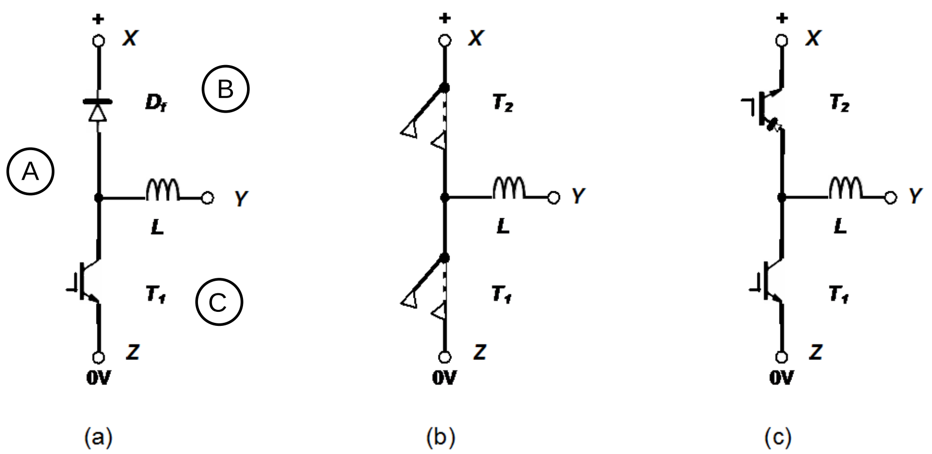
CHAPTER 20

Advanced DC to DC Converters
- Switched-Mode

Chapter 19 introduced a single-switch, single-diode, first quadrant, dc-to-dc converter that employed two inductors and two capacitors, specifically the boost-buck Cuk converter. Many other converter variations exist that employ multiple capacitors and inductors, for example the zeta, ±Luo, CSC, Landsman, and sepic converters. A circuit based systematic concept can unifying the thirty three existing single switch/diode topologies, where three of the thirty three are the basic buck, boost, and buck-boost topologies considered in Chapter 19. The extra reactive components yield topologies having new transfer functions, for example the zeta converter produces a non-inverting buck-boost voltage output function.
Other converter properties considered in this chapter are the discontinuous/continuous conduction boundary of the boost converter and current source dc to dc converters.

20.1 Basic generic smps transfer function mapping

The three basic smps, viz., the buck, boost and buck-boost converters, utilise a switch, diode and inductor, as shown in figure 20.1a, to realise their fundamental dc-to-dc conversion function.
Figure 20.1b shows a general form of the circuit in figure 20.1a, where the function of the two switching elements have not been prejudged to be a diode and a unidirectional voltage and current switch.
If the switch T_1 in the configuration of the circuit in figure 20.1a is controlled with an on-state duty cycle of *δ*, then the transfer functions associated with the buck, boost and buck boost converters are realised, as decoded in the table in figure 20.1. Although each transfer function is fixed, the output function can be modified by mapping the control parameter. For example, if the complement of the duty cycle *δ* is used to control T_1, namely 1-*δ*, then in the case of the boost converter, the output voltage tracks -*δ* /1-*δ*, the buck-boost transfer function. The mapped transfer functions of the three basic converters, when controlled by the duty cycle complement 1-*δ*, are shown in Table 20.1 and are plotted in figure 20.2.



converter	ports	transfer function	nodes
forward /buck	B/A	δ	xy/xz
boost	A/C	1 / (1- δ)	xz/yz
buck-boost	-B/C	-δ/ (1- δ)	yx/yz

Figure 20.1. *Circuit elements of basic smps:*
(a) circuit diagram; (b) generalised functional circuit; and (c) specific circuit components.

Generally, if the duty cycle is encoded by *f(δ),* any effective transfer function can be generated within the 0≤*δ*≤1 voltage range of the basic converter. For example, in the case of the buck converter, any monotonically increasing output voltage profile can be produced in the range between zero volts and the input voltage magnitude. A lookup table mapping approach provides total flexibility.

Table 20.1: Mapped transfer functions

duty cycle mapping	$0 < \delta < 1$	$0 < 1-\delta < 1$	$0 < f(\delta) < 1$
buck	δ	$1-\delta$	$f(\delta)$
boost	$\frac{1}{1-\delta}$	$\frac{1}{\delta}$	$\frac{1}{1-f(\delta)}$
buck-boost	$-\frac{\delta}{1-\delta}$	$-\frac{1-\delta}{\delta}$	$-\frac{f(\delta)}{1-f(\delta)}$

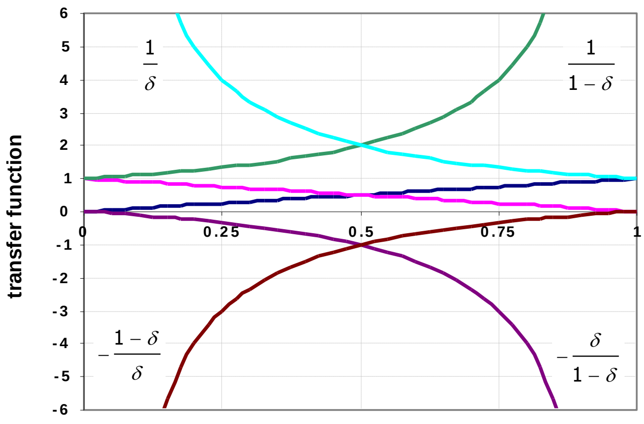


Figure 20.2. *The transfer functions of the three basic converters in terms of δ and their complementary transfer functions in terms of 1- δ.*

20.2 Basic generic current sourced smps

The smps considered in chapter 19 are based on a voltage source input, and are termed *voltage-sourced* converters. A switch, diode and inductor are *T* (tee) connected between the voltage source on the input and the capacitor-shunt decoupled load at the output, as shown in figure 20.3a. The three smps in figure 20.3b are *current-sourced* equivalents (duals) to the considered *voltage-sourced* buck, boost, and buck-boost converters and each has the same corresponding voltage and current transfer ratio. The current transfer functions are shown plotted in figure 20.4, with various circuit operating conditions shown in Table 20.2. In these circuits, figure 20.3b, the smps capacitor C is equivalent to the inductor L in a voltage-sourced smps, and the switch, diode and capacitor are *π* (pi) connected between the input current source and the series-inductor filtered output load. Just as the inductor is designed for a specific ripple current and continuous or discontinuous current conduction, the capacitor C in figure 20.3b is the dual, being designed to have a specific ripple voltage and may or may not reach zero charge within each cycle. The current boost circuit in figure 20.3b-*ii* is complementary to the voltage boost circuit in figure 20.3a-*iii*, in the sense of continuous input power. The voltage boost smps circuit can ensure continuous current (hence continuous input power) from a dc voltage (energy) source, while the current boost smps circuit can ensure a continuous non-zero voltage (hence continuous input power) at the output of a current (energy) source.

Just as the three basic *voltage-sourced* smps produce a *voltage-sourcing* output (due to output shunt capacitor C_o in figure 20.3a), the three dual *current-sourced* smps in figure 20.3b produce a *current-sourcing* output, because of the series output inductor L_o. In each *current-sourcing* case, bidirectional energy flow is achieved by using parallel diode/switch combinations, just as with the *voltage-sourced* converters, where bipolar capacitor voltages arise in the boost-buck case. In observing power conservation, the voltage transfer function is the inverse of the current transfer function.

The current boost-buck converter in figure 20.3b-*iv* is the generic root of the Cuk converter (better generically called the boost-buck converter), where the current source input in figure 20.3b-*iv* is replaced by an equivalent series voltage source and inductor, as shown in figure 20.3c-*iv*. This same input conversion from a current source input to a voltage source input (with a series inductor) can be applied to the other two current source converters, specifically giving figures 20.3c.

