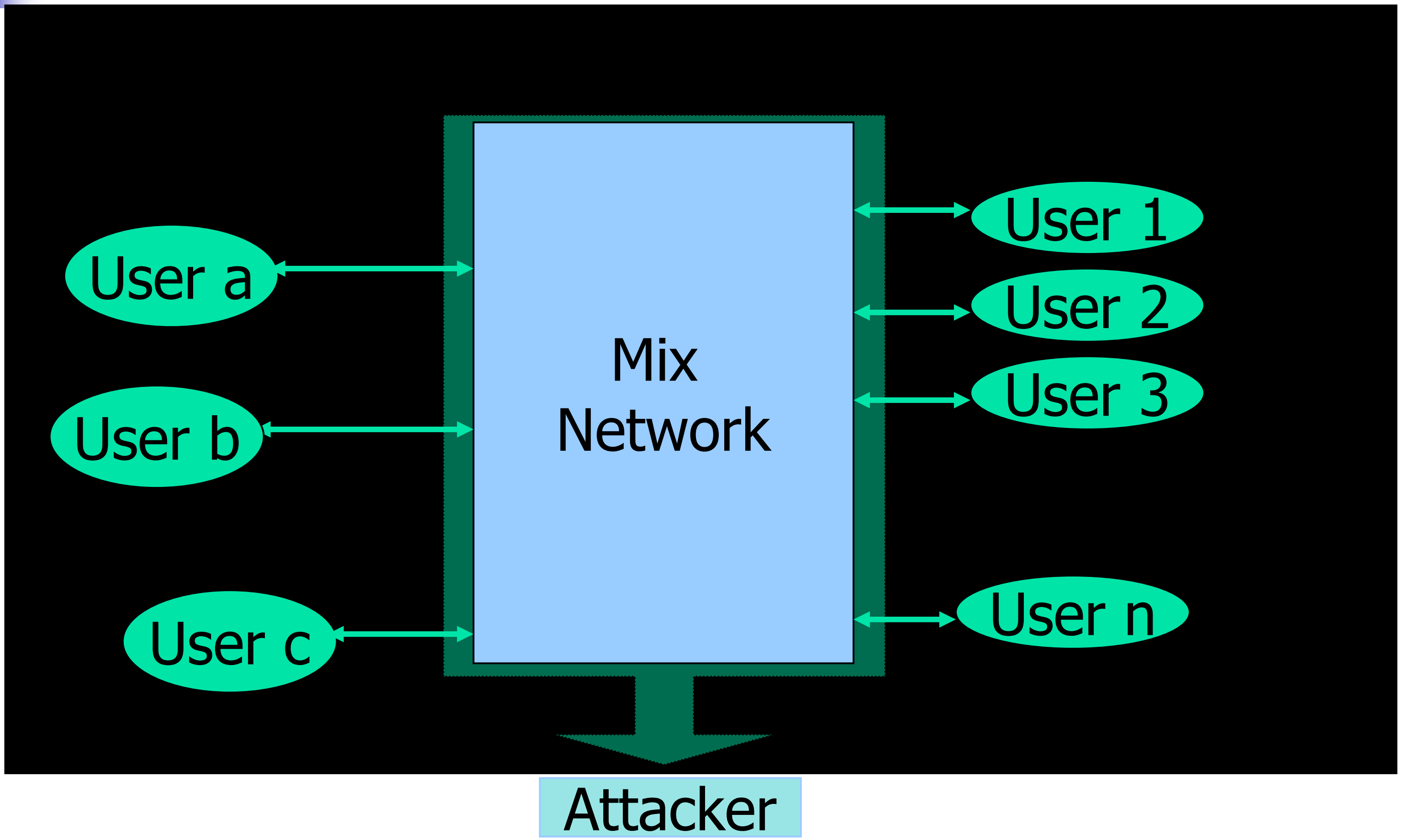
# Mixes on unstructured topologies

Shishir Nagaraja
Computer Lab
University of Cambridge

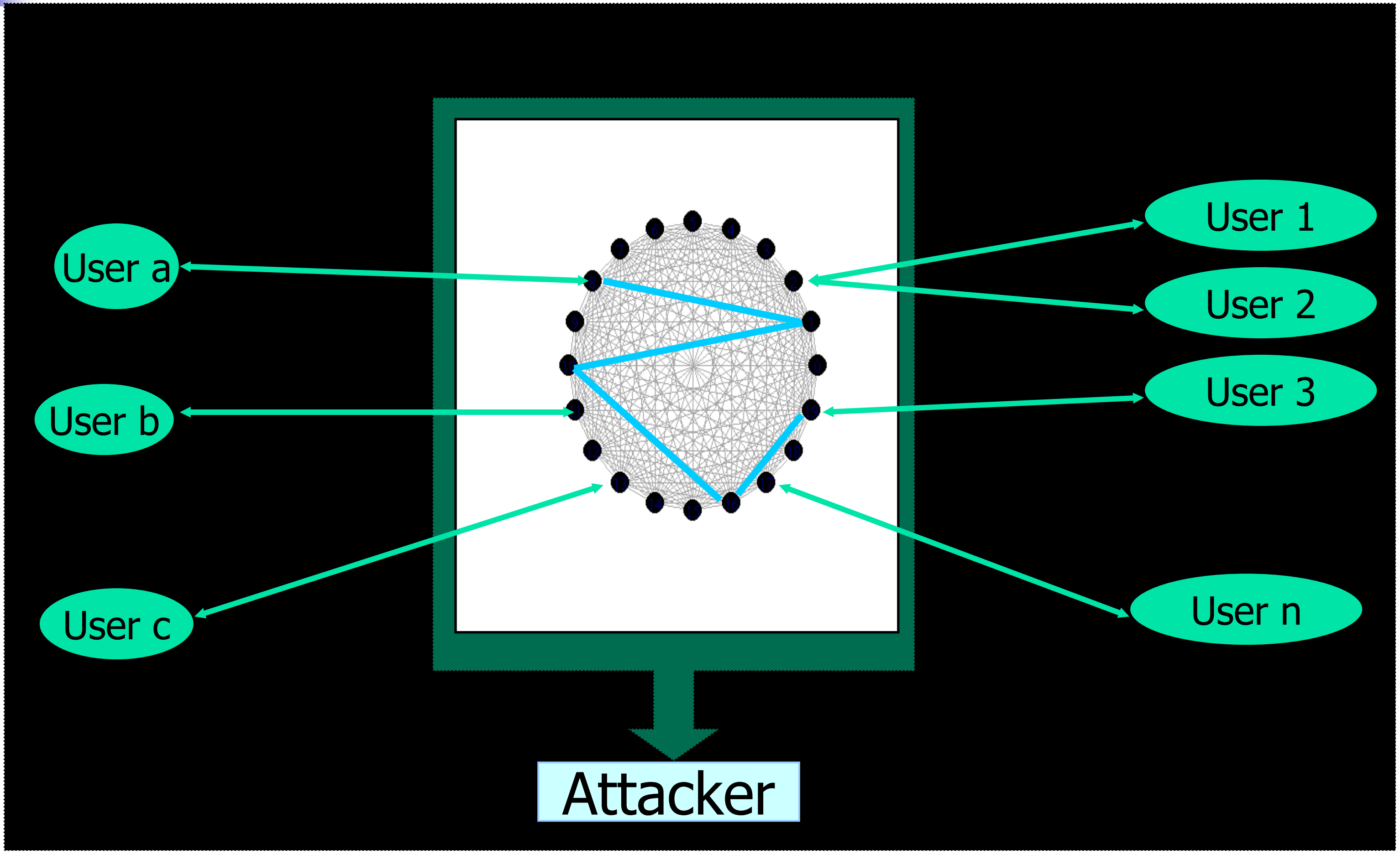
# what does anonymity mean?

- Unlinkability: Hide the connection between the senders and the recipients.
- Untraceability: Hide the connection between actions of the same sender.
- Unobservability: Hide the fact that the user is talking.

# Mix-network


User a
User b
User c
Mix
Network
User 1
User 2
User 3
User n
Attacker

# Mix-network topology and mix-route



# Desirable properties of the mix-network

- 'High' traffic analysis resistance.
- Size: The larger the network, greater the anonymity set or maximal anonymity.
- Traffic: More traffic means better anonymity
- Robustness:
  - Liability management in anonymous communication
  - Clear incentives for carrying traffic under legal pressure

# Evaluation framework

- Is the given topology any good?
  - Figure out the efficiency of the mixing process.
  - Analyze the traffic-analysis resistance of the mix-network.
