NM317/318 Exam Paper 11/12
Three Questions for the Statistical Part

Three questions in this part are compulsory and each carries 25 marks. Battery-powered pocket calculators may be used. However, answers without any working are not acceptable, so full marks will not be awarded, even if the answer is correct, unless an outline of the calculations carried out is clearly shown.

1.(a) Three parts of a machine are manufactured independently. They have probabilities p_1, p_2 and p_3 respectively of failing. Find the probability that (i) at least one part fails, (ii) at most one part fails.

(8 marks)

(b) A machine for filling soap powder into boxes is installed in a production line. A random sample of eight 1.5kg boxes is selected and weighted with the following results (in kg):

$$1.51 \quad 1.53 \quad 1.49 \quad 1.54 \quad 1.50 \quad 1.51 \quad 1.51 \quad 1.53$$

Assume that the weights are normally distributed. Do the data provide sufficient evidence to indicate that the population mean μ differs from 1.5kg? Test using $\alpha = 0.05$. Also find a 95% confidence interval for μ.

(9 marks)

(c) Assume that a random variable X is uniformly distributed between 0 and 2. Write down the probability density function of X and find its mean and variance.

(8 marks)

2.(a) While automobile insurance is required of drivers in most states, the American Automobile Association claims that 5% of all drivers do not carry any liability, medical or collision insurance. To check on compliance with a state's mandatory automotive insurance coverage law, 25 cars are randomly stopped by a policeman and the driver of each is asked to show evidence of insurance coverage. Find the following probabilities: (i) All 25 drivers possess insurance coverage. (ii) No more than 2 drivers are found to be without insurance coverage.

(7 marks)

(b) The following data represent the measurements on a controlled variable x and its dependent variable y:

x	1	2	3	4	5	6
y	7	5	5	3	3	1

Find the linear regression equation of y on x. Calculate the correlation coefficient and show that it is significantly different from zero at $\alpha = 0.05$ level of significance.

(9 marks)

(c) Bags of sugar of nominal weight 1kg, have weights which are normally distributed with mean 1.006kg and standard deviation 6g. What is the probability that a randomly selected bag of sugar is underweight? If the firm wishes to ensure that only 1% of bags are underweight, what should the standard deviation of bags be decreased to? (Assume that the mean 1.006kg remains unchanged.)

(9 marks)

3.(a) There are four types of relay (electronic switch) to be used in a telephone exchange. From random samples we have the following measurements on the response times (milli-seconds) of these relays:

Type A	Type B	Type C	Type D
7	6	11	12
8	8	10	10
7	9	12	9
10	7	9	10

Construct an ANOVA table for the data. Is there any evidence to suggest that mean response times differ between types? Test using $\alpha = 0.05$.

(10 marks)

(b) Assume that the random variable X obeys the exponential distribution with the probability density function

$$f(x) = \begin{cases} 3e^{-3x} & \text{if } x \geq 0; \\ 0 & \text{if } x < 0. \end{cases}$$

Find (i) $P(X \leq 1)$, (ii) $P(1 < X \leq 3)$, (iii) $P(X > 3)$.

(6 marks)

(c) A multinomial experiment consisting of 6 possible outcomes was conducted 600 times, resulting in the following observations:

Outcome	1	2	3	4	5	6
Frequency	85	102	96	97	121	99

Do the data provide sufficient evidence to indicate that the six outcomes are equally likely? Test using $\alpha = 0.05$.

(9 marks)

Solutions to NM317/318 Exam Paper 11/12

1(a) Define $E_i = \{\text{part } i \text{ fails}\}$ $i = 1, 2, 3$.

(i) P(at least one part fails)

$= 1 - P(\text{all 3 parts function})$

$= 1 - P(\bar{E}_1 \bar{E}_2 \bar{E}_3)$

$= 1 - P(\bar{E}_1) P(\bar{E}_2) P(\bar{E}_3)$

$= 1 - (1-p_1)(1-p_2)(1-p_3)$

(ii) P(at most one part fails)

$= P(\text{all 3 parts function}) + P(\text{exactly one part fails})$

$= P(\bar{E}_1 \bar{E}_2 \bar{E}_3) + P(E_1 \bar{E}_2 \bar{E}_3) + P(\bar{E}_1 E_2 \bar{E}_3) + P(\bar{E}_1 \bar{E}_2 E_3)$