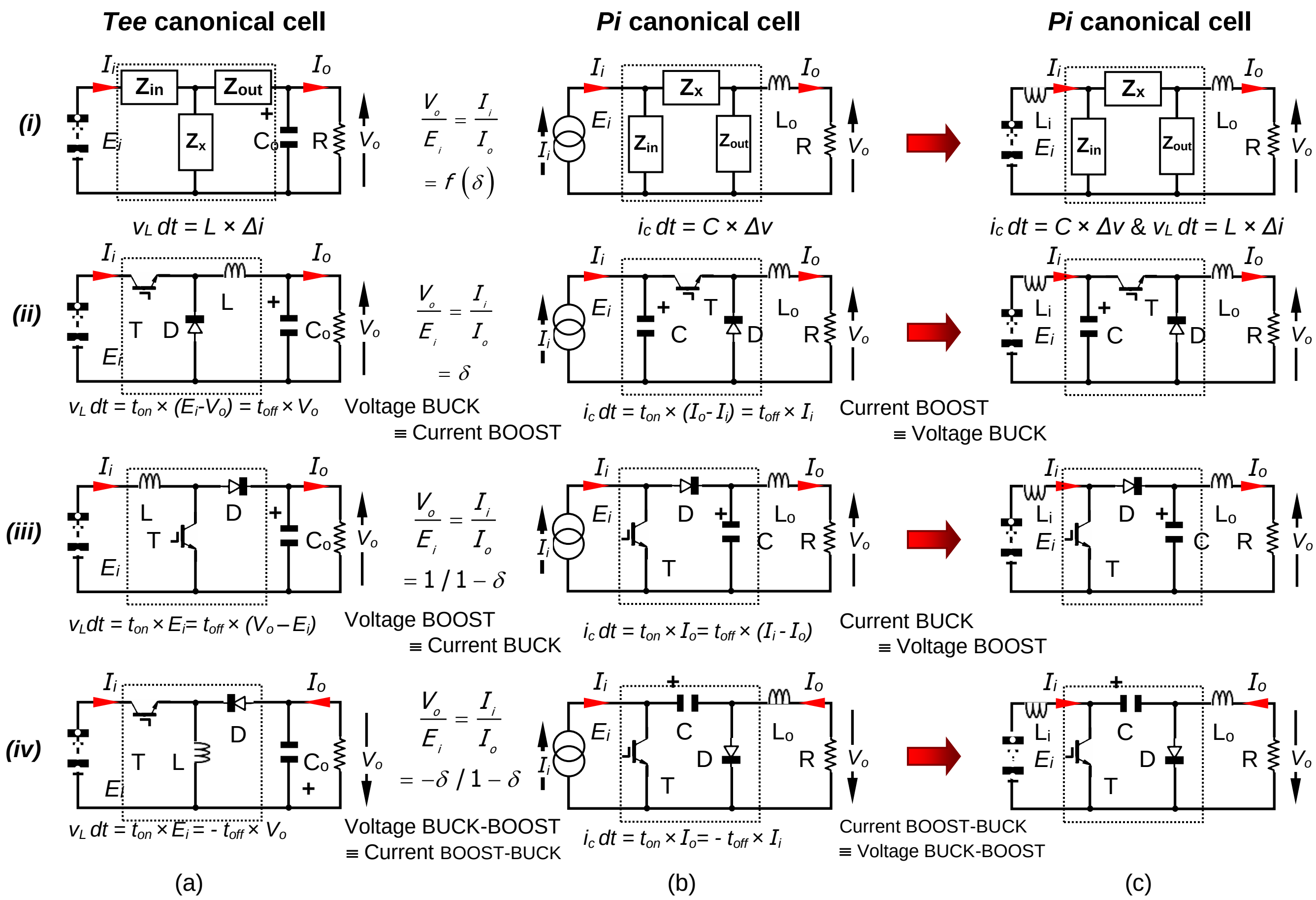


Figure 20.3. *Three basic converters: (a) voltage sources, (b) current sources, and (c) current source converted to equivalent voltage source plus series inductor.*

Table 20.2: Three current-sourced single switch smps

Characteristic	***Current-sourced*** converters Current step-up Buck voltage	Current step-down Boost voltage	Current reversal step up/down
$\Delta V_C =$ $t_T + t_{off} = \tau$	$\frac{I_o - I_i}{C} t_T = \frac{I_i}{C}(\tau - t_T)$	$\frac{I_o}{C} t_T = \frac{I_i - I_o}{C}(\tau - t_T)$	$\frac{I_o}{C} t_T = \frac{-I_i}{C}(\tau - t_T)$
$\frac{I_o}{I_i} = \frac{E_i}{V_o} =$ $0 \le \delta \le 1$ $E_i \bar{I}_i = V_o \bar{I}_o$	$\frac{\tau}{t_T} = \frac{1}{\delta}$	$1 - \delta$	$-\frac{1-\delta}{\delta}$
	$I_o \ge I_i$ thus $V_o \le V_i$	$I_o \le I_i$ thus $V_o \le V_i$	$I_o \le 0$
$\left.\begin{array}{l}\hat{v}_C\\ \check{v}_C\end{array}\right\} = \bar{V}_C \pm ½\Delta V_C$	$= E_i \pm ½\frac{I_i}{C}(\tau - t_T)$ $= I_i\left[\frac{R}{\delta^2} \pm \frac{(1-\delta)\tau}{2C}\right]$	$= \bar{V}_o \pm ½\frac{I_o}{C} t_T$ $= \bar{I}_i(1-\delta)\left[R \pm \frac{\delta\tau}{2C}\right]$	$= \frac{V_o}{\delta} \pm ½\frac{I_o}{C} t_T$ $= I_i\frac{1-\delta}{\delta}\left[\frac{R}{\delta} \pm \frac{\delta\tau}{2C}\right]$
$R_{crit} \ge \frac{V_o}{\bar{I}_o}$ $= \frac{I_i}{I_0} \times \frac{(1-\delta)\delta\tau}{2C}$ when $\check{v}_C = 0$	$\frac{(1-\delta)\delta^2\tau}{2C}$ $= ½\delta^2\frac{\Delta V_C}{I_i}$	$\frac{\delta\tau}{2C}$ $= ½\frac{1}{1-\delta}\frac{\Delta V_C}{I_i}$	$\frac{\delta^2\tau}{2C}$ $= ½\frac{\delta^2}{1-\delta}\frac{\Delta V_C}{I_i}$
input/output voltage	continuous input voltage discontinuous output voltage	discontinuous input voltage continuous output voltage	discontinuous input voltage discontinuous output voltage
current reversibility voltage	$\frac{1}{\delta} \leftrightarrow 1-\delta$ $\delta \leftrightarrow \frac{1}{1-\delta}$	$1-\delta \leftrightarrow \frac{1}{\delta}$ $\frac{1}{1-\delta} \leftrightarrow \delta$	$-\frac{1-\delta}{\delta} \leftrightarrow -\frac{1-\delta}{\delta}$ $-\frac{\delta}{1-\delta} \leftrightarrow -\frac{\delta}{1-\delta}$
apparent load resistance $R_i = \left(\frac{I_o}{I_i}\right)^2 R_o$	$\frac{1}{\delta^2} R_o$	$(1-\delta)^2 R_o$	$\frac{(1-\delta)^2}{\delta^2} R_o$

Figure 20.4 shows that the output current magnitude monotonically decreases as the duty cycle δ increases. If required, monotonic current magnitude increase with increasing δ can be achieved by mapping δ with $1 - \delta$, as considered in section 20.1. The current transfer functions for the current-sourced converters are then the same as for the voltages sourced converter voltage transfer functions.
Nine voltage sourced dc-to-dc converters, offering continuous input and output current, can be found in section 20.6, figure 20.7.

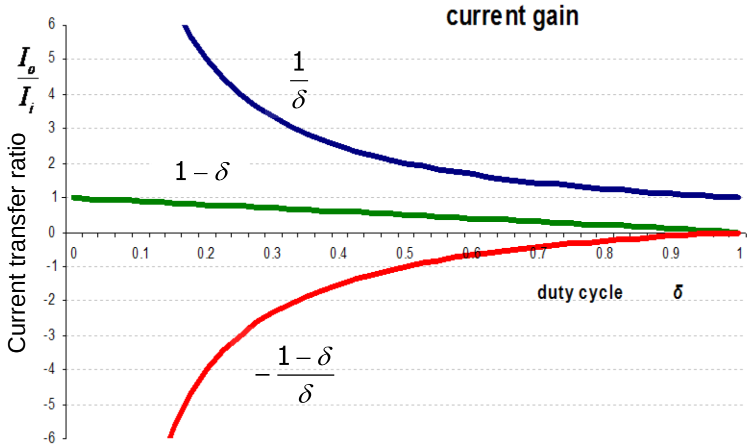


Figure 20.4. *The current transfer functions of three basic current sourced converters in terms of δ.*

20.3 Generic current sourced converters, converted to voltage sourced converters

The inputs of the three basic current source converters in figure 20.3b can be made compatible with voltage sources by interposing inductance as shown in figure 20.3c. The three resulting converters offer continuous input and output current operation due to series inductance on the input and output. This continuous input energy possibility is essential for renewable energy source interfacing, such as with photovoltaic arrays, if large, unreliable electrolytic capacitor decoupling is to be avoided.