- Modeling mix network operation
  - Markovian random walks
- What we are not interested in:
  - Side channel analysis
  - Variation in protocol behaviour across topologies

# Measuring anonymity

- Number of bits the attacker is missing to uniquely link an actor to an action – (Serjantov and Danezis, PET 2002).

$$\mathcal{A} = \mathcal{E}[\alpha_i] = -\sum_i Pr[\alpha_i] log_2 Pr[\alpha_i]$$

# Evaluation recap

Under conditions of maximal anonymity:

- Minimum mix-route length required.
- Amount of traffic needed to prevent intersection attacks – traffic load patterns.
- Resistance to corrupt nodes.

# Theory

1.

Walk of length $t$

Infinite length Walk

$$\Delta(t) = max_i \frac{|q_i^t - \pi_i|}{\pi_i}$$

Convergence

$$\Delta(t) \leq n^{1.5}(\lambda_2)^t$$

Second eigenvalue of the transition matrix $T_{ij}=(1/k_i)\, A_{ij}$

**A** Adjacency matrix
**$k_i$** Degree of node i

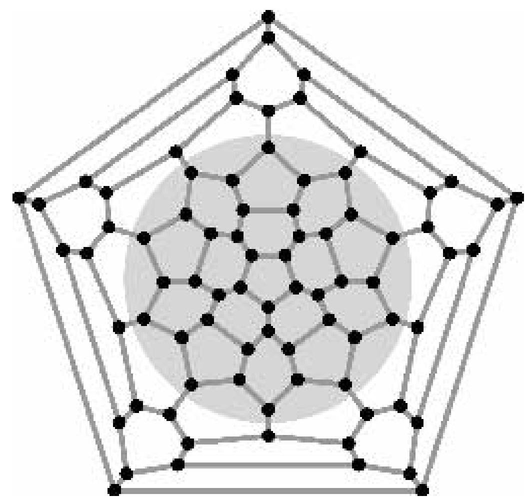
2. $q^t = q^{t-1}T$

See the paper for a proof of why the second eigenvalue is a constant for varying network size (n).

