Example 13

Suppose the joint distribution of X and Y is as given below

		X 0	1	2	*Marginal distribution of* Y
	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$
Y	1	$\frac{1}{4}$	0	$\frac{1}{8}$	$\frac{3}{8}$
	2	$\frac{1}{8}$	$\frac{1}{8}$	0	$\frac{1}{4}$
Marginal distribution of X		$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	

The grand total of the joint probabilities is one. The row totals form the marginal distribution of Y: the column totals form the marginal distribution of X.

If $Y = 0$ the conditional distribution of X is given by

$$P(0|0) = \frac{1/8}{3/8} = 1/3,$$

$$P(1|0) = \frac{1/8}{3/8} = 1/3,$$

$$P(2|0) = \frac{1/8}{3/8} = 1/3.$$

NM318/3.7: Do Ex. 1, 4, 5, 8, 11

NM309. Do Ex. 1, 5, 8

These conditional probabilities also add up to one. The other conditional distributions of both X and Y can be found in a similar fashion. Since the above conditional distribution is not the same as the marginal distribution of X, the two random variables are not independent.

Exercises

1. Packets of food are filled automatically and the proportion of packets in a very large batch which are underweight is p. A sample size n is selected randomly from the batch and the probability that the sample contains exactly r defective packets ($r = 0, 1, 2, \ldots, n$) follows a certain probability distribution. Name this distribution and write down the probability that the sample contains exactly r defective packets.

For one particular process it has been found in the past that 2 per cent of the packets are underweight. An inspector takes a random sample of ten packets. Calculate

(a) the expected number of packets in the sample which are underweight,

~~(b)~~ the probability that none of the packets in the sample are underweight.

(c) the probability that more than one of the packets in the sample is underweight.

2. Compute and plot the binomial distributions when $n = 4$ for

(a) $p = \frac{1}{4}$,

(b) $p = \frac{1}{2}$,

(c) $p = \frac{3}{4}$.

3. It has been found in the past that 4 per cent of the screws produced in a certain factory are defective. A sample of ten is drawn randomly from each hour's production and the number of defectives is noted. In what fraction of these hourly samples would there be at least two defectives? What doubts would you have if a particular sample contained six defectives?

4. One per cent of a certain type of car has a defective tail light. How many cars must be inspected in order to have a better than even chance of finding a defective tail light?

5. An electronic component is mass-produced and then tested unit by unit on an automatic testing machine which classifies the unit as 'good' or 'defective'. But there is a probability 0·1 that the machine will mis-classify the unit, so that each component is in fact tested five times and regarded as good if so classified three or more times. What now is the probability of a mis-classification?

6. A recent court decision supported a pilot's decision to continue to his destination with a four-engine jet aircraft after an engine failure at midrange on a two-hour flight, rather than land at the midway point. His argument was that the aircraft will fly on two engines and that the probability of two additional failures was quite small. If the one-hour reliability of a single engine is 0·9999, do you agree with the pilot's

decision? If the aircraft was a three-engine jet that will fly on one engine would you agree? Comment on the comparative reliability of the three- and four-engine jets in this situation.

7. The new 'Tigercat' sports car has an idle loping problem as about 10 per cent of the Tigercats have an unstable fluctuating engine speed when they are idling. An engineering 'fix' is put in a production pilot lot of a hundred cars.

(a) If the fix has no effect on the problem, how many cars would you expect to have the fault?

(b) If only two of the pilot lot have the idling fault, and the other ninety-eight cars are not defective, would you conclude that the fix has a significant effect? (Hint: Show that the probability of getting two or less defectives in a sample size of a hundred, given that the fix has no effect, is very small indeed.)

8. The average number of calls that a hospital receives for an ambulance during any half-hour period is 0·3. Considering a reasonable cost per ambulance and crew and presuming that any ambulance will return to the hospital in half an hour, how many ambulances would you recommend for this hospital? Comment on the idea of ambulance pools which are shared by several hospitals.

9. A manned interplanetary space vehicle has four engines each with reliability 0·99. Each engine has a failure detection system which may itself fail. If the engine does fail there is a conditional probability of 0·02 that a success will be signalled. If an engine fails and is not detected the result is catastrophic. However the mission can be completed with three engines if one engine fails and is detected. What is the probability of mission success? If there is no abort system or escape system, what action would you recommend if two failures are signalled? (Hint: Calculate probability of no engine failures plus one detected failure.)

10. It has been found in the past that one per cent of electronic components produced in a certain factory are defective. A sample of size one hundred is drawn from each day's production. What is the probability of getting no defective components by using

(a) the binomial distribution,

(b) the Poisson distribution approximation.

11. A construction company has a large fleet of bulldozers. The average number inoperative at a morning inspection due to breakdowns is two. Two standby bulldozers are available. If a bulldozer can always be mended within twenty-four hours of the morning inspection find the probability that at any one inspection

(a) no standby bulldozers will be required,

(b) the number of standby bulldozers will be insufficient.

12. There are many other discrete distributions apart from the binomial and Poisson distributions. For example the *hypergeometric* distribution arises as follows. A sample size n is drawn without replacement from a finite population size N which contains Np successes and $N(1-p)$ failures. Show that the probability of getting exactly x successes in the sample is given by

$$P(x) = \frac{{}^{Np}C_x \, {}^{N(1-p)}C_{n-x}}{{}^{N}C_n} \qquad (x = 0, 1, 2, \ldots, \text{minimum of } n \text{ and } Np).$$

If N is very large compared with n, it can be shown that this distribution can be approximated by the binomial distribution with

$$P(x) \simeq {}^{n}C_x p^x (1-p)^{n-x} \qquad (x = 0, 1, 2, \ldots, n).$$

13. A series of items is made by a certain manufacturing process. The probability that any item is defective is a constant p, which does not depend on the quality of previous items. Show that the probability that the rth item is the first defective is given by

$$P(r) = p(1-p)^{r-1} \qquad (r = 1, 2, \ldots).$$

This probability distribution is called the *geometric* (or *Pascal*) distribution. Show that the sum of the probabilities is equal to one as required: also show that the mean of the distribution is equal to $1/p$.

Reference

HOEL, P. G. (1984). *Introduction to Mathematical Statistics*, 5th edn. Wiley.