Discrete Distributions

Basic Definitions

- *Discrete random variable (r.v.)* : a function X that can take values $x_1, x_2, \cdots, x_N$ randomly.
- *Discrete probability distribution*: If a discrete r.v. X can take values $x_1, x_2, \cdots, x_N$ with probabilities
$$p_i = P(X = x_i), \quad i = 1, 2, \cdots N,$$
where
$$p_i \geq 0 \quad \text{and} \quad \sum_{i=1}^{N} p_i = 1.$$
Then this defines a discrete probability distribution for X.
- *Mean (or expectation)*:
$$\mu = E(X) = \sum_{i=1}^{N} x_i p_i.$$
- *Variance*:
$$\sigma^2 = Var(X) = \sum_{i=1}^{N} (x_i - \mu)^2 p_i = \sum_{i=1}^{N} x_i^2 p_i - \mu^2.$$
- *Standard deviation*:
$$\sigma = \sqrt{\sigma^2}.$$

The Binomial Distribution $B(n, p)$

The binomial experiment:

- n independent identical trials,
- each trial has only two outcomes, say “success” and “failure”, with probability p and $1 - p$, respectively,
- $X =$ the number of successes.

X may take values $0, 1, 2, \cdots, n$ with probabilities

$$p_r = P(X = r) = {}^nC_r p^r (1 - p)^{n-r}, \quad 0 \leq r \leq n,$$

where

$${}^nC_r = \frac{n!}{r!(n - r)!}.$$

This discrete probability distribution is called a binomial distribution and is denoted by $B(n, p)$. In this case we say X follows $B(n, p)$ or write $X \sim B(n, p)$. It can be shown that

$$\mu = E(X) = np \quad \text{and} \quad \sigma^2 = Var(X) = np(1-p).$$

The Poisson Distributions

If X takes values $0, 1, 2, \cdots$ with probabilities

$$p_r = P(X = r) = \frac{e^{-\mu}\mu^r}{r!} \quad (r = 0, 1, 2, \cdots, \ \mu > 0)$$

then X follows a Poisson distribution with parameter μ. It can be shown that

$$E(X) = Var(X) = \mu,$$

i.e. the parameter μ is both mean and variance of X.