# Structured graph topologies

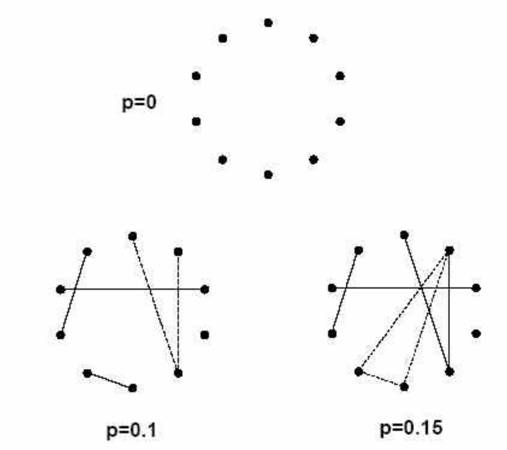
- Optimal mixing properties are obtained in expander graphs such as Ramanujan graphs.
- For N=5000 nodes, we have, $\lambda_2 \geq 0.5527$
- Hence, we can calculate mix-route length as approximately 4 hops.

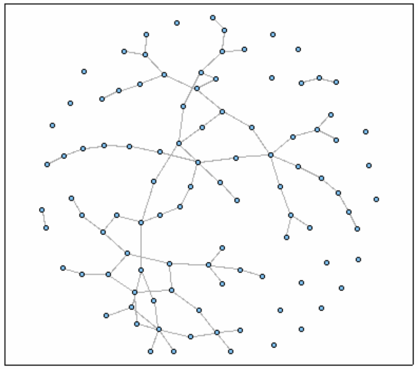
(source:www.ams.org)

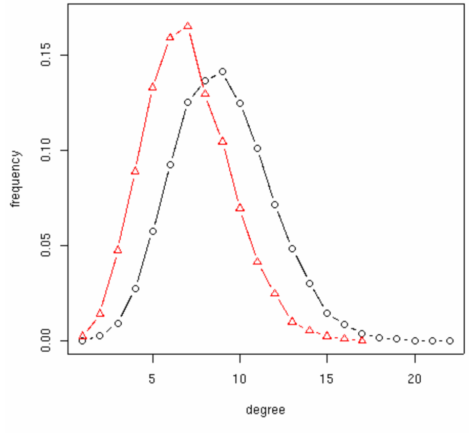


# Unstructured network topologies

- Erdős-Rényi random graph topology



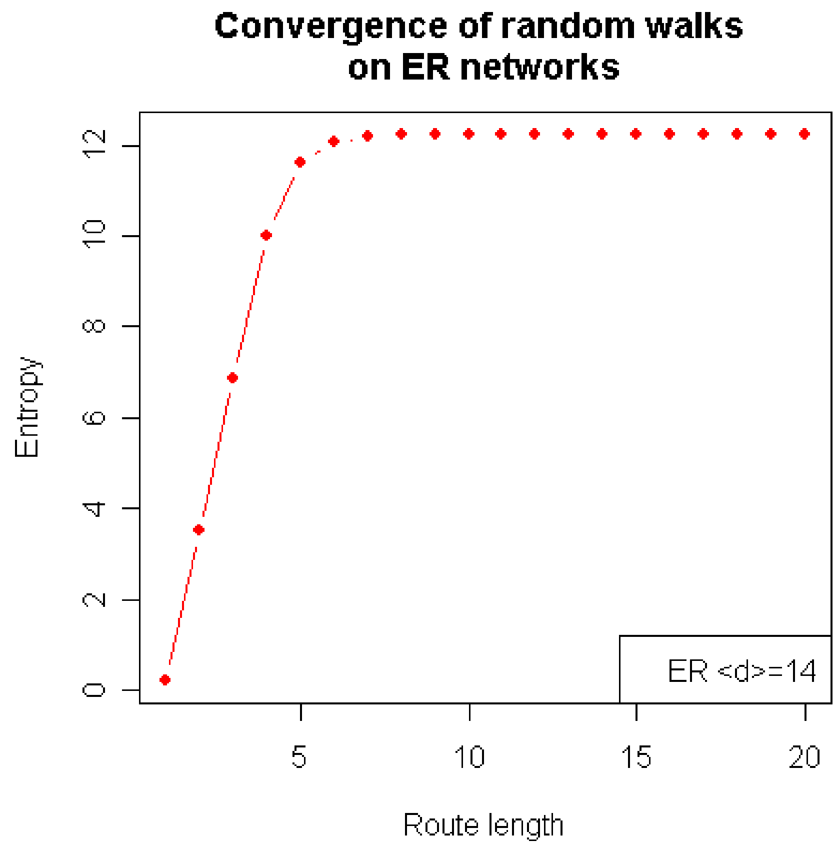



$$P(k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$\lambda = \binom{N-1}{k} p_{ER}{}^k (1 - p_{ER})^{N-1-k}$$

