Outline Solutions of Honours Class 11.949 Mathematics of Financial Derivatives Section 5

1. If $S \geq E$, then $\max(E-S,0)=0$, $\max(S-E,0)=S-E$, we get

$$S+\max(E-S,0)-\max(S-E,0)=S+0-(S-E)=E.$$

If $S<E$, then $\max(E-S,0)=E$, $\max(S-E,0)=0$, we get

$$S+\max(E-S,0)-\max(S-E,0)=S+E-0=E.$$

2. Since today $S+P-C=100=E>Ee^{-0.05}$, you sell one asset to get 80 and sell one put option to get 30, you buy one call option at 10. The volume of the portfolio is 100, putting this in the bank you will get 105 at $T=1$ (more than E at T=1). The profit is 5.

3. (a) If $V(S,t)=AS$, then

$$\frac{\partial V}{\partial t}=0,\ \frac{\partial V}{\partial S}=A,\ \frac{\partial^2 V}{\partial S^2}=0.$$

So the right side of PDE (8) $=0+0+rSA-rSA=0$. $\Delta=A$. That the number of stocks that you need to hold in a portfolio which earns the risk-free rate is constant throughout the life-time of the option.

(b) If $V(S,t)=Ae^{rt}$, then

$$\frac{\partial V}{\partial t}=Are^{rt},\ \frac{\partial V}{\partial S}=0,\ \frac{\partial^2 V}{\partial S^2}=0.$$

So the right side of PDE (8) $=Are^{rt}+0+0-rAe^{rt}=0$. $\Delta=0$. This is the case when you save A in a deposit account with the fixed interest rate r.