The output voltage V_o (and current I_o) of each converter is controlled by varying the switch T on-state duty cycle, $\delta = t_T / \tau$, usually assuming a fixed (voltage or current) energy-source, and continuous energy flow (continuous conduction mode, CCM) in the canonical pi or tee cell reactive component is also assumed.
Analysis of the voltage-sourced circuits in figure 20.3a, in determining their voltage transfer function, V_o / E_i, in a CCM, involves equating the volt-second (Vxt) of the inductor during switch T on-period, t_T, and off-period, $\tau - t_T$. The average inductor voltage $\int v.dt$ is zero if the instantaneous current is the same at the beginning and the end of the integration period τ, even if there is a constant dc bias current through the inductor. The inductor L is referred to as the controlling element in the sense that it controls the ripple current magnitude, hence energy transfer magnitude, whilst the switch T induced inductor voltage duty cycle specifies the energy transfer rate, hence the voltage transfer function, f(δ).

Analysis, I_o / I_i, of the current-sourced converters in figures 20.3b and 20.3c, is centred on the capacitor charge (Ixt) over one cycle, τ, under steady-state switch duty cycle δ and input/output conditions. The average capacitor current $\int i.dt$ is zero if the instantaneous voltage is the same at the beginning and the end of the integration period τ, even if the capacitor holds a dc bias voltage. The capacitor C is referred to as the controlling element since it controls the ripple voltage magnitude, hence energy transfer magnitude; whilst the switch T induced capacitor current duty cycle specifies the energy transfer rate, hence the current transfer function, $1/f(\delta)$.

The result of the mesh analysis is that each voltage-source transfer function has a current source equivalent (a dual) with the identical transfer function, as shown in figure 20.3. In each corresponding case, the voltage transfer function is the reciprocal of the current transfer function, that is $f(\delta) = V_o / V_i = I_i / I_o$, since $E_i \times I_i = V_o \times I_o$, that is, the average power in (from the energy source) is equal to average power out (into the load R). The instantaneous powers need not be equal since the canonical pi or tee cell reactive L/C temporarily and cyclically stores energy during the energy transfer process.

20.4 Thirty-three dc-to-dc voltage source converters

Figure 20.5 tabulates thirty-three dc-to-dc converters, which use one switch, one diode and no more than two capacitors and two inductors. Family ′A′ is the three basic converters, namely the buck – A1, boost - A2 and buck-boost – A5 converters considered in Chapter 19. Family ′C′ are the three converters in figure 20.3c, derived from the current source converters in figure 20.3b. Family ′G′ produces transfer functions other than the basic three functions, amongst the converters, the zeta converter, G6, which produces the positive (non-inverting) buck-boost function. Family ′P′ comprises converters that produce the basic three transfer functions with zero average capacitor voltage, as considered in section 20.5. Nine of the thirty-three converters in figure 20.5 offer continuous input and output current (topologies including all of family ′D′). Just as continuous output voltage topologies usually employ a shunt capacitor C_o across the output (as with family ′A′, the basic three converters) continuous output current topologies usually employ a series output inductor, L_o.
If two diode//switch pairs are used, all thirty-three converters are reversible, whilst the converters in series G (other than the zeta converter G6) require the reversible pairs for operation over the full duty cycle range, $0 \le \delta \le 1$, in either direction of power flow.

Topologies in any column are cyclically related by repeatedly flipping (output port becomes the second port) and inversing (input and output ports are interchanged). The mathematical transfer function is also generated by this process (see section 20.10), without recourse to circuit analysis.

By duality, thirty-three current source converter equivalent circuits exist.

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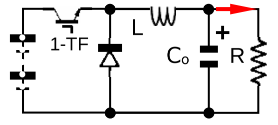
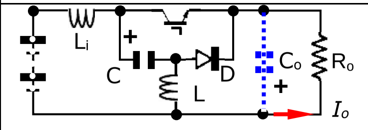
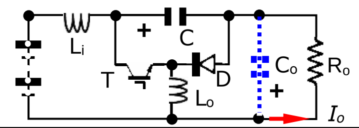
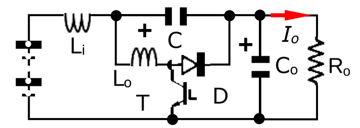
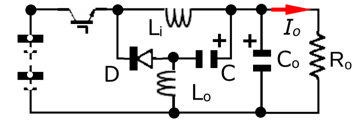
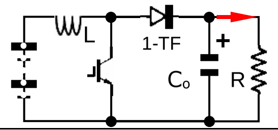
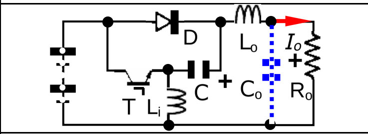
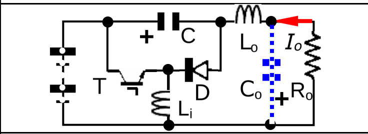
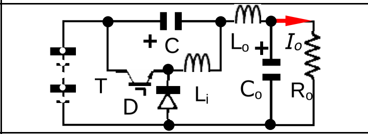
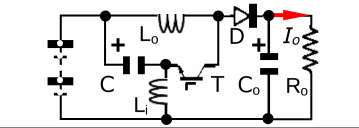
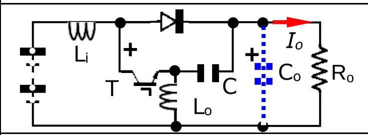
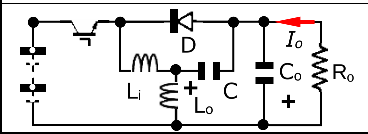
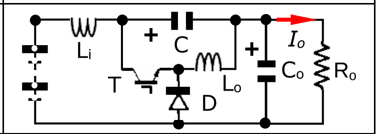
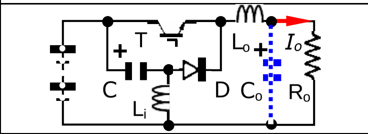
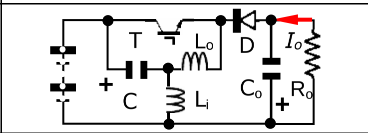
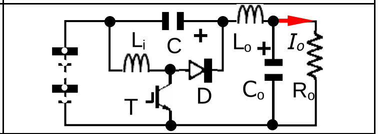
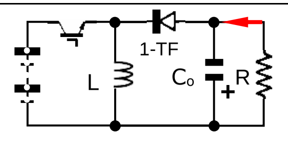
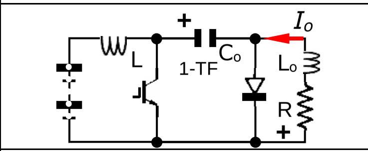
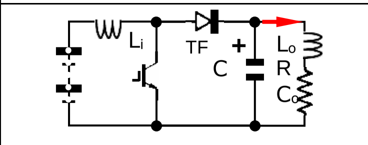
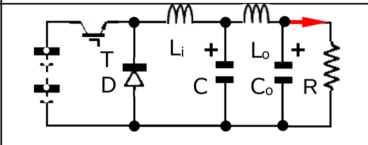
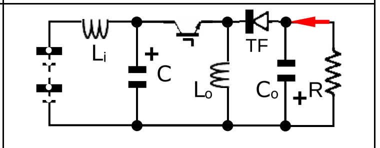
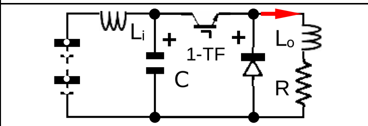
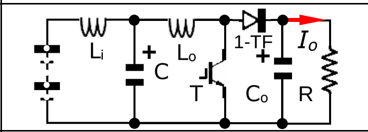
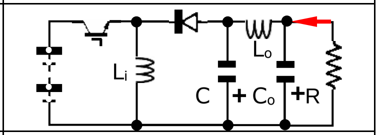
1					
	A1: Buck δ	**C1:** δ	$-\delta/1-\delta$	$1/1-\delta$	δ
2					
	A2: Boost $1/1-\delta$	**C2:** $1/1-\delta$	$-\delta/1-\delta$	δ	$1/1-\delta$
3					
			$1/1-\delta$	$-\delta/1-\delta$	δ
4					
			δ	$-\delta/1-\delta$	$1/1-\delta$
5					
	A5: Buck-Boost $-\delta/1-\delta$	**C5: Cuk** $-\delta/1-\delta$	$1/1-\delta$	δ	$-\delta/1-\delta$
6					
			δ	$1/1-\delta$	$-\delta/1-\delta$ **-ve Luo**