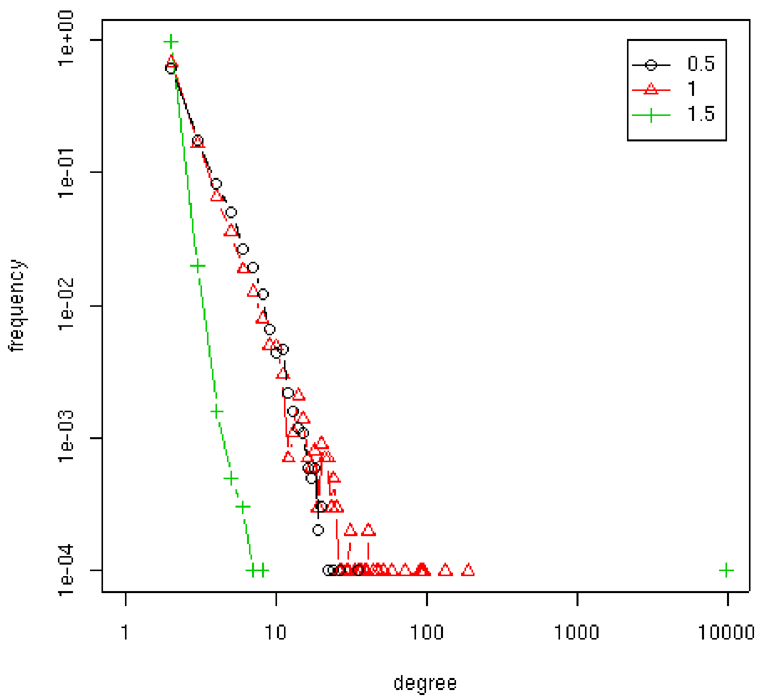
We chose $p$ such that the |biggest component| ~ |V|=5000 nodes

# Mixing efficiency of ER graph topology

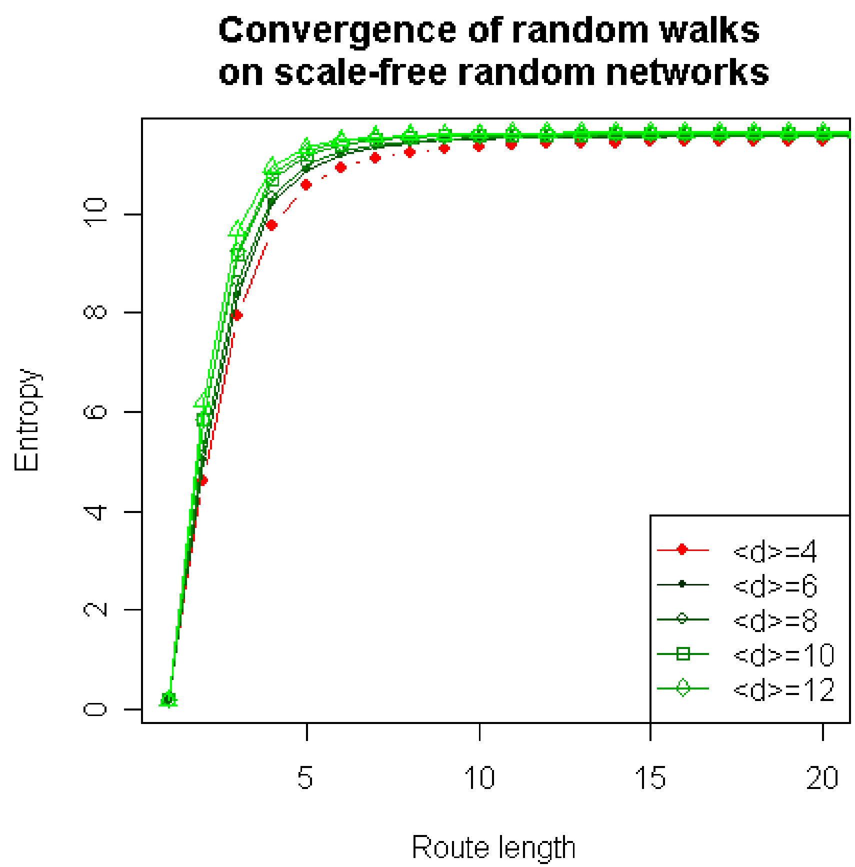


# Graph topologies continued…

- Scale-free topologies
- Power law
  - Heavy-tailed distribution
    - $P(X>x) \sim x^{-a}$, $0<a<2$
  - Zipf distribution / Zeta distribution
    - $P(k) = Ck^{-(a+1)}$
  - Pareto distribution
    - $f(x) = ab^a x^{-(a+1)}$



