 & 1 are expected to alternate between all repetitions of the events. One possible *task execution sequence* that obeys these recurrence constraints is 3 1 2 5 1 4 3 5 4 2. Another scenario is building a conveyer by placing copies of n types of robots in a line and respecting a set of given requirements defining the *specification graph*, where, apart from *initial setup* requiring human input, robot i cannot do its job before robot j completed its job, and vice versa, for some pairs (i, j), $1 \leq i < j \leq n$.

The roots of word-representable graphs are in [87] by Seif and myself, where similar graphs not only allowed to find the *free spectrum* of the celebrated *Perkins semigroup* (which has played a central role in semigroup theory since 1960, particularly as a source of examples and counterexamples), but also to solve the *word problem* for this semigroup. However, a systematic study of word-representable graphs was first done in [74] in collaboration with Pyatkin, where several important properties of these graphs were obtained. For example, it was shown that any word-representable graph can be represented uniformly (different letters occur the same number of times), which gives a basis for more efficient computer experiments on such graphs. Another result is that the neighbourhood of each vertex in a word-representable graph is a comparability graph, which shows that the *Maximal Clique problem* is polynomially solvable on these graphs [58]. It should be noted that many optimization problems are NP-hard on word-representable graphs, e.g. *Independent Set* and *Graph Coloring* [38]. Also note that there are non-word-representable graphs in which the neighbourhood of each vertex is a comparability graph, and the smallest such graph is on 7 vertices [58].

The paper [38] is of special importance as it contains a fundamental result — a characterization of word-representable graphs in terms of certain graph orientations. That is, it was shown that a graph is word-representable if and only if it admits a *semi-transitive orientation*. Semi-transitive orientations generalize *transitive orientations*. Obtaining the characterization result led to showing that the recognition problem of graph word-representability is in NP; recently, it was shown that this problem is actually NP-hard [58]. An application of the characterization result is that answering questions on word-representability can now be done via finding orientations on graphs instead of dealing with words, and the majority of papers in the area use semi-transitive orientations.

A far-reaching generalization of the notion of a word-representable graph [43] is in observing that alternation of letters in a word is defined by the absence of the *consecutive pattern* 11. However, edges/non-edges can be defined by any other binary pattern. In particular, letting the pattern be of length at least 3, we can represent any graph [55], which is potentially useful. Another extension of the notion of a word-representable graph that allows to represent *any* graph is discussed in [18].

4. Planar maps

Tutte [94] founded the enumeration theory of planar maps in a series of papers in the 1960s. Planar maps are a natural model of discrete surfaces. They form the combinatorial (non-geometric) part of meshes (which gives an approximate representation of a surface), and as such, efficient coding of maps is important for mesh compression. Planar maps are also used as a discrete model for *2D quantum gravity*.

Description trees related to planar maps were introduced by Cori, Jacquard and Schaeffer [24] in 1997. Of special interest to us are $\beta(a, b)$*-trees* that are rooted plane trees labeled with positive integers such that (1) leaves have label a; (2) the root has label equal to the sum of children's labels; (3) any other nodes have labels no greater than the sum of their children's labels. It turns out that $\beta(0, 0)$-trees, $\beta(1, 0)$-trees, $\beta(0, 1)$-trees, $\beta(1, 1)$-trees, and $\beta(2, 2)$-trees are in bijection with, respectively, *rooted plane trees*, *rooted non-separable planar maps*,*bicubic maps*, 3-*connected cubic planar maps*, and *cubic non-separable planar maps*.

A particular result in [23] coauthored by me is that the vector of statistics (**sub**, **leaves**, **root**, **lpath**, **rpath**, **lsub**, **beta**) has the same distribution on $\beta(1, 0)$-trees as (**comp**, 1+**asc**, **lmax**, **lmin**, **rmax**, **ldr**, **lir**) on the reverse of non-separable permutations, where, for instance, **sub** = the number of children of the root, **leaves** = the number of leaves, **root** = the root label, **lpath** = the number of edges on the path from the root to the leftmost leaf, **rpath** = the number of edges on the path from the root to the rightmost leaf, etc.

I was involved in finding involutions on $\beta(1, 0)$- and $\beta(0, 1)$-trees [21, 23] that were used to obtain non-trivial equidistribution results on planar maps, description trees and pattern-avoiding permutations. These involutions allow to control 8 statistics on $\beta(1, 0)$-trees, 5 statistics on the reverse of non-separable permutations, and 2 statistics on $\beta(0, 1)$-trees/bicubic maps. The key idea to discover such involutions was to study different ways to generate the trees in question.

A specialization of the involution on $\beta(1, 0)$-trees to trees where the label of each node, possibly except the root, is 1, gave a recipe to find non-trivial equidistribution results on *any* objects counted by the *Catalan numbers* [23]. For example, on rooted plane trees, a classic structure counted by the Catalan numbers, the vector of statistics (**sub**, **rpath**, **lpath**, **mleaf**, **leaves**) is equidistributed with (**rpath**, **sub**, **mleaf**, **lpath**, **non-leaves**), where **non-leaves** = the number of non-leaves (internal vertices) and **mleaf** is related to the rightmost leaf either with no siblings or having only leaves as siblings to the right of it.

The fixed points of the involution on $\beta(1,0)$-trees were enumerated in [69] by de Mier and me (there are $\frac{1}{n}\binom{3n-2}{n-1}$ fixed points with $2n$ nodes) and they turned out to be equinumerous with *self-dual rooted maps* studied by me in collaboration with de Mier and Noy [70]. On the other hand, Claesson et al. [21] conjectured that the fixed points of the involution on $\beta(0,1)$-trees are equinumerous with a number of combinatorial objects including certain *weighted Motzkin lattice paths.*

Finally, [84], coauthored by me, studied, with the aid of $\beta(1,0)$-trees, rooted non-separable planar maps with restricted faces, as well as those with no multiple edges. This is also interesting in connection with *plane bipolar orientations* and *Baxter permutations*

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