1. When A plays B at tennis the probability that A wins any one game is p and different games are independent. Find the probability that A wins a set given that the score has reached 4-4 and there are no tie breaks. (The winner is the first to have won six games and to be two games ahead.)

2. A simple random walk has absorbing barriers at 2 and -3 and initial position 0, and $p = 0.5$, $q = 0.3$, $r = 0.2$. Find the probability
(a) of absorption at 2,
(b) that -2 is never reached and absorption occurs at 2,
(c) that absorption at -3 occurs and 1 is reached exactly once,
(d) that absorption at -3 occurs and 1 is never reached.
Also find the mean time till absorption and the mean step size.

3. Consider a simple random walk on n equally spaced points round a circle. Find the probability that the first return to the starting points is after a complete circuit.

4. Show that for a simple random walk with absorbing barriers and starting position 0, T.E(step length) = E(final position), where T denotes the expected time till absorption.

5. A Markov chain on the state space $S = \{1, 2, 3\}$ has the transition probability matrix

$$P = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}$$

Determine the mean time to reach state 3 starting from state 1 by using a first-step analysis.

6. A Markov chain on the state space $S = \{1, 2, 3, 4\}$ has the transition probability matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Find the probability that the absorption occures at state 1 starting from state 2 by using a first-step analysis.

7. A gambler has £9 and has the opportunity of playing a game in which the probability is 0.4 that he wins an amount equal to his stake, and probability 0.6 that he loses his stake. He is allowed to decide how much to stake at each game (in multiple of 10p). How should he choose the stakes to maximise his chances of increasing his capital to £10?