BWW

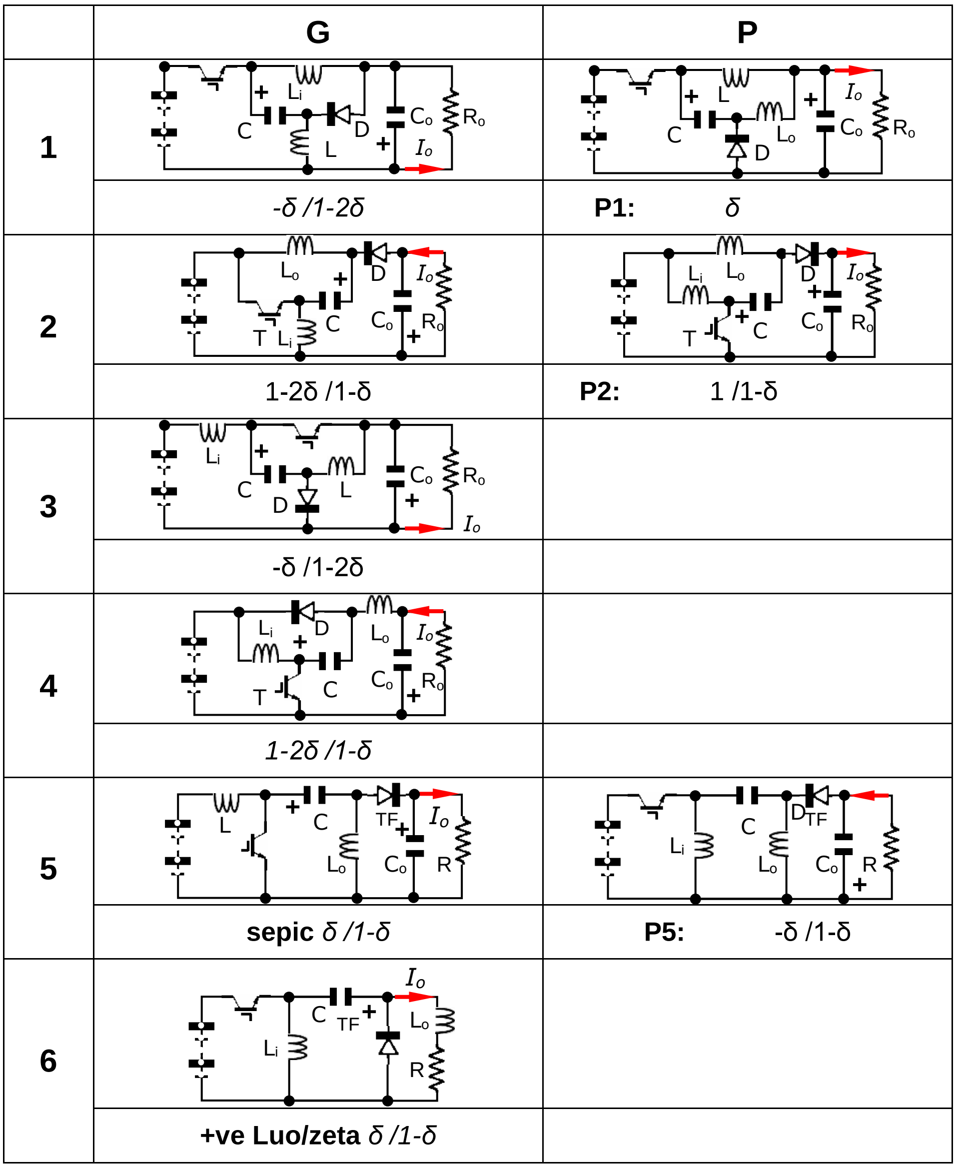


Figure 20.5. *The thirty-three possible single-switch, single-diode dc to dc converters.*

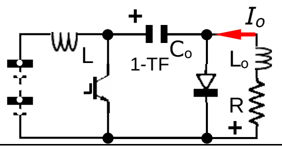
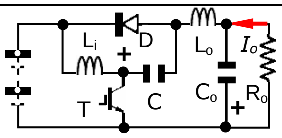
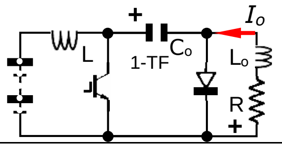
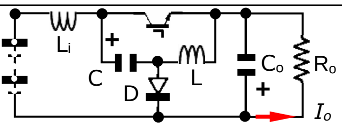
SMPS operate on ac circuit theory properties, specifically the dc input, dc output and zero voltages are at the same ac potential (short circuit all dc voltage sources). Thus specific topology remain operational (with a voltage transfer function change), when component connection to a dc potential is changed to another dc potential, including 0V. This concept is exploited in example 19.4 and can be applied to the C5 Cuk converter, as in example 20.1. The ground connected input switch can be connected to the output to create the G3 converter. If the output diode is connected to the input, the G4 converter is formed. Example 20.1 shows that factors, such as voltage transfer function, reversibility, etc. remain internal consistent.

Example 20.1: ***C5 (Cuk) converter topological conversion to G3 and G4 topologies***

The ground connection of the switches in the C5 converter are reconnected as follows.

- i The output diode ground connection is moved to the input forming a G4 converter
- ii The input switch ground connection is moved to the output forming a G3 converter

Based on the C5 voltage transfer function, establish the G3 and G4 transfer functions

		***Connect o/p diode to input***		
	C5: Cuk *-δ /1-δ*	→	**G4:** *1-2δ /1-δ*	
		***Connect input switch to o/p***		
	C5: Cuk *-δ /1-δ*	→	**G3:** *-δ /1-2δ*	

Solution

The voltage transfer function of the C5 buck-boost converter is

$$\frac{v_o}{E_i} = -\frac{\delta}{1-\delta}$$

i If the output side diode of C5 is connected to the input, then the G4 topology is created.

The C5 transfer function can be rearranged as

$$v_o = -E_i \frac{\delta}{1-\delta}$$

The output diode reconnection to the input E_i, increases the output voltage by the input voltage. That is

$$v_o = E_i \frac{-\delta}{1-\delta} + E_i = E_i\left(\frac{-\delta}{1-\delta}+1\right) = E_i\left(\frac{1-2\delta}{1-\delta}\right)$$

That is

$$\frac{v_o}{E_i} = \frac{1-2\delta}{1-\delta}$$

which the G4 converter transfer function. The output inductor in both topologies filters the discontinuous voltage at its input and creates an output current source.

ii If the input switch of C5 is connected to the output, then the G3 topology is created.

The C5 transfer function can be rearranged as

$$E_i = v_o \frac{1-\delta}{-\delta}$$

The input side switch reconnection to the output v_o, increases the input voltage by the output voltage, viz.

$$E_i = v_o \frac{1-\delta}{-\delta} + v_o = v_o\left(\frac{1-\delta}{-\delta}+1\right) = v_o\left(\frac{1-2\delta}{-\delta}\right)$$

That is

$$\frac{v_o}{E_i} = \frac{-\delta}{1-2\delta}$$

which the G3 converter transfer function. The input inductor in both topologies filters the discontinuous voltage at the inductor's output and creates an input current source.

