Probability

Basic Definitions

- *Sample space*: the set of all possible outcomes of an experiment.
- *Sample point*: a possible outcome.
- *Event*: a collection of sample points with a common property.
- *Probability of a sample point*: the probability of a sample point x is the proportion of occurrences of the sample point in a long series of experiments and is denoted by $P(x)$.
- *Probability of an event*: the probability of an event A is the sum of the probabilities of the sample points which constitute the event and is denoted by $P(A)$, that is

$$P(A) = \sum_{x \in A} P(x).$$

- *Equally likely sample points*: all sample points in the sample space have the same probability. In this case,

$$P(A) = \frac{\text{the number of sample points in } A}{\text{the total number of sample points in the sample space}}$$

Types of Events

- *Union (or sum) of events* $E_1 + E_2 = E_1 \cup E_2$: at least one of the events occurs.
- *Intersection of events* $E_1 E_2 = E_1 \cap E_2$: both events occurs. It is useful to know that

$$P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 E_2).$$

- *Mutually exclusive events* $E_1 E_2 = \emptyset$ (empty set): the events have no common sample points. In this case,

$$P(E_1 + E_2) = P(E_1) + P(E_2).$$

Conditional Probability

The conditional probability of E_1, given that E_2 has occurred, is written $P(E_1|E_2)$ and read as $P(E_1 \text{ given } E_2)$. The conditional probability is computed by

$$P(E_1|E_2) = \frac{P(E_1 E_2)}{P(E_2)}.$$

If all sample points are equally likely to occur, then

$$P(E_1|E_2) = \frac{\text{the number of sample points in } E_1 E_2}{\text{the total number of sample points in } E_2}.$$

Independent Events Two events, E_1 and E_2 are said to be *independent* if and only if

$$P(E_1E_2) = P(E_1)P(E_2)$$

or $P(E_1|E_2) = P(E_1)$ or $P(E_2|E_1) = P(E_2)$. Thus the knowledge that event E_2 has occurred has no effect on the probability of event E_1 and vice versa.

Dependent Events Two events, E_1 and E_2 are said to be *dependent* if they are not independent. It is useful to know that

$$P(E_1E_2) = P(E_1)P(E_2|E_1) = P(E_2)P(E_1|E_2).$$

Opposite Events (or Complements)

The opposite event (or the complement) of A is denoted by $\bar{A}$, and

$$P(\bar{A}) = 1 - P(A).$$