A scientific choice of the sample size is also desirable when H_0 is not precise or when the choice of a precise H_0 conflicts with the idea that the burden of proof should be on H_1. For example suppose that a chemist wishes to decide if a certain method of analysis gives unbiased measu ins of known concer ition namely:
(a) Th
(b) Th
Here t iat hypothesis
(b) sh would choose
hypot procedure can
be rer gh to give a
guarar

NM318/317. Do Ex. 1, 3, 5

NM309 Do Ex 1, 3

(All tests using $\alpha = 0.05$)

Exercises

1. Test the hypothesis that the random sample

12·1 12·3 11·8 11·9 12·8 12·4

came from a normal population with mean 12·0. The standard deviation of the measurements is known to be 0·4.

Also construct a 95 per cent confidence interval for the true mean, μ.

2. Repeat question one without assuming that the standard deviation is known to be 0·4. In other words estimate the population variance from the sample measurements and use a t-test.

3. A manufacturer claims that the percentage of phosphorus in a fertilizer is at least 3 per cent. Ten small samples are taken from a batch and the percentage of phosphorus in each is measured. The ten measurements have a sample mean of 2·5 per cent and a sample standard deviation of 0·5 per cent. Is this sample mean significantly below the claimed value? State the null hypothesis and the alternative hypothesis and say if a one- or two-tailed test is required.

4. The strength of paper used by a certain company is approximately normally distributed with mean 30 p.s.i. and standard deviation 3 p.s.i. The company decides to test a new source of paper made by a different manufacturer. If this paper is significantly stronger, then the company will switch its trade to this new manufacturer. A batch of paper is

obtained from this manufacturer and a series of measurements are to be made on the strength of different pieces of paper from the batch. Assuming that the standard deviation of these measurements is also 3 p.s.i., how large a sample size should be chosen so as to be 95 per cent certain of detecting a mean increase in strength of 2 p.s.i. with a one-tailed test at the 5 per cent level?

5. For a certain chemical product it is thought that the true percentage of phosphorus is 3 per cent. Ten analyses give $\bar{x} = 3{\cdot}3$ per cent and $s = 0{\cdot}2$ per cent. Is the sample mean significantly different from 3 per cent? (This question differs from question 3 because we are interested in departures from 3 per cent in either direction.)

6. One sample of fifteen observations has $\bar{x}_1 = 82$ and $s_1 = 5$. A second sample of ten observations taken by a different scientist has $\bar{x}_2 = 88$ and $s_2 = 7$. Is there a significant difference between the two sample means at the (a) 0·05 and (b) 0·01 level of significance? (You may assume that the two populations have equal variances.)

7. Test the hypothesis that the following set of 200 numbers are 'random digits', that is, each number is equally likely to be 0, 1, 2, . . . , 9.

r	0	1	2	3	4	5	6	7	8	9
Frequency	22	16	15	18	16	25	23	17	24	24

8. The following figures show the number of accidents to 647 women in a period of five weeks while working on the manufacture of shells. (Source: M. Greenwood and G. U. Yule, *Journal of the Royal Statistical Society*, 1920.)

Number of accidents	0	1	2	3	4	5	6+
Frequency	447	132	42	21	3	2	0

Find the Poisson distribution with the same mean. Test the hypothesis that the Poisson distribution gives a good fit to the data.

9. In order to test the effectiveness of a new drug in treating a particular disease, seventy patients suffering from the disease were randomly divided into two groups. The first group was treated with the drug and the second group was treated in the standard way. The results were as follows.

	Recover	*Die*
Drug	20	15
No drug	13	22

Test the hypothesis that the drug has no effect.