RSE Saltire Facilitation Network
on Stochastic Differential Equations: Theory, Numerics and Applications

Workshop
Monday 29 – Wednesday 31 August 2022

http://personal.strath.ac.uk/x.mao/RSE_Network/Workshops/

Programme

Venue: LT908, Livingstone Tower, 26 Richmond Street, Glasgow G1 1XH.
The University map can be found at
https://www.strath.ac.uk/maps/

Date: Monday 29 August 2022

Chair	Professor Xuerong Mao, Strathclyde University
09:28-09:30	Welcome address
90:30-10:20	Prof John Appleby, City University of Dublin, Ireland Oscillatory dynamics are almost impossible for the mean square of two dimensional linear SDEs
10:20-10:45	QA and Discussions
10:45-11:15	Discussions over tea/coffee
11:15-12:05	Prof Mireille Bossy, Universite Cote d'Azur, France Strong rate of convergence of approximation scheme for SDEs with superlinear coefficients.
12:05-12:30	QA and Discussions
12:30-14:00	Discussions over lunch
Chair	Prof Gabriel J. Lord, Radboud University, Netherlands
14:00-14:50	Prof Evelyn Buckwar, Johannes Kepler University, Austria A couple of ideas on splitting methods for SDEs
14:50-15:15	QA and Discussions
15:15-15:45	Discussions over tea/coffee
15:45-16:35	Prof Tomas Caraballo, Universidad de Sevilla, Spain Approximations to stochastic time fractional Navier-Stokes equations with delays
16:35-17:00	QA and Discussions
17:30-19:00	Discussions over dinner

Date: Tuesday 30 August 2022

Chair	Prof Evelyn Buckwar, Johannes Kepler University, Austria
90:30-10:20	Prof Michael Tretyakov, Nottingham University, England Weak approximation of stochastic differential equations driven by Levy processes
10:20-10:45	QA and Discussions
10:45-11:15	Discussions over tea/coffee
11:15-12:05	Dr Conall Kelly, University College Cork, Ireland Numerical methods applied to stochastic differential equations with finite-time explosions
12:05-12:30	QA and Discussions
12:30-14:00	Discussions over lunch
Chair	Prof John Appleby, City University of Dublin, Ireland
14:00-14:50	Prof Raphael Kruse, Martin Luther University Halle-Wittenberg, Germany Finite p-variation of solutions to (stochastic) evolution equations and applications in numerical analysis
14:50-15:15	QA and Discussions
15:15-15:45	Discussions over tea/coffee
15:45-16:35	Prof Anne Kvarno, Norwegian University of Science and Tech., Norway An SDE wind model applied to a hydrodynamic lake model
16:35-17:00	QA and Discussions
18:00-20:00	Conference dinner for speakers

Date: Wednesday 31 August 2022

Chair	Prof Tomas Caraballo, Universidad de Sevilla, Spain
90:30-10:20	Prof Gabriel J. Lord, Radboud University, Netherlands GBM based integrators for S(P)DEs
10:20-10:45	QA and Discussions
10:45-11:15	Discussions over tea/coffee
Chair	Prof Raphael Kruse, Martin Luther University Halle-Wittenberg
11:15-11:40	Discussions on the DIEHARD database
Chair	Prof Xuerong Mao, Strathclyde University
11:45-12:15	Review of the themes of the Network
12:15-13:30	Discussions over lunch

Titles and Abstracts

John Appleby, School of Mathematical Sciences, DCU

Oscillatory dynamics are almost impossible for the mean square of two dimensional linear SDEs

This talk reports some preliminary findings for a very simple class of SDEs, namely two dimensional linear autonomous stochastic differential equations. The question addressed in the talk is classical: what is the asymptotic behaviour of the mean square of (the components of) the solution?

In a sense, the question has long been settled. It is well-established that the mean square of each component, as well as the mean of the products of the components, can be written as the solution of a four-dimensional linear ODE with constant coefficients. Thus questions of asymptotic stability, or rates of convergence, in the mean square, essentially reduce to finding the eigenvalues of a 4 by 4 matrix. However, since that matrix could seemingly have almost any structure, it would appear, without further investigation, that the mean square dynamics could be as general as those exhibited by four dimensional linear ODEs. Hence, we might expect sinusoidal dynamics, resulting from dominant complex eigenvalues, to be very typical.