♣

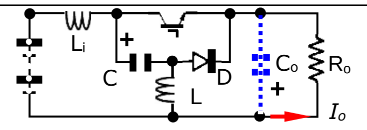
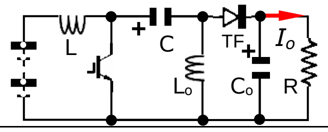
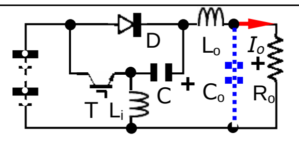
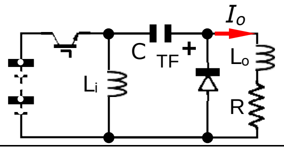
Example 20.1 illustrates two cases where the port dc voltage is additive. But the port voltage can also be subtractive, depending on the relative polarity of the input and output voltage and when reversing the connection changing process. Example 20.2 illustrates the zeta G6 and sepic G5 converters reconfigured to create converters C1 and C2.

Example 20.2: ***C1 and C2 converter topological conversion to G5 and G6 topologies***

The switch and diode connection in the C1 and C2 converters are reconnected as follows.

i C1 converter:- the switch terminal connected to the output is grounded to form G5
ii C2 converter:- the diode anode connected to the input is grounded, forming a G6 converter

Based on the C1 and C2 voltage transfer function, establish the G5 and G6 transfer functions, respectively.

		***Connect switch to ground***		
	C1: δ	→	**G5: sepic** $\delta/1\text{-}\delta$	
		***Connect diode to ground***		
	C2: $1/1\text{-}\delta$	→	**G6: zeta** $\delta/1\text{-}\delta$	

Solution

The voltage transfer function of the C1 converter is

$$\frac{v_o}{E_i} = \delta$$

i If the switch in C1 is connected to ground, then the G5 sepic topology is created.

The C1 transfer function can be rearranged as

$$E_i = v_o \frac{1}{\delta}$$

The switch reconnection means the input is no longer added to the output, that is, it is subtracted:

$$E_i = v_o \frac{1}{\delta} - v_o = v_o \left(\frac{1}{\delta} - 1 \right) = v_o \left(\frac{1-\delta}{\delta} \right)$$

That is

$$\frac{v_o}{E_i} = \frac{\delta}{1-\delta}$$

which the G5 sepic converter transfer function.

ii If the diode of C2 is connected to ground, then the G6 zeta topology is created.

The C2 transfer function can be rearranged as

$$v_o = E_i \frac{\delta}{1-\delta}$$

With the diode grounded, the input component no longer opposes the output voltage. That is

$$v_o = E_i \frac{1}{1-\delta} - E_i = E_i \left(\frac{1}{1-\delta} - 1 \right) = E_i \left(\frac{\delta}{1-\delta} \right)$$

That is

$$\frac{v_o}{E_i} = \frac{\delta}{1-\delta}$$

which the G6 zeta converter transfer function.

♣

20.5 Converters with zero average capacitor voltage

The last family of three dc-to-dc converter topologies in figure 20.6, (family 'P' in figure 20.5), with buck, boost, and buck-boost voltage transfer functions, offer operation with zero average voltage across the controlling capacitor C. These three single-switch, single-diode, converters offer the same features as basic dc-to-dc converters, such as the buck function with continuous output current and the boost function with continuous input current, but both with zero average capacitor voltage.
Figure 20.6 shows the two switch states, namely the current loops when the switch T is alternating on and off. Transfer function analysis is based on the capacitor C voltage ripple *Δv_C*, assuming constant current in the two circuit inductors L_i and L_o.
In each case, the average capacitor voltage is zero, since the capacitor is in a Kirchhoff voltage loop involving two inductors, which each have an average voltage of zero. The average inductor currents are related to the input and output currents. Thus as the load current decreases, the input current ripple decreases, whence the inductor currents decrease. As the energy transfer decreases, the capacitor ripple voltage (energy transfer) decreases, eventual reaching DCM, when the capacitor voltage has zero voltage periods.

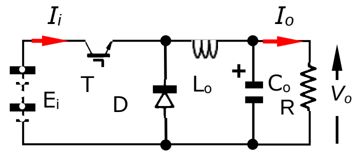
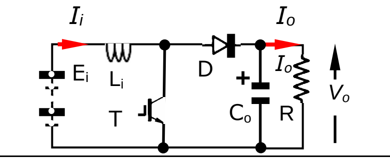
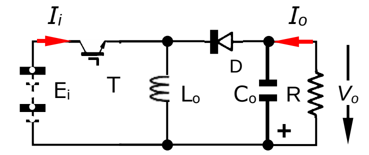
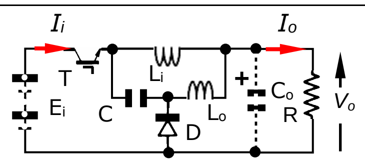
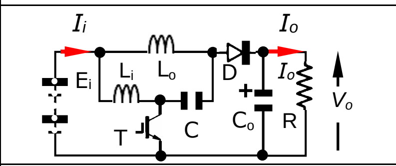
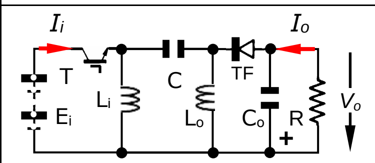
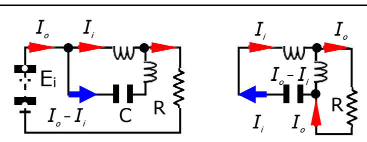
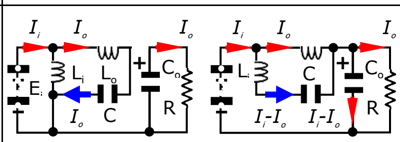
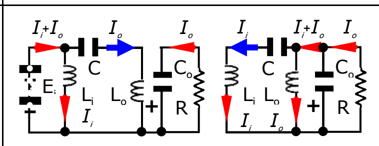
Basic voltage sourced converters				cct **O**
$f_v(\delta)=\frac{1}{f_i(\delta)}$	**A1** (**a**) Voltage BUCK ≡ Current BOOST **P1**	**A2** (**b**) Voltage BOOST ≡ Current BUCK **P2**	**A5** (**c**) Voltage BUCK-BOOST ≡ Current BOOST-BUCK **P5**	cct
New voltage sourced converters				cct **A**
switch T state	switch T ON / switch T OFF	switch T ON / switch T OFF	switch T ON / switch T OFF	
Two operating stages				cct **B**
Loop equations	$C\times\Delta v_C = i_C\,dt = t_{on}\times(I_o - I_i) = t_{off}\times I_i$	$C\times\Delta v_C = i_C\,dt = t_{on}\times I_o = t_{off}\times(I_i - I_o)$	$C\times\Delta v_C = i_C\,dt = t_{on}\times I_o = t_{off}\times I_i$	*eqn 1*
current transfer function, $f_i(\delta)$	$f_i(\delta)=\frac{E_i}{V_o}=\frac{I_o}{I_i}=\frac{1}{\delta}=\frac{1}{f_v(\delta)}$	$f_i(\delta)=\frac{E_i}{V_o}=\frac{I_o}{I_i}=1-\delta=\frac{1}{f_v(\delta)}$	$f_i(\delta)=\frac{E_i}{V_o}=\frac{I_o}{I_i}=-\frac{1-\delta}{\delta}=\frac{1}{f_v(\delta)}$	*eqn 2*
voltage transfer function, $f_v(\delta)$	$f_v(\delta)=\frac{V_o}{E_i}=\frac{I_i}{I_o}=\delta$	$f_v(\delta)=\frac{V_o}{E_i}=\frac{I_i}{I_o}=\frac{1}{1-\delta}$	$f_v(\delta)=\frac{V_o}{E_i}=\frac{I_i}{I_o}=\frac{-\delta}{1-\delta}$	*eqn 3*
	P1 **(a)**	**P2** **(b)**	**P5** **(c)**	

Figure 20.6. *Three dc-to-dc converters with zero average capacitor voltage: topologies, operating stages, and transfer functions.*

Figure 20.6B shows the two states created by operation of the switch T, namely the current loops when the switch T is on, t_{on} and when T is off, t_{off}, (such that $t_{on}+t_{off}=\tau$). Energy transfer (voltage and current transfer function) analysis is based on the capacitor C voltage ripple Δv_c, specifically $C\times\Delta v_c = i_c\,dt$, (eqn 1 in figure 20.6), assuming constant current in the two circuit inductors L_i and L_o (large inductance). The three basic converter transfer functions result, viz., buck, boost, and buck-boost, which are only switch on-state duty cycle δ dependent, as shown by equations 2 and 3 in figure 20.6. The average input inductor L_i current is always equal to the average input current, while the average output inductor L_o current is always equal to the average output current.