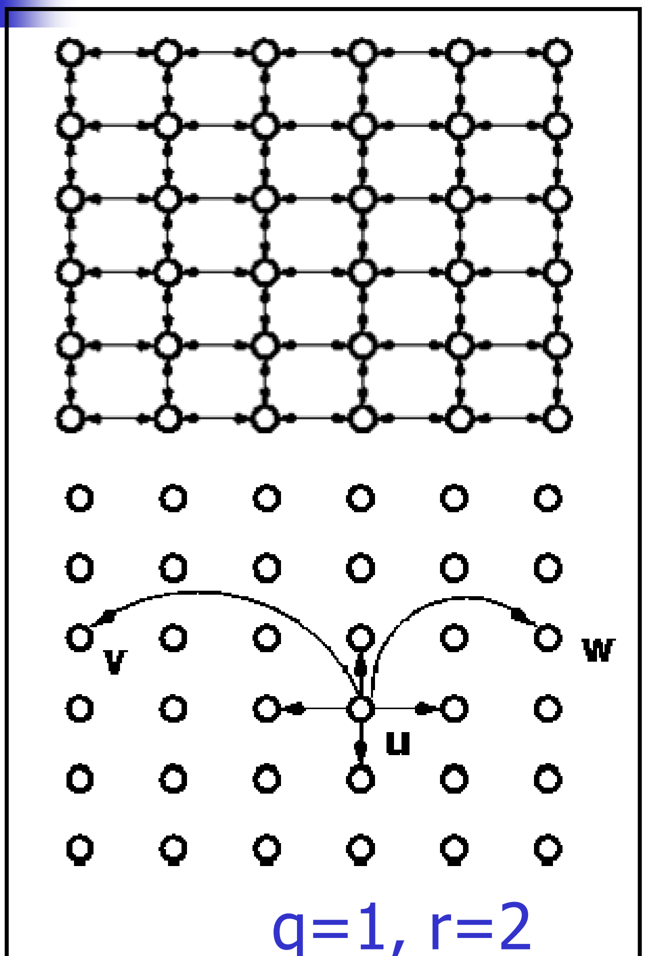
# Mixing efficiency of SFR graph topology

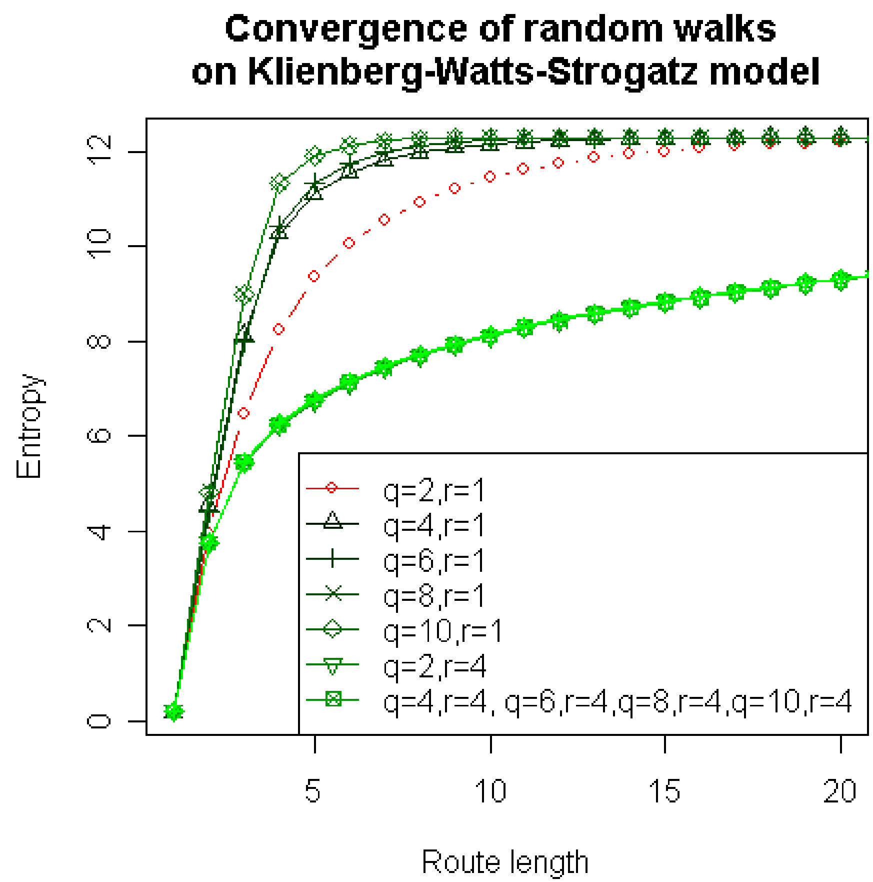



SFR models the massive AT&T call graph

| Network ($N = 5000$) | $\langle d \rangle or D$ | $t$ | $A_{network}$ |
|---|---|---|---|
| SFR | 2 | 8 | 11.4383 |
| | 3 | 7 | 11.5626 |
| | 4 | 6 | 11.5958 |
| | 5 | 6 | 11.6135 |
| | 6 | 5 | 11.6351 |
| ER | 14 | 7 | 12.2339 |
| Expander | 14 | 4 | 12.2877 |