$= (1-p_1)(1-p_2)(1-p_3) + p_1(1-p_2)(1-p_3) + (1-p_1)p_2(1-p_3)$
$+ (1-p_1)(1-p_2)p_3$

1(b) $H_0: \mu = 1.5$, $H_1: \mu \neq 1.5$

Compute

$\bar{x} = \frac{1}{8}[1.51 + 1.53 + \cdots + 1.51 + 1.53] = 1.515$

$s^2 = \frac{1}{8-1}[1.51^2 + 1.53^2 + \cdots + 1.53^2 - 8 \times 1.515^2]$

$= 0.0002857$

$s = 0.0169$

$$t = \frac{\bar{x} - 1.5}{s/\sqrt{n}} = \frac{1.515 - 1.5}{0.0169/\sqrt{8}} = 2.51$$

Given $\alpha = 0.05$, d.f. $= n - 1 = 7$, we find $t_{\alpha/2, n-1} = t_{0.025, 7} = 2.365$.

Since $|t| = 2.51 > t_{\alpha/2, n-1}$, we reject H_0 and accept $H_1: \mu \neq 1.5$. The 95% C.I. for μ is

$\bar{x} \pm t_{0.025, 7} \frac{s}{\sqrt{n}}$

$= 1.515 \pm 2.365 \times \frac{0.0169}{\sqrt{8}}$

$= 1.515 \pm 0.014 = (1.501, 1.529$

2(b) $\bar{x} = \frac{1}{6}(1+2+\cdots+6) = 3.5$

$$\bar{y} = \frac{1}{6}(7+5+\cdots+1) = 4$$

$$S_{xx} = 1^2+2^2+\cdots+6^2 - 6\times 3.5^2 = 17.5$$

$$S_{xy} = 1\times 7 + 2\times 5 + 3\times 5 + 4\times 3 + 5\times 3 + 6\times 1$$

$$- 6\times 3.5\times 4$$

$$S_{yy} = 7^2+5^2+5^2+3^2+3^2+1^2 - 6\times 4^2$$

$$= 22$$

$$\hat{a}_1 = \frac{S_{xy}}{S_{xx}} = -\frac{19}{17.5} = -1.08571$$

$$\hat{a}_0 = \bar{y} - \hat{a}_1\bar{x} = 4 + (1.08571)\times 3.5 = 7.8$$

$$\therefore \hat{y} = 7.8 - 1.08571\, x$$

The correlation coefficient

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{-19}{\sqrt{17.5\times 22}} = 0.9683$$

Given $\alpha = 0.05$ find

$$t_{\alpha/2, n-2} = t_{4, 0.025} = 2.776$$

Since

$$|r| \sqrt{\frac{n-2}{1-r^2}} = 7.7529 > 2.776$$

We conclude that the correlation coefficient is significantly different from 0 at $\alpha = 0.05$ level of significance

2(c) Let X = the weight of a bag of sugar. Then

$X \sim N(1.006, 0.006^2)$

P(a bag of sugar is underweight)

$$= P(X < 1) = P\left(\frac{X-1.006}{0.006} < \frac{1-1.006}{0.006}\right)$$

$$= P(Z < -1) = P(Z > 1) = 1 - P(Z < 1)$$

$$= 1 - 0.8413 = 0.1587$$

If we require

$$P(X < 1) = 0.01$$

where $X \sim N(1.006, \sigma^2)$, then

$$P(X<1) = P\left(\frac{X-1.006}{\sigma} < \frac{1-1.006}{\sigma}\right)$$

$$= P\left(Z < -\frac{0.006}{\sigma}\right) = 0.01$$

Hence $$P\left(Z > \frac{0.006}{\sigma}\right) = 0.01$$

$$P\left(Z \leq \frac{0.006}{\sigma}\right) = 0.99$$

But, by the normal distribution table, we have

$$P(Z \leq 2.3263) = 0.99$$

So $$\frac{0.006}{\sigma} = 2.3263$$

$$\sigma = \frac{0.006}{2.3263} = 0.00258 \text{ (kg)}$$
$$= 2.58 \text{ (g)}$$

Namely, the standard deviation should be decreased to 2.58 g.