As best we can judge, an exhaustive and general description of the eigenstructure in the two dimensional case has not been carried out, at least as far as determining the asymptotically dominant dynamics is concerned. Thus, the seemingly sensible conjecture that the dynamics are similar to the deterministic case, in which both oscillatory or real exponential behaviour are typical, does not appear to have been systematically tested.

However, our investigations, which are presently confined to the cases where there is small or large noise, suggest a completely different situation for the mean square, compared with deterministic ODEs. It transpires, in almost all circumstances, that the dominant dynamics are real exponential, and the rare cases when this does not happen can be completed described. In particular, oscillation in the mean square of either component of the two dimensional system is, to all intents and purposes, impossible.

The degree to which these results can be generalised to higher dimensions, general noise intensity, difference equations (or simulations), and equations with memory, is open. It thus forms an potentially interesting topic for discussion.

The investigation is part of a joint project involving Emmet Lawless (DCU), as well as Conall Kelly and Evelyn Buckwar.

Prof Mireille Bossy, Universite Cote d'Azur, France

Strong rate of convergence of approximation scheme for SDEs with superlinear coefficients.

In this talk I would like to give some ideas and proof arguments that are rather generic, and particularly useful for the strong error control of a scheme when the polynomial behaviour of the coefficients only increases the number of moments to be bound in the dynamics of the error itself. I will apply these arguments to the case of the exponential scheme, underlying the required properties, which may be applicable to other cases. This talk is based on a joint work with Kerlyns Martinez (University of Valparaiso).

Prof Evelyn Buckwar, Johannes Kepler University, Austria

A couple of ideas on splitting methods for SDEs

Abstract: We discuss developing splitting methods for stochastic differential equations. Splitting methods are a well-known type of numerical methods in the context of Geometric Numerical Integration of ordinary differential equations, in particular they are known to be structure preserving schemes in various situations. Extensions of these methods to the case of stochastic differential equations exist for considerable time already and they currently appear to become quite popular. In this talk I will present examples illustrating some benefits of splitting methods for SDEs. Illustrative examples include SDEs employed in neuroscience and chemical kinetics.

Prof Tomas Caraballo, Universidad de Sevilla, Spain

Approximations to stochastic time fractional Navier-Stokes equations with delays

In this talk we analyze the well-posedness of a stochastic Navier-Stokes model with delay and time fractional derivative. The deterministic version without delay was successfully studied several years ago, and it was proved the existence of global solution for small initial data as well as the local existence of solutions for arbitrary initial value. However the stochastic version with delay exhibit challenging difficulties which so far have not been solved. In our talk, we describe two approximations to the problem by considering first the Stokes model, and later the case in which the noise is given by the so called colored noise which is a particular case of Wong-Zakai approximation of our problem.

Dr Conall Kelly, University College Cork, Ireland

Numerical methods applied to stochastic differential equations with finite-time explosions

Consider an It\^o-type stochastic differential equation (SDE) with locally Lipschitz continuous drift and diffusion coefficients. Protter (1977) then guarantees the existence of a unique strong solution up to a (possibly infinite) explosion time. If the explosion time is finite with probability one, we say that an explosion has occurred.

We are interested in the design and analysis of numerical methods that can reliably determine if an explosion has occurred over an interval $[0,T]$, for some fixed $T<\infty$, and if so to approximate the explosion time.

In this talk, we start to develop a framework for this task. First, we classify SDE solutions across a parameter range that describes the relative asymptotic properties of the drift and diffusion. Second, we examine some candidate numerical approaches that make use of adaptive timestepping to either reproduce finite time explosions or to capture the closely related property of pathwise instability. Finally, for the purposes of assessing the performance of these approaches, and comparing to commonly used fixed step methods, we motivate a suitable test equation and discuss some preliminary findings.

