Local solutions of Optimal Power Flow Problem

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Abstract

The existence of locally optimal solutions to the AC optimal power flow problem (OPF) has been a question of interest for decades. We have shown the existence of local solutions on a variety of test networks including slightly modified versions of common networks. Standard local optimization techniques are shown to converge to these local optima if started close enough to them. These test cases are available in an online archive [1] and can be used to test local or global optimization techniques for OPF.

Introduction

Optimal power flow (OPF) is a well studied optimization problem in power systems. The objective of OPF is to find a steady state operating point that minimizes the cost of electric power generation while satisfying operating constraints and meeting demand.

Mathematically

The OPF can be formulated as nonlinear programming problem, in which some constraints and possibly the objective function are nonlinear.

$$\min \sum_{g\in\mathcal{G}} f(p_g^{\mathsf{G}}) \qquad (1)$$

subject to

$$\sum_{g\in\mathcal{G}_b} p_g^{\mathsf{G}} = \sum_{d\in\mathcal{D}_b} P_d^{\mathsf{D}} + \sum_{b'\in\mathcal{B}_b} p_{bb'}^{\mathsf{L}} + G_b^B v_b^2 \qquad (2)$$

$$\sum_{g\in\mathcal{G}_b} q_g^{\mathsf{G}} = \sum_{d\in\mathcal{D}_b} Q_d^{\mathsf{D}} + \sum_{b'\in\mathcal{B}_b} q_{bb'}^{\mathsf{L}} - B_b^B v_b^2 \qquad (3)$$

$$p_{bb'}^{\mathsf{L}} = \quad v_b^2 G_{bb} + v_b v_{b'}(G_{bb'}\cos\delta_{bb'} + B_{bb'}\sin\delta_{bb'}) \qquad (4)$$

$$q_{bb'}^{\mathsf{L}} = -v_b^2 B_{bb} + v_b v_{b'}(G_{bb'}\sin\delta_{bb'} - B_{bb'}\cos\delta_{bb'}) \qquad (5)$$

$$\theta_{b_0} = 0, \quad \theta_b - \theta_{b'} = \delta_{bb'} \qquad (6)$$

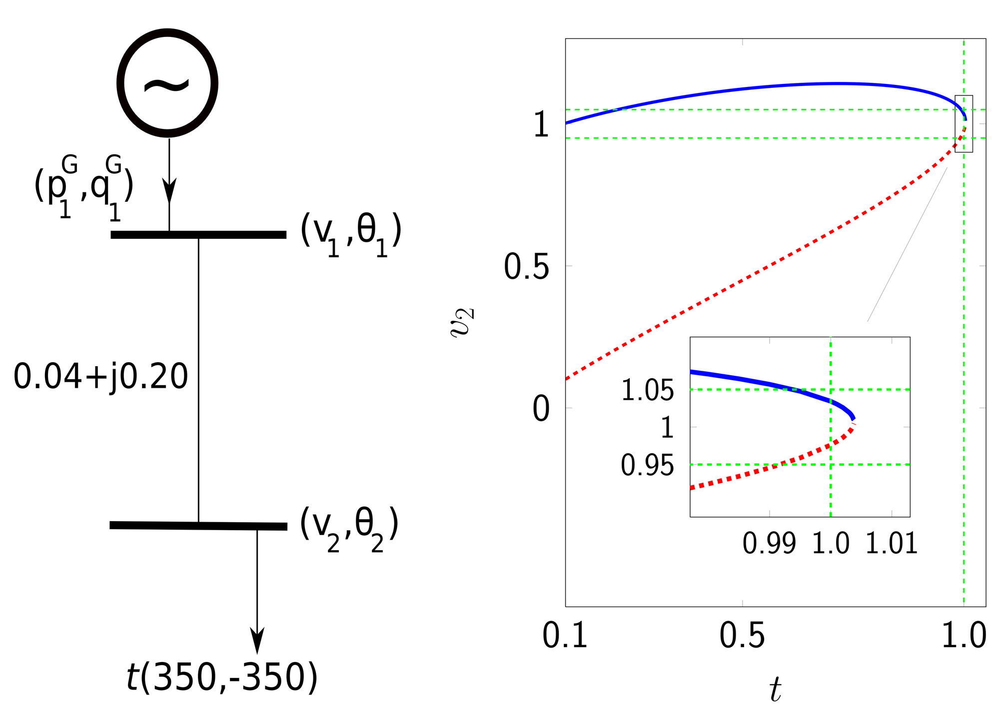
$$v_b^{\mathsf{LB}} \le v_b \le v_b^{\mathsf{UB}}, \quad {p_{bb'}^{\mathsf{L}}}^2 + {q_{bb'}^{\mathsf{L}}}^2 \le (S_{bb'}^{\mathsf{max}})^2 \qquad (7)$$

$$P_g^{\mathsf{LB}} \le p_g \le P_g^{\mathsf{UB}}, \quad Q_g^{\mathsf{LB}} \le q_g \le Q_g^{\mathsf{UB}} \qquad (8)$$

Equation (1) is the objective function and usually it is convex. Equations (2-5) are Kirchhoff's laws and they are main source of nonconvexity in the OPF problem. Equations (6-8) are convex constraints.