Circuit component average voltages and currents are given in Table 20.3. In practice, the average capacitor voltage depends on the voltage difference of the resistive voltage drops associated with the inductor windings. Although the two inductor resistive voltages counter each other, they only cancel to zero at specific conditions, including duty cycle.

This near zero capacitor voltage can be exploited to create a transformer isolated version of the buck-boost converter P5 by splitting the capacitor into series capacitors and inserting a shunt transformer, as is done with the transformer coupled Cuk converter (see section 20.7). But unlike the Cuk transformer coupled version, large dc decoupling capacitors are needed to minimise any core dc bias. The important coupled circuit feature is that energy is transfer by transformer action, with no energy temporarily stored in the core, as is the case with the basic isolated buck-boost topology. Therefore energy transfer is not limited by the magnitude of energy that can be stored in the core volume.

If the capacitance C is large, ac wise, for analysis purposes, the two inductors L_i and L_o are in parallel and each of the three P sequence converters effectively reduce (degenerate) to the corresponding basic converters in sequence A.

Table 20.3: DC-to-dc converter normalized component ratings

			voltage	buck	boost	buck-boost
			Figure 20.6 / cct	A (a)	A (b)	A (c)
			topology	**P1**	**P2**	**P5**
transfer function	Voltage	TF_v	V_o / E_i	δ	$\frac{1}{1-\delta}$	$\frac{-\delta}{1-\delta}$
	Current	TF_i	I_o / I_i	$1/\delta$	$1-\delta$	$\frac{1-\delta}{-\delta}$
Switch T	T (ave)	voltage	V_T / E_i	$1-\delta$	1	1
		current	I_T / I_o	δ^2	δ^2	$\frac{\delta}{1-\delta}$
	T (max)	voltage	V_T / E_i	1	$\frac{1}{1-\delta}$	$\frac{1}{1-\delta}$
		current	I_T / I_o	1	$\frac{1}{1-\delta}$	$\frac{1}{1-\delta}$
Diode D	D (ave)	voltage	V_D / E_i	δ	$\frac{\delta}{1-\delta}$	$\frac{\delta}{1-\delta}$
		current	I_D / I_o	$\delta(1-\delta)$	1	1
	D (max)	voltage	V_D / E_i	1	$\frac{1}{1-\delta}$	$\frac{1}{1-\delta}$
		current	I_D / I_o	1	$\frac{1}{1-\delta}$	$\frac{1}{1-\delta}$
Capacitor C	current	t_{on}	I_C / I_o	$1-\delta$	1	1
		$\tau - t_{on}$	I_C / I_o	δ	δ	$\frac{\delta}{1-\delta}$
	voltage	average	V_C / E_i	0	0	0
		ripple	$C\Delta v_C / \tau I_o$ $C\Delta v_C / \tau I_i$	$\delta(1-\delta)$ $1-\delta$	δ $\delta(1-\delta)$	δ $1-\delta$
Inductor current I_L	average current	L_i	I_{Li} / I_o I_{Li} / I_i	δ 1	$\frac{\delta}{1-\delta}$ δ	$\frac{\delta}{1-\delta}$ 1
		L_o	I_{Lo} / I_o I_{Lo} / I_i	$1-\delta$ $\frac{1-\delta}{\delta}$	1 $1-\delta$	1 $\frac{1-\delta}{\delta}$
	Δ inductor voltages	$V_{Li} - V_{Lo}$	$\frac{I_{Li} - I_{Lo}}{I_i}$	$\frac{2\delta - 1}{\delta}$	$2\delta - 1$	$\frac{2\delta - 1}{\delta}$
	dc losses	$L_i + L_o$	$\frac{I_{Li}^2 + I_{Lo}^2}{I_i^2}$	$\frac{2\delta^2 - 2\delta + 1}{\delta^2}$	$2\delta^2 - 2\delta + 1$	$\frac{2\delta^2 - 2\delta + 1}{\delta^2}$
	ripple current	L_i	$L_i \Delta I_{Li} / \tau E_i$	$\delta(1-\delta)$	δ	δ
		L_o	$L_o \Delta I_{Lo} / \tau E_i$	$\delta(1-\delta)$	δ	$\approx \delta$
input/output ripple current	input	I_i	$L_i \Delta I_i$	Discontinuous $0, I_o$	Continuous $2\delta\tau E_i$	Discontinuous $0, I_o/(1-\delta)$
	output	into C_o // R	$L_o \Delta I_o$	Continuous $2(1-\delta)\tau V_o$	Discontinuous $0, I_o/(1-\delta)$	Discontinuous $0, I_o/(1-\delta)$

20.6 Converters with continuous input and output current (continuous power)

Of the three basic voltage-sourced dc-to-dc converters (buck A1, boost A2, and buck-boost A5) shown in the three parts of figure 20.7A, only the boost converter, figure 20.7A-b, offers continuous input current, but with a voltage sourcing output. Many renewable interfacing stages employ this topology or an energy conversion stage based on its functionality concept, or alternatively resort to a converter large input filtering stage. The three basic voltage-sourced converters (A1, A2, and A5) in figure 20.7A have current sourced duals, as shown in figure 20.7B. Both converter source types (current and voltage) have the same transfer function, relating the input voltage E_i and current I_i to the output voltage V_o and current I_o, based on a lossless converter, that is, power conservation gives $E_i \times I_i = V_o \times I_o$. In the dc voltage source case, the input current for energy conservation is the average input current, whilst in the dc current source case, the input voltage for energy conservation is the average input voltage.

Analysis to derive the current transfer function, $f_i(\delta) = I_o / I_i$, of the current-sourced converters in figure 20.7B, is based on the capacitor C charge $I \times t$ over one cycle, τ, under steady-state switch on-state duty cycle, δ, and input/output conditions. The average capacitor current $\int i.dt$ is zero if the instantaneous voltage is the same at the beginning and the end of the integration period τ, even if the capacitor holds a dc bias voltage. The switch T and diode D on/off energy transfer paths (whence transfer function) are summarized in the three parts of figure 20.7C. The capacitor C is referred to as the controlling element in the sense that its ripple voltage magnitude Δv_c, controls the energy transfer magnitude; whilst the switch T induced capacitor current duty cycle specifies the energy transfer rate, hence current (whence voltage) transfer function. Because the transfer function is based on dc circuit conditions and the energy transfer is an ac function, any of or combination of the reactive components, the input inductance L_i, the output inductance L_o, and C, can be controlling elements. That is, irrespective of the inductor ac voltage wave shape, in a steady-state inductor CCM, the dc current at the beginning of a cycle period is the same as at the end of the cycle. The same applies to a capacitor, but with voltage and current interchanged. This means any reactive component can be used to track any desired input current reference or profile.

Continuous current alone from an energy source is insufficient to ensure continuous power from that source. Short-circuiting a current source means zero delivered power, although the source remains functional current wise. Depending on the topology, an electromagnetic alternative is to temporarily stored the source energy in a shunt capacitor (for a current source) or a series inductor (in the case of a voltage source). A PV cell exhibits both current and voltage source property, in which case either or both reactive component types can be used to temporarily store energy.

The controlling element, that is the element experiencing the ripple associated with ac energy being transferred from the input to the output, is the capacitor C. The capacitor voltage ripple Δv_c, is given by eqn 1 in figure 20.7 and is dependent on the load current magnitude. But if the input inductance L_i is low such that it experiences a significant current ripple, then the transfer function becomes dependent on both L_i and C, whence the input current can vary so as to track the current profile necessary to follow the energy source operational maximum power point. One of the two energy transfer stages (T on, period t_{on} or T off, period $\tau - t_{on}$, with CCM) in figure 20.7C remains dependent only on C, while the other stage will be dependent on both L_i and C and their energy variations. Both controlling elements produce the same transfer function, so both stably contribute to the energy being transferred at the same transfer function level. Alternatively, if C is increased to experience minimal ripple, operational properties are dependent on L_i, as in basic voltage source dc-to-dc converter analysis. Any input current profile can be tracked, with a response related to the inverse of the input inductance, L_i.