# Mixing efficiency of KWS topology (weak and strong ties)



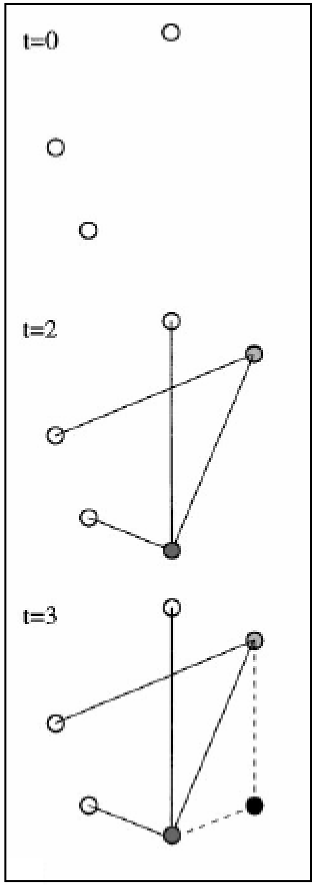



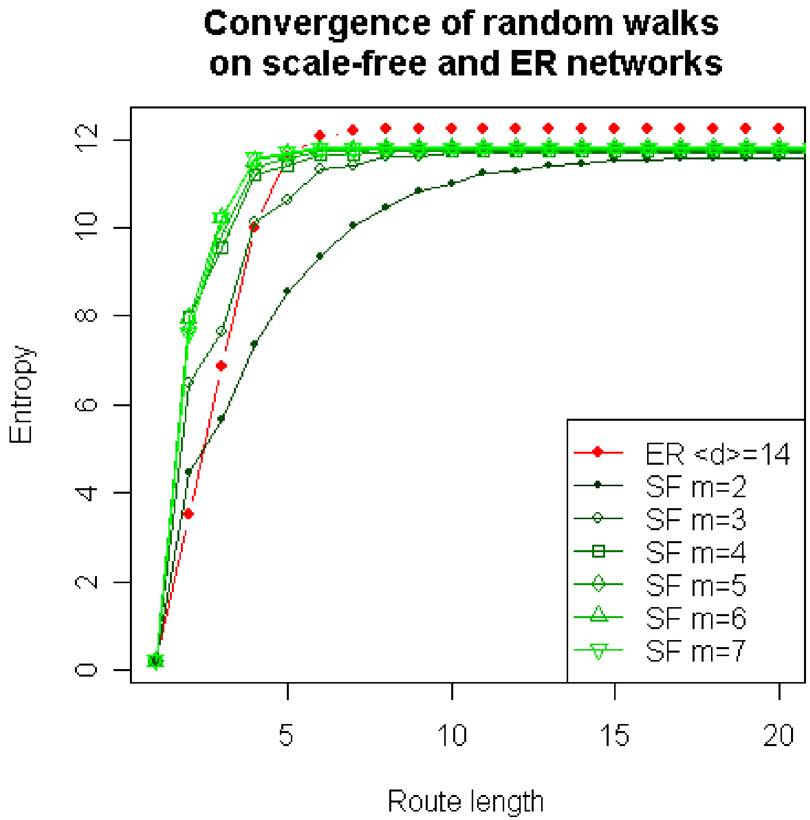
# Mixing efficiency of SF-BA graph topology

- Growth
  - Start with $m_0$ nodes, and then add a node with m edges at every time step.
  - $m=m_0$
- Preferential attachment

$$\prod(k_i) = \frac{k_i}{\sum_j k_j}$$

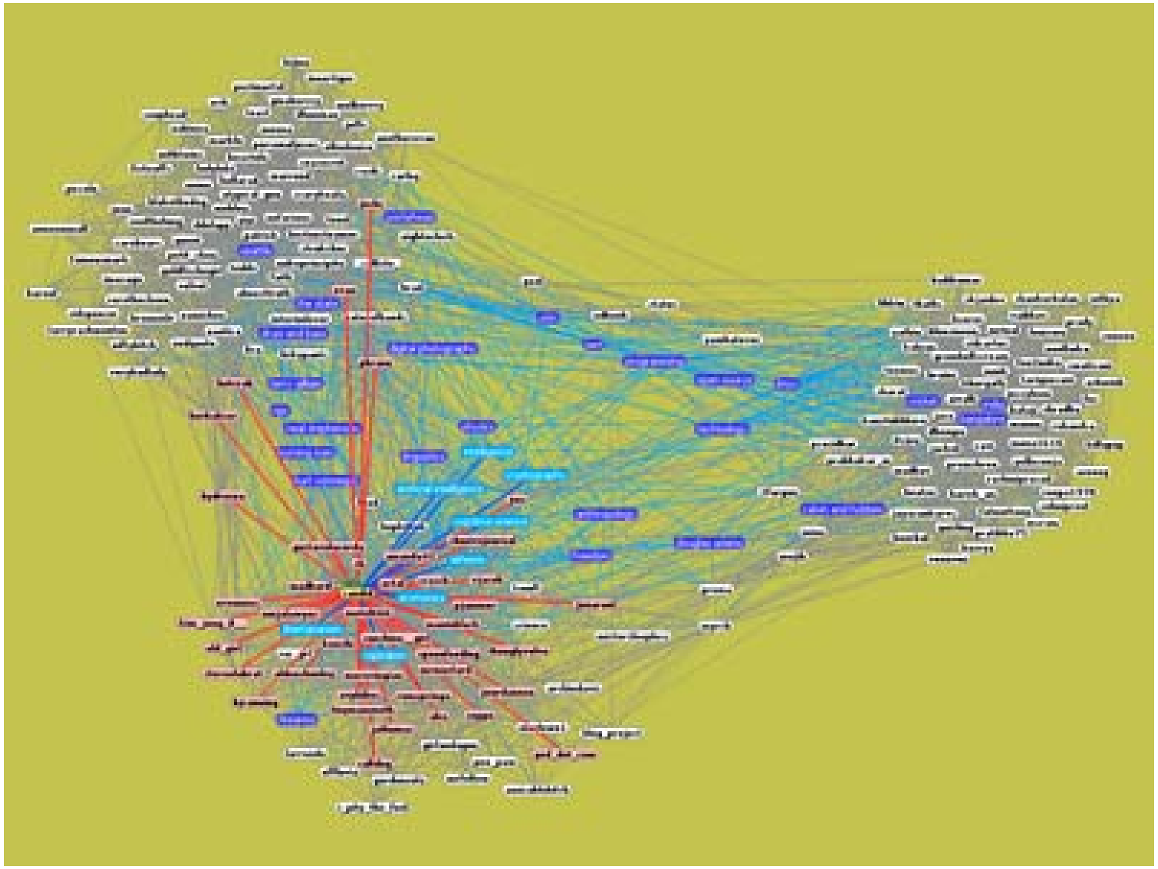
- It is a simple model but…
  - Fixed exponent = 3