Relation of OPF to Load Flow Problem

If we fix all demands, and the voltages at all generator buses, and the generator outputs at all generator buses except one (referred to as slack bus), and also fix the phase angle at the slack bus, then we can use equations (2-5) to solve for the remaining variables. This problem is known as load flow problem and it is well known that it can have 0, 1 or multiple solutions. The solutions of load flow problem are feasible solutions of OPF problem provided they satisfy all the constraints.



(a) 2 Bus Case (b) Voltage

Figure 1: Alternate load flow solutions as the demand parameter, t, varies.

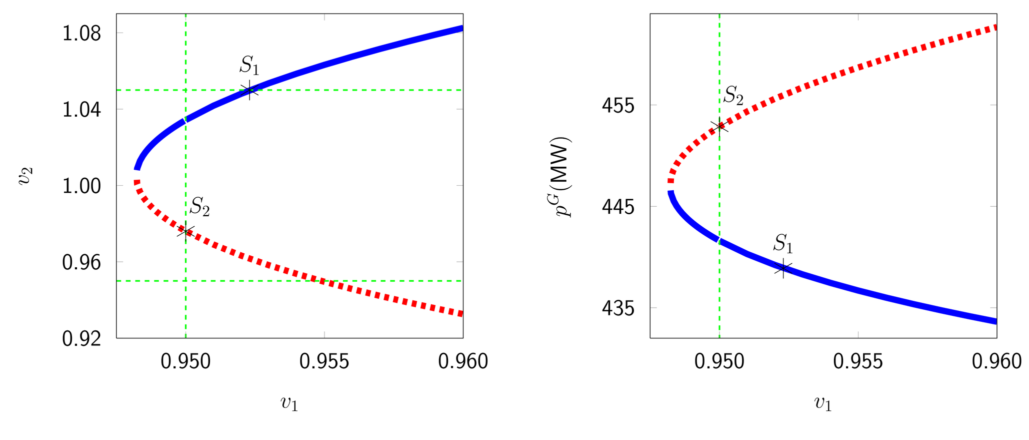
Two bus network is shown in Fig. 1(a). Bus 1 is the generator bus and slack bus, and for it we set $v_1 = 0.95$ and $\theta_1 = 0$. If we know the load at Bus 2 then it is possible to find the remaining variables $(p_g, q_g, v_2, \theta_2)$. There are at most two possible load flow solutions. Fig. 1(b) shows these solutions when the load at Bus 2 is $(P_2^{\mathsf{D}}, Q_2^{\mathsf{D}}) = t(350, -350)$ for $0.1 \le t$. As the load increases the two solutions get closer and eventually coalesce at a point.

Examples of Local Solutions of OPF

Here we present examples of power systems networks with local optima. All these examples have voltage limits within $\pm 5\%$ off-nominal or the default voltage bounds of networks from which they are derived.

2 Bus Example

Consider the two bus example with $t = 1$, and $\pm 5\%$ voltage bounds. The full feasible region is shown in Fig 2. S_1 is the global solution and S_2 is the local solution.



(a) Voltage (b) Real Power

Figure 2: Effect of Voltage

	Bus	v (p.u.)	θ (deg)	p^{G} (MW)	q^{G} (MVAr)
S_1	1	0.952	0.00	438.89	94.44
	2	1.050	−57.14		
S_2	1	0.950	0.00	452.86	164.32
	2	0.976	−64.94		

Table 1: Two OPF solutions for the 2 bus problem

9 Bus Example

In the standard 9 bus case, when the reactive power generation lower bounds on all 3 generators were raised from -300 MVAr to -5 MVAr and all loads scaled to 60%, then the 4 optimal solutions given in Table 2 were found.