Although circuit topologies E in figure 20.7 have no inductor at the input, as with the other topologies in figure 20.7, the input current is continuous. This transferable continuous current property is readily understood from Kirchhoff's current law. In figure 20.8, the sum of the three currents at the canonical cell terminals x, y, and z must be zero. Therefore if the currents at any two terminals are continuous, then the current must be continuous (or continuously zero) at the third terminal. All the topologies in figure 20.7 have inductance at two of the three terminal, hence current can be continuous at the third terminal.

The topologies in figure 20.7E (and figure 20.7F) indicate an output voltage sourcing mode by the addition of the output filter capacitor C_o. For a current-source output, the presence of L_o allows C_o to be removed. Similarly, adding the same load shunt capacitance C_o across R in the topologies in figure 20.7D, converts the current sourcing output into a voltage sourcing output. Current sourcing outputs are amenable to stable parallel connection, while voltage sourcing outputs are applicable to stable series connection.

20.6.1 Converter component ratings

Based on the transfer function $V_o/E_i = I_i/I_o$, component average voltages and currents can be normalized with respect to the input voltage E_i and output current, I_o, (or V_o and $= I_i$) as appropriate. The average inductor voltage and average capacitor current are both zero in steady state, so in conjunction with Kirchhoff's laws, the various component dc voltages and currents in tables 20.4 and 20.5 can be expressed in terms of the voltage transfer function, and more generally in terms of the switch T on-state duty cycle, δ.

For a given transfer function, independent of topology, the switch and diode average voltages and currents are the same, as are the capacitor dc and ripple voltages, provided the inductor ripple currents are minimal. Given the capacitor ripple voltage is the same, as is the dc offset, the maximum load resistance, specifically critical resistance, is the same for a given transfer function independent of topology, as summarized in Table 20.4. Transposition of the inductors in the different topologies, for a given transfer function, results in different inductor average currents (and ripple), as shown in Table 20.5. When capacitor voltage ripple dominates (large inductances for L_i, and L_o), the component peak currents are similar to their average currents but the peak voltages can significantly differ from component average voltages. This is particularly true for low or high duty cycles. The peak inductor current being similar to the average current implies minimal input and output current ripple.

The converse is applicable when inductor ripple current dominates, with large C hence minimal ripple voltage. The capacitor peak voltages are similar to their average voltages but the peak currents can significantly differ from capacitor average currents within the switch T on and off periods.

Each converter design is the same as for its generic converter since each topology with the same transfer function, has the same component ratings, stresses, and maximum resistance for CCM, as shown in Table 20.4. The inductors are dimensioned to produce the required ripple currents and/or I^2R losses, according to Table 20.5.

The performance factors of efficiency, voltage and current regulation, and current ripple are strongly related to converter transfer function. With reference to tables 20.4 and 20.5:

i. *Efficiency*: Boost converters have the highest efficiency because for a given power they have the lowest output current hence lowest diode on-state voltage and copper I^2R losses, whilst the buck converter has the highest output current whence the highest diode conduction voltage and I^2R losses. Converters where inductor current comprises the input and output current have the highest I^2R losses, hence lowest efficiency.

The copper losses of converter pairs (D5, D2), (C2, C5), and pair (D3, D1) would be expected to be the same because they are the same converter, but with power drawn from the other port. That is, the I^2R loss function of D5 is the same as D2 ($\left(2-\delta^2\right)/\left(1-\delta^2\right)$ from Table 20.4 since they are the same converter but with power drawn from complementary output ports P_o and P_1. On this basis the overall efficiency of the converter pairs would be expected to be the same.

ii. *Voltage and current regulation*: Similarly for output voltage regulation; the buck converter has the lowest output voltage (and increased output current), whence the diode voltage which is the largest of the three basic converters, represents a more significant proportion of the output voltage. Hence buck converters should employ silicon Schottky diodes. Mitigating the poor output voltage regulation of the buck converter, it has the best current regulation, with the boost converter having the poorest current regulation at low current. That is, in open loop, the buck converter is best for accurately tracking the current transfer ratio, (with an efficiency sacrifice), which is a fundamental control aspect of current source input and output converters. Generally good current regulation is expected because the current transfer function is independent of input and output voltages, which are effectively reduced due to series component voltage drops, thus affect voltage regulation.

iii. *Ripple current*: Also because of the higher circuit voltages, the boost converter tends to have the highest ripple current, whilst the buck converters have the lowest voltages, hence the lowest ripple currents. Although converters where the two inductors are effectively in series with the output or input tend to have a lower ripple current. Constant input ripple current, for a given δ, occurs when an inductor experiences only the input voltage, otherwise ripple current is regulation and/or capacitor voltage ripple dependent. Converters with significant ripple current increase at higher currents, have a capacitor voltage swing which tends to become exponentially shaped, which affects open loop regulation.

Generally the boost converter is used when continuous source current is required, without the use of energy source decoupling capacitance. It is therefore commonly interfaced to renewable sources. The disadvantage of the boost converter is that the controlled output voltage is always greater than the input voltage. The presented buck-boost converters offer not only continuous input and output current but controlled voltage down to zero, but with voltage polarity inversion. Nevertheless the buck-boost topologies offer an alternative first stage for PV and fuel cell applications. The buck converters, in a reversible form, may be viable in battery dc link backup systems of grid connected PV and fuel cell applications.

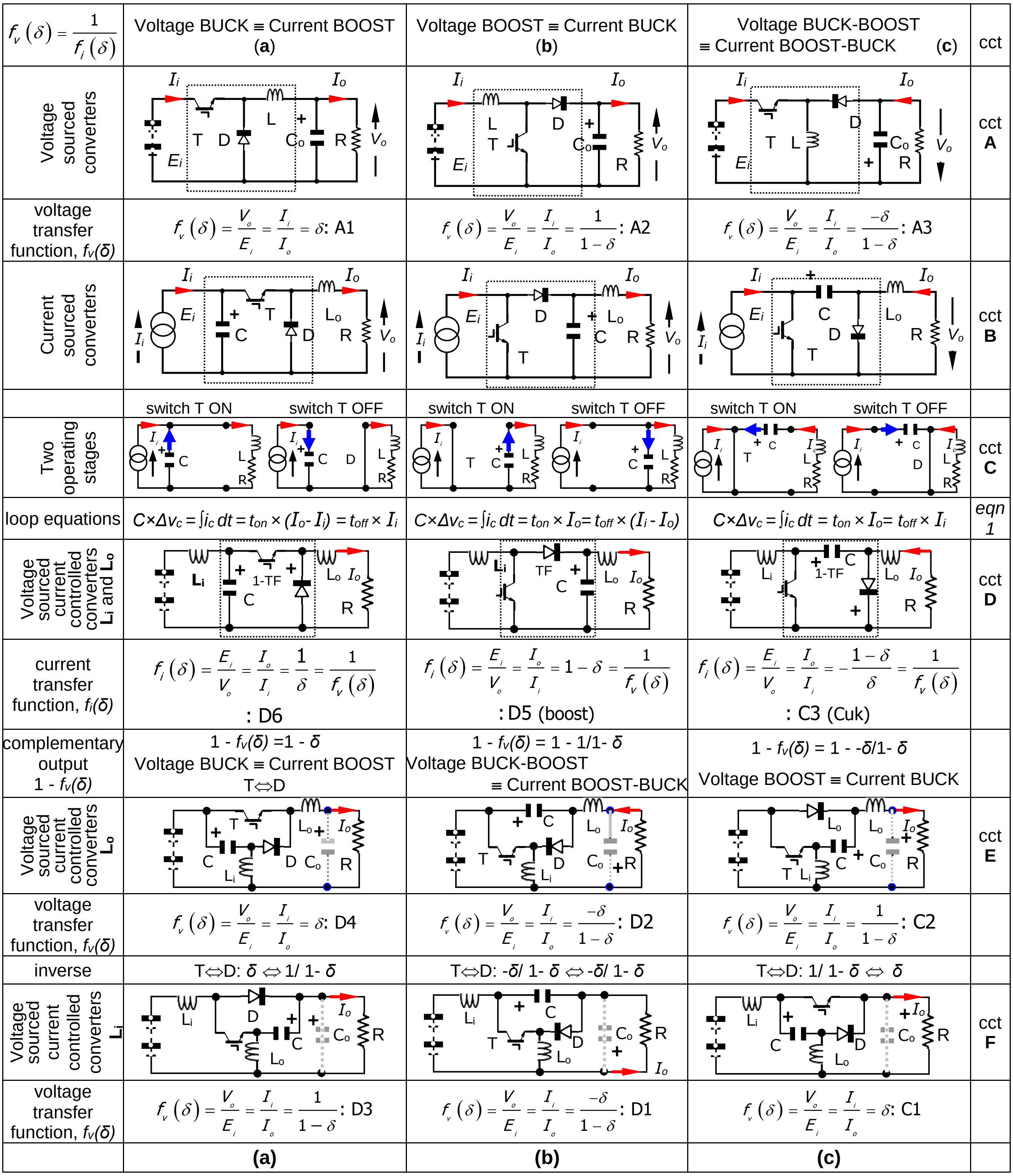


Figure 20.7. *DC-to-dc current-sourced and voltage-sourced topologies, operating stages, and conversions.*