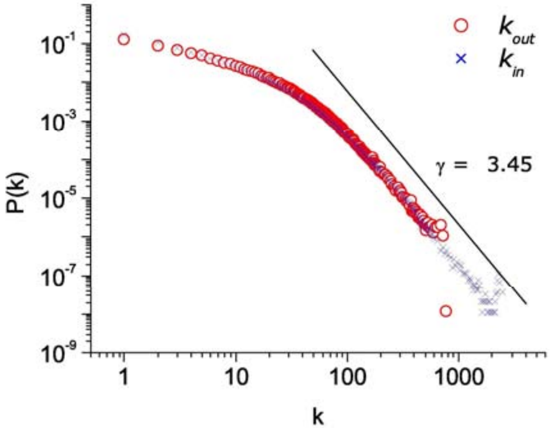


# LiveJournal

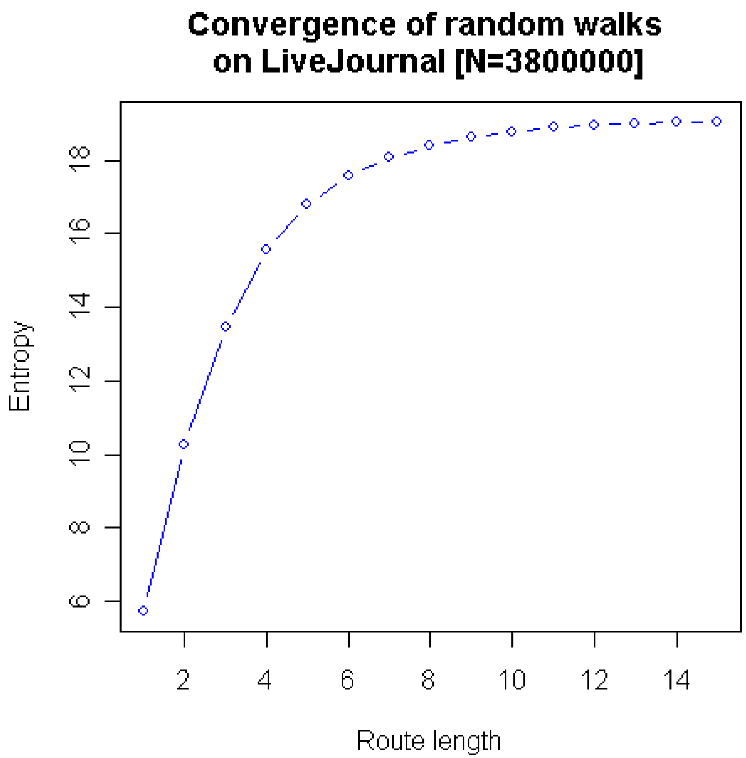
Source: Trejkaz Xaoza, Touchgraph





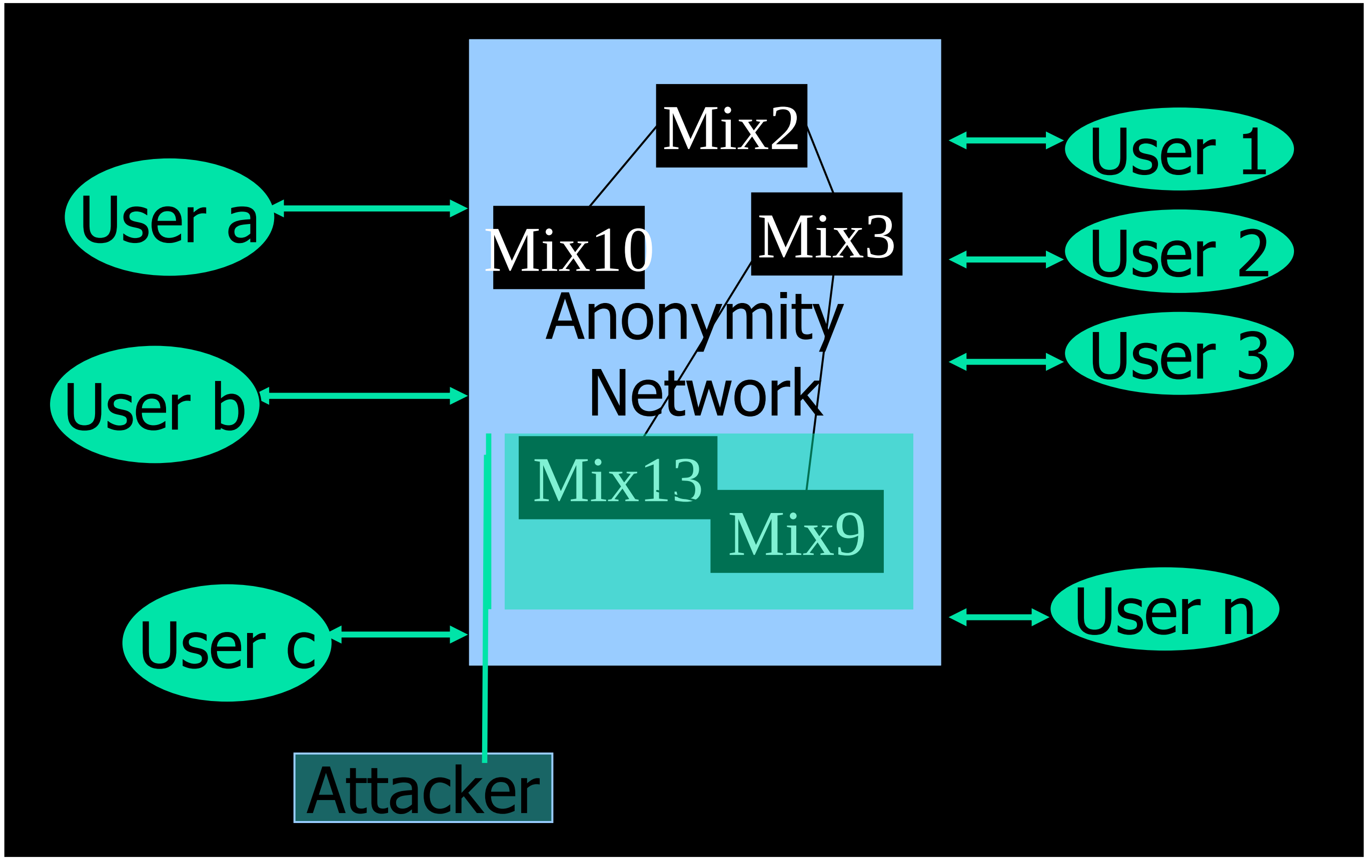

Pavel Zakharov, Thermodynamic approach for community discovering within the complex networks: LiveJournal study. e-print on arxiv.org: physics/0602063.

# Mixing efficiency of LiveJournal topology




| Network | $m$ | $t$ | $A_{network}$ |
|---|---|---|---|
| SF | 2 | 15 | 11.5852 |
| | 3 | 10 | 11.6961 |
| | 4 | 6 | 11.7293 |
| | 5 | 6 | 11.7687 |
| | 6 | 6 | 11.7953 |
| | 7 | 6 | 11.8090 |
| KWS | $q = 2, p = 1$ | 11 | 12.2945 |
| | $q = 10, p = 1$ | 5 | 12.2939 |
| | $q = 2, p = 4$ | 63 | 11.6440 |
| | $q = 10, p = 4$ | 63 | 11.6380 |
| ER | $\langle d \rangle = 14$ | 7 | 12.2339 |
| Expander | $D = 14$ | 4 | 12.2877 |

# Corrupt nodes



# Intersection attacks

- Pr[any unused link] ~ 0
- Attacker: is the traffic from Alice proceeding along (i,j)?
  - Mean volume vs observed volume of traffic
  - L (confidence parameter) – number of standard deviations from the mean.
  - b – batch size, $p_i = 1/d_i$, k = 2 mixing rounds.

$$:k > 4l^2 \frac{p_i}{(1-p_i)}(b-1)$$

| Network | $\langle d \rangle$ | $p_{min}$ | Batch size |
|---|---|---|---|
| SFR | 2 | 0.0344 | 10.08 |
| | 3 | 0.0222 | 15.84 |
| | 4 | 0.0243 | 14.4 |
| | 5 | 0.0192 | 18.36 |
| | 6 | 0.0135 | 26.28 |
| | 7 | 0.0125 | 28.44 |
| KWS | 27 $(q = 1, r = 1)$ | 0.0294 | 11.88 |