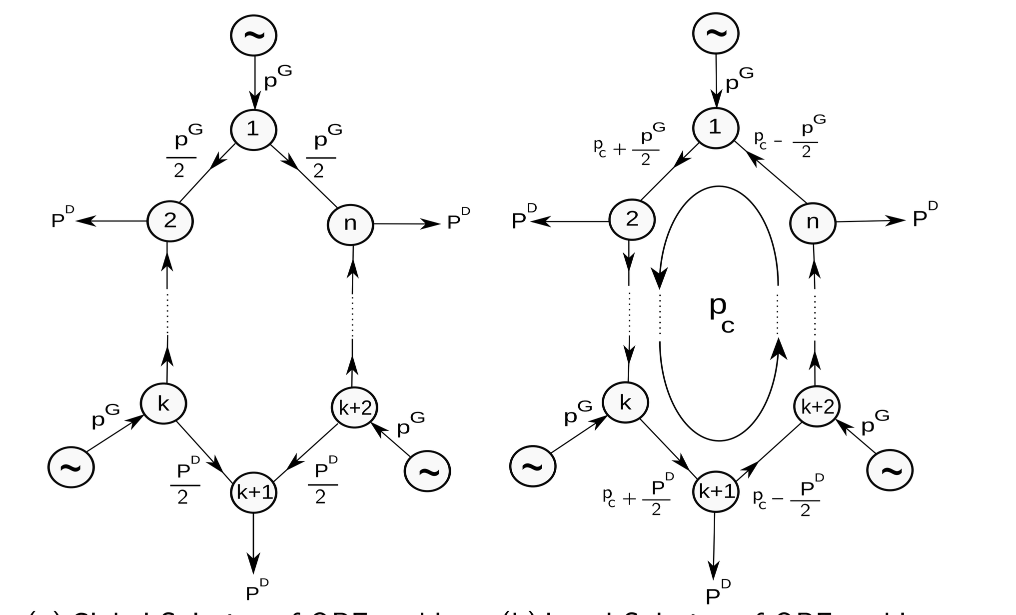
	1	2	3	4
Objective Value	3087.84	3398.03	4265.15	4246.48
Loss (MW)	3.40	3.58	2.01	2.03

Table 2: Four OPF Solutions for the 9 bus network

Note that in this case the line losses in global solution is higher then two of the local solution. This is because more of the power comes from cheaper generators.

Loop Networks

In power transmission networks there is usually more than one path between pairs of buses, and such networks contain loops.



(a) Global Solution of OPF problem (b) Local Solution of OPF problem

Figure 3: Solutions in n Bus Loop Network (showing only real flows)

The network shown Fig. 3 consists of single loop of n buses. p_c represents the circulating power. Fig. 3(a) shows the global solution and Fig. 3(b) shows the local solution. In the local solution $p_c \neq 0$ and there is a circulation power.

Summary

Table 3 gives a summary of the test cases we found with local optima.

Case	n^B	# of Solutions	% Difference
case2	2	2	3.18
case3	3	2	0.21
case5	5	2	14.34
case9mod	9	4	38.12
case30loop	30	2	33.69
case39mod1	39	2	115.48
case39mod2	39	16	0.53
case118mod	118	3	50.97
case300mod	300	7	2.52

Table 3: Summary of Results

Why Local solutions matter?

Following are some of the points, which highlights the importance of this issue:

- Local solutions are expensive
- Unfair electricity market awards
- Performance of local optimization techniques

Reasons for local optima in OPF

Cases of local optima seems to be uncommon: indeed none was found in any of the standard test cases. However after modifying load or generator bounds local optima were found in all of the test cases. The examples of local optima presented in the paper [2] are either due to one of the following reasons:

- Disconnected feasible space
- Loop flows
- Excess of reactive/real power in the network
- Negative locational marginal prices (LMPs)

Test Case Archive

In order to support current research interest in optimization techniques for OPF problems, it is important to have test cases with known local optima. The data for the examples and the local solutions are publicly available at [1] and can be used in testing local and global optimization techniques.

Intelligent initial guess

We obtained the local solutions using many random starting points. It is more natural to start the local optimization technique from the midpoint of the bounds. We generated a large number of problems by taking the above examples with local optima and randomly perturbing their costs and tested how often the global minimum was found starting from the central point and from random points. We found that the central starting point was significantly better than random points, however there were some cases where the central point produced a local optima.

Conclusions

We have shown the existence of local optima of OPF problems. All our examples have either $\pm 5\%$ voltage bounds or the same bounds as standard cases from which they are derived. We have shown that the local optima can occur because of the disconnection in the feasible region and/or because of the nonlinearities in the constraints.

References

[1] W. A. Bukhsh, A. Grothey, K. M. McKinnon, P. A. Trodden, "Test Cases for Optimal Power Flow Problems", [Online] Available: `http://www.maths.ed.ac.uk/optenergy/LocalOpt`

[2] W. A. Bukhsh, A. Grothey, K. M. McKinnon, P. A. Trodden, "Local Solutions of Optimal Power Flow Problem," Submitted in *IEEE Transactions on Power Systems*, Feb. 2013.