1(c) The p.d.f. of X is

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } 0 \le x \le 2 \\ 0 & \text{if } x > 2. \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x) = \int_0^2 \frac{1}{2} x \, dx = \left[\frac{1}{4}x^2\right]_{x=0}^{2}$$

$$= \frac{1}{4} \times 2^2 - \frac{1}{4} \times 0^2 = 1$$

$$Var(X) = \int_{-\infty}^{\infty} x^2 f(x) \, dx - (E(X))^2$$

$$= \int_0^2 \frac{1}{2} x^2 \, dx - 1^2$$

$$= \left[\frac{1}{6}x^3\right]_{x=0}^{2} - 1$$

$$= \frac{8}{6} - 1 = \frac{1}{3}$$

2(a) Let X = # of the drivers without insurance

Then $X \sim B(25, 0.05)$

(i) $P(X = 0) = \binom{25}{0} 0.05^0 (1 - 0.05)^{25} = 0.95^{25}$

$= 0.277$

(ii) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \binom{25}{0} 0.05^0 \, 0.95^{25} + \binom{25}{1} 0.05^1 \times 0.95^{24} + \binom{25}{2} 0.05^2 \times 0.95^{23}$

$= 0.95^{25} + 25 \times 0.05^1 \times 0.95^{24} + \frac{25 \times 24}{2 \times 1} 0.05^2 \times 0.95^{23}$

$= 0.873$

3(a)

	A	B	C	D
	7	6	11	12
	8	8	10	10
	7	9	12	9
	10	7	9	10
Total	32	30	42	41
Mean	8	7.5	10.5	10.25

Compute

$$T = 145$$

$$CF = \frac{145^2}{4\times 4} = 1314.06\cancel{25}$$

$$\sum_{ij} x_{ij} = 7^2 + 8^2 + \cdots + 10^2 = 1363$$

$$SS_{total} = 1363 - 1314.06 = 48.94$$

$$SS_{treat} = \frac{1}{4}\left[32^2 + 30^2 + 42^2 + 41^2\right] - CF$$

$$= 1342.25 - 1314.06 = 28.19$$

$$SS_{error} = 20.75$$

ANOVA

Source	SS	d.f.	MS	F
Types	28.19	3	9.40	5.43
Error	20.75	12	1.73	
Total	48.94	15		

Hypothesis H_0: The means of 4 types are the same

H_1: ~ ~ ~ ~ ~ are different

Given $\alpha = 0.05$, find

$$F_{\alpha, 3, 12} = F_{0.05, 3, 12} = 3.49$$

$\because 5.43 > 3.49$ $\therefore$ reject H_0 and accept H_1.

3(b) (i) $P(X \leq 1) = \int_{-\infty}^{1} f(x)\,dx = \int_{0}^{1} 3e^{-3x}\,dx$

$= \left[-e^{-3x}\right]_{x=0}^{1} = 1 - e^{-3}$

(ii) $P(1 < X \leq 3) = \int_{1}^{3} 3e^{-3x}\,dx$

$= \left[-e^{-3x}\right]_{x=1}^{3} = e^{-3} - e^{-9}$

(iii) $P(X > 3) = \int_{3}^{\infty} 3e^{-3x}\,dx$

$= \left[-e^{-3x}\right]_{x=3}^{\infty} = e^{-9}$

3(c) H_0: 6 outcomes are equally likely

H_1: - - - - - - NOT - - - -

Under H_0, we should have

Outcome	1	2	3	4	5	6
Prob	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

Compute

$$X^2 = \sum_{i=1}^{6} \frac{(n_i - np_i)^2}{np_i}$$

where $n = 600$, $p_i = \frac{1}{6}$, $np_i = 100$, n_i's are the frequencies given. Hence

$$X^2 = \frac{(85-100)^2}{100} + \frac{(102-100)^2}{100} + \frac{(96-100)^2}{100} + \frac{(97-100)^2}{100} + \frac{(121-100)^2}{100} + \frac{(99-100)^2}{100}$$

$= 6.96$

Given $\alpha = 0.05$, d.f. $= 6-1 = 5$, we find

$\chi_5^2(0.05) = 11.07$

As $X^2 < \chi_5^2(0.05)$, we accept H_0 and conclude that the 6 outcomes are equally likely.