Prof Raphael Kruse, Martin Luther University Halle-Wittenberg, Germany

Finite p-variation of solutions to (stochastic) evolution equations and applications in numerical analysis

In numerical analysis of deterministic and stochastic evolution equations one often depends on estimates of the temporal regularity of the exact solution to derive the optimal order of convergence for discretization methods. For instance, the order of convergence of the backward Euler method typically agrees with the exponent $\gamma \in (0,1]$ of Hölder continuity of the exact solution.

In this talk, we discuss how to measure the temporal regularity of the exact solution in terms of the p-variation semi-norm. As it turns out, this notion of regularity is weaker than that of Hölder continuity.
In particular, if the solution is γ-Hölder continuous then it is also of finite p-variation for any $p \in [\frac{1}{\gamma},\infty)$. In addition, it allows to treat stochastic evolution equations with non-smooth initial values or whose regularity is measured in terms of fractional Sobolev spaces $W^{\sigma,q}(0,T;H)$, $\sigma \in (\frac{1}{q},1)$, in a unified setting. At the same time, the p-variation semi-norm

still allows to derive the optimal order of convergence of numerical methods for the temporal discretization of deterministic and stochastic evolution equations.

This is joint work with Johanna Weinberger and Rico Weiske (both MLU Halle-Wittenberg).

Prof Anne Kvarno, Norwegian University of Science and Tech., Norway

An SDE wind model applied to a hydrodynamic lake model

Accurate modelling of lake water temperature is important in multiple contexts. Firstly, empirical time series and thermal imagery data have found a trend of increasing lake surface temperature around the globe during the last decades. Lake water temperature warming alters the lake stratifcation period, changes the thermocline depth and reduces ice cover. In the ecological context, increased lake water temperature alters biomass size, contributes to cyanobacteria bloom formation and alters thermal refuge conditions, threatens vulnerable freshwater fish faunas, alters greenhouse gas uxes, and a_ects metabolic rate and balance.

In our studies, the popular 1-dimensional lake model Simstrat [1] have been applied to the very well surveilled lake Arungen near Oslo, with focus on how the stratifcation of the lake is influenced by the input data (weather conditions), in particular the effect of the wind.

The aim of our ongoing project is to develop an SDE model for the wind components, and use this as an input to the Simstrat model. So far, a rather simple linear SDE model has been tested.
In this talk, we will describe the model and some preliminary tests. Preliminary results will be presented, as well as some options for future work.

[1] The Simstrat project: https://www.eawag.ch/en/department/surf/projects/simstrat/
[2] Alemayehu Adugna Arara, Exponential integrators for stochastic differential equations: order theory and applications , PhD thesis, Hawassa University, 2019.

Prof Gabriel J. Lord, Radboud University, Netherlands

GBM based integrators for S(P)DEs

We introduce a class of exponential time integrators that exploit linear terms in the drift and diffusion for SDEs. In particular we consider a one sided Lipschitz drift and examine a tamed version of the scheme (and numerically an adaptive version).
We summarize results on strong convergence and illustrate the efficiency of the

methods. We then present recent work proving weak convergence of order 1 for both the GBM and standard tamed exponential methods. We illustrate convergence numerically (comparing different MLMC methods to estimate the weak error). This highlights a disadvantage of a standard exponentially tamed method not observed for the GBM method.

Prof Michael Tretyakov, Nottingham University, England

Weak approximation of stochastic differential equations driven by Levy processes

Stochastic differential equations driven by general Levy processes (SDEs) with infinite activity and the related, via the Feynman-Kac formula, Dirichlet and Cauchy problems for parabolic integro-differential equation (PIDE) are considered. Solutions of the PIDE problems are approximated using a random walk algorithm for the SDEs. The numerical method is based on the three ingredients: (i) small jumps are approximated by a diffusion; (ii) restricted jump-adaptive time-stepping is used; and (iii) between the jumps a weak Euler approximation is exploited. Weak convergence of the considered algorithm is proved, and an in-depth analysis of how its error and computational cost depend on the jump activity level is provided. Results of some numerical experiments are presented. Practical variance reduction is also discussed. The talk is based on joint works with George Deligiannidis (Oxford) and Simon Maurer (former Nottingham) and with Piers Hinds (Nottingham).