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Example Sheet 4 : Branching Processes

1. In a neutron chain reaction neutrons collide with nuclei and are absorbed. Each collision produces 0, 1 or 2 new neutrons with the following probabilities:

m	0	1	2
$p(m)$	0.5	0.2	0.3

Different neutron collisions are independent. Initially there is just one neutron. Find the expected number of neutrons at the nth generation.

2. A population starts off with just one male. Each male lives for a fixed time T and then dies. The number of male offspring of a single male is given by the following probability distribution:

m	0	1	2	3
$p(m)$	0.1	0.4	0.3	0.2

Different males are independent. Find the expected number of males at the nth generation.

3. In Example 1 find the probabilities of extinction by the nth generation for n = 1,2,3,4,5 and the probability of ultimate extinction.

4. In Example 2 find the probabilities of extinction by the nth generation for n = 1,2,3,4,5 and the probability of ultimate extinction.

5. A population of fish each independently survives to reproduce with probability $\frac{2}{3}$ when each fish surviving produces two young fish. Initially there is just one fish. Different fish are independent. If the probability generating function of the number of fish in the first generation is

$$G(s) = \frac{1}{3} + \frac{2}{3}s^2,$$

show that the probability generating function of the number of fish in the third generation is

$$G^{(3)}(s) = G(G(G(s))).$$

Hence determine the probability distribution of the number of fish in the third generation.