| | 43 $(q = 10, r = 1)$ | 0.0169 | 20.88 |
| | 26 $(q = 1, r = 4)$ | 0.0333 | 10.44 |
| | 28 $(q = 10, r = 4)$ | 0.0294 | 11.88 |
| SF-linear | 4 | 0.0048 | 74.16 |
| | 6 | 0.0048 | 74.16 |
| | 8 | 0.0041 | 86.04 |
| | 10 | 0.0038 | 93.6 |
| | 12 | 0.0037 | 96.12 |
| | 14 | 0.0031 | 112.32 |
| LJ | 7.3221 | 0.00857 | 41.64 |
| ER | 14 | 0.0333 | 10.44 |
| Expander | 14 | 0.0714 | 4.68 |

Table 1: Batch sizes required to prevent intersection attacks

# structured vs unstructured…

- Traffic analysis resistance – Comparable
- Maximal anonymity – Comparable
- Topological robustness in the face of litigation pressure – this depends on the social capital in the network.
  - Friends process each other's traffic.
  - Processing "3rd party" traffic - indirect reciprocity - encourages a diverse user base which brings its benefits (Anonymity loves company, Dingledine and Mathewson 2006).

# Conclusions

- A successful mix-network design needs to consider the issues of liability management.
- Tapping social capital in a network to enhance topological robustness is an attractive proposal,
- And we have established the essential technical feasibility of this if this means using an unstructured mixnet topology
- Specifically:
  - Mix-route length is not a problem
  - Corrupt hubs are not a problem either
  - Batch sizes are however a challenge. Where do we bring 8 times the amount of dummy traffic from when compared to expanders? – will social chatter suffice?