Example Sheet 5 : The Poisson Process

1. Strikes in a factory occur according to a Poisson process of rate 2 per year. Find the probability that there is exactly one strike in the first 3 months and exactly 3 strikes in the next year.

2. Components in a machine fail and are replaced according to a Poisson process of rate 3 a month.
   (a) Find the probability that exactly 3 fail in the first month and exactly 5 fail in the next two months.
   (b) Find the probability that exactly 3 fail in the first month given that exactly 8 fail in the first three months.
   (c) Find the probability that at least 2 fail in the first month and at least 4 fail in the first two months.
   (d) Find the probability that at least 2 fail in the first month given that at least 4 fail in the first two months.

3. An electron gun fires electrons at a target at a rate of one per minute. Each electron independently hits the target with probability 0.2.
   (a) What is the arrival process of electrons at the target?
   (b) What is the probability that exactly 2 electrons hit the target in 10 minutes?

4. Three independent types of flaw are distributed along a length of wire. Type A flaws occur according to a Poisson process of rate one per metre. Type B flaws occur according to a Poisson process of rate two per metre. Type C flaws occur according to a Poisson process of rate three per metre.
   (a) What is the distribution of the total number of type A and B flaws in the wire?
   (b) What is the distribution of the total number of flaws in the wire?
   (c) What is the probability that there is exactly one type A flaw, exactly one type B flaw and exactly one type C flaw in three metres of wire?
   (d) What is the probability that there are exactly three flaws in three metres of wire?
   (e) What is the probability that there is one flaw of each type in three metres of wire given that there are three flaws in total?

5. Red cars pass a point on a road according to a Poisson process of rate λ and blue cars according to an independent Poisson process of rate μ.
   (a) What is the distribution of the total number of (red or blue) cars passing the point in time T?
   (b) Given that exactly one car passes the point in time T what is the probability that it is red?