are the marginal probability density functions of the two random variables.

The two random variables are said to be independent if

$$f(x, y) = f_X(x) f_Y(y) \qquad \text{for all } x, y.$$

## Exercises

1. Find the area under the standard normal curve ($\mu = 0$, $\sigma = 1$)
(a) outside the interval $(-1, +1)$,
(b) between $-0{\cdot}5$ and $+0{\cdot}5$,
(c) to the right of $1{\cdot}8$.

2. If $X$ is $N(3, \sigma^2 = 4)$ find
(a) $P(X < 3)$,
(b) $P(X \leqslant 5)$,
(c) $P(X \leqslant 1)$.

3. The mean weight of 500 students at a certain college is 150 lb and the standard deviation 15 lb. Assuming that the weights are approximately normally distributed, estimate the proportion of students who weigh
(a) between 120 and 180 lb,
(b) more than 180 lb.

4. The lengths of a batch of steel rods are approximately normally distributed with mean 3·1 ft and standard deviation 0·15 ft. Estimate the proportion of rods which are longer than 3·42 ft.

5. If $X$ is normally distributed with $\mu = 10$ and $\sigma = 2$, find numbers $x_0$, $x_1$ such that
(a) $P(X > x_0) = 0{\cdot}05$,
(b) $P(X > x_1) = 0{\cdot}01$.
Also find $k$ such that

$$P(|X - \mu| > k) = 0{\cdot}05.$$

6. Suppose that the lifetimes of a batch of radio components are known to be approximately normally distributed with mean 500 hours and standard deviation 50 hours. A purchaser requires at least 95 per cent of them to have a lifetime greater than 400 hours. Will the batch meet the purchaser's specifications?

7. An examination paper consists of twenty questions in each of which the candidate is required to tick as correct one of the three possible answers. Assume that a candidate's knowledge about any question may be represented as either (i) complete ignorance in which case he ticks at random or, (ii) complete knowledge in which case he ticks the correct answer.

How many questions should a candidate be required to answer correctly if not more than 1 per cent of candidates who do not know the answer to any question are to be allowed to pass? (Use the normal approximation to the binomial distribution.)

8. The lengths of bolts produced in a certain factory may be taken to be normally distributed. The bolts are checked on two 'go–no go' gauges so that those shorter than 2·983 in. or longer than 3·021 in. are rejected as 'too-short' or 'too-long' respectively.

A random sample of $N = 300$ bolts is checked. If they have mean length 3·007 in. and standard deviation 0·011 in., what values would you expect for $n_1$, the number of 'too-short' bolts and $n_2$, the number of 'too-long' bolts?

A sample of $N = 600$ bolts from another factory is also checked. If for this sample we find $n_1 = 20$ and $n_2 = 15$, find estimates for the mean and standard deviation of the length of these bolts.

NM318/317: Do Ex. 2, 3, 5, 6, 8

NM309: Do Ex. 2, 3, 8