Table 20.4: DC-to-dc converter normalized component ratings

Figure 1 / cct	topology	Voltage transfer function	Critical load resistance	Switch T (ave)		Diode D (ave)		Capacitor C			
								current		voltage	
		TFv	R_c	voltage	current	voltage	current	t_{on}	$T - t_{on}$	average	ripple
		V_o / E_i	$\tau R_c / 2L\text{TF}_v$	V_T / E_i	I_T / I_o	V_D / E_i	I_D / I_o	I_C / I_o	I_C / I_o	V_C / E_i	$C\Delta v_C / \tau I_o$
D-a E-a F-c	D6 D4 C1	δ	δ(1-δ)	$1-\delta$	δ	δ	$1-\delta$	$1-\delta$	δ	1	$\delta(1-\delta)$
D-b E-c F-a	D5 C2 D3	$\frac{1}{1-\delta}$		1	$\frac{\delta}{1-\delta}$	$\frac{\delta}{1-\delta}$	1	1	$\frac{\delta}{1-\delta}$	$\frac{1}{1-\delta}$	δ
D-c E-b F-b	C5 D2 D1	$\frac{-\delta}{1-\delta}$									

Ratings for boost and buck-boost converters are the same since one topology is the flip of the other, as seen in figure 20.8 (complementary outputs of the same topology).

Table 20.5: DC-to-dc converter normalized inductive component ratings

Figure 1 / cct	topology	Voltage transfer function	Current transfer function	Inductor average current		Total inductor Copper loss	Inductor L_i L_o ripple current		Input/Output ripple current	
		TFv	TFi	L_i	L_o	$L_i + L_o$	ΔI_{Li}	ΔI_{Lo}	ΔI_i	$L_o/R \gg \tau$
		V_o / E_i	I_i / I_o	I_{Li} / I_o	I_{Lo} / I_o	$\left(I_{Li}^2 + I_{Lo}^2\right) / I_o^2$	$L_i\, \Delta I_{Li} / \tau$	$L_o\, \Delta I_{Lo} / \tau$	$L_i\, \Delta I_i / \tau$	$L_o \Delta I_o / \tau$
Cct D-a	D6	δ	$1/\delta$	δ	1	$\delta^2 + 1$	$½(1-\delta)\Delta V_c$	$(1-\delta)V_o$	$½(1-\delta)\Delta V_c$	$(1-\delta)\ V_o$
Cct E-a	D4			$1-\delta$	1	$\delta^2 - 2\delta + 2$	$½(1-\delta)\Delta V_c$	$(1-\delta)V_o$	$\delta(1-\delta)E_i$	$(1-\delta)V_o$
Cct F-c	C1			δ	$1-\delta$	$\delta^2 - 2\delta + 1$	$\delta(1-\delta)E_i$	$(1-\delta)V_o$	$\delta(1-\delta)E_i$	$\delta\delta'\ \Sigma 1/L$
Cct D-b	D5	$\frac{1}{1-\delta}$	$1-\delta$	$\frac{1}{1-\delta}$	1	$\left(2-\delta^2\right) / \left(1-\delta^2\right)$	δE_i	$¼(1-\delta)\Delta V_c$	δE_i	$¼(1-\delta)\Delta V_c$
Cct E-c	C2			$\frac{\delta}{1-\delta}$	1	$1 / \left(1-\delta^2\right)$	δE_i	$(1-\delta)V_o$	$\delta\ \Sigma 1/L$	$(1-\delta)V_o$
Cct F-a	D3			$\frac{1}{1-\delta}$	$\frac{\delta}{1-\delta}$	$\left(1+\delta^2\right) / \left(1-\delta^2\right)$	δE_i	$¼(1-\delta)\Delta V_c$	δE_i	$(1-\delta)V_o$
Cct D-c	C5	$\frac{-\delta}{1-\delta}$	$\frac{1-\delta}{-\delta}$	$\frac{\delta}{1-\delta}$	1	$1 / \left(1-\delta^2\right)$	δE_i	$(1-\delta)\ V_o$	δE_i	$(1-\delta)V_o$
Cct E-b	D2			$\frac{1}{1-\delta}$	1	$\left(2-\delta^2\right) / \left(1-\delta^2\right)$	δE_i			
Cct F-b	D1			$\frac{\delta}{1-\delta}$	$\frac{1}{1-\delta}$	$\left(1+\delta^2\right) / \left(1-\delta^2\right)$	$\delta E_i\, \Sigma L$	$(1-\delta)\ V_o$	$\delta E_i\, \Sigma L$	$(1-\delta)\ V_o$

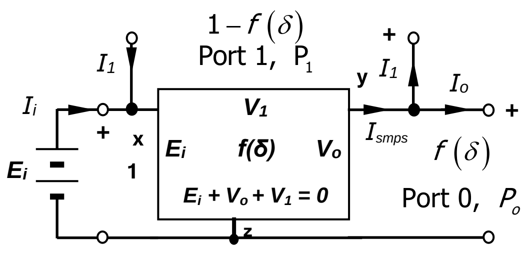
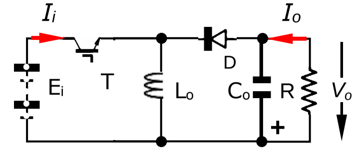
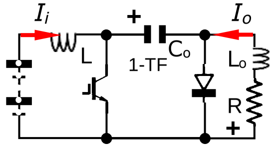
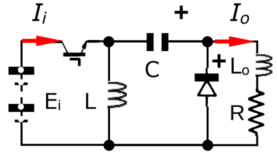
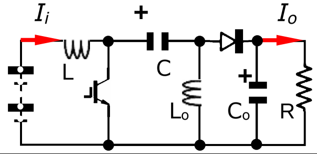
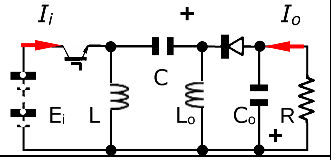
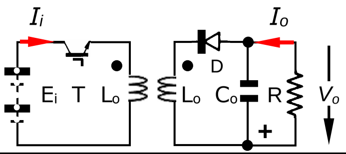
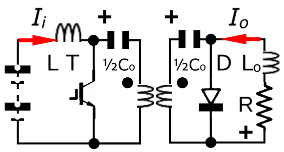
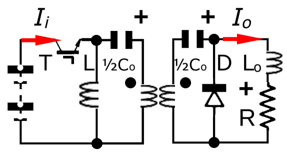
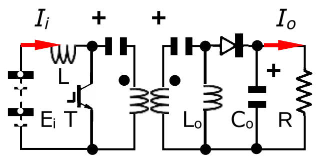
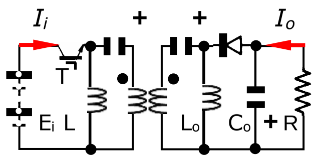


Figure 20.8. *Basic converter functional block shown as a three-port block with two outputs P_o and P_1 shown.*

Table 20.6: Transformer isolated buck-boost dc-dc converters

voltage sourced converters					
Classification name voltage transfer function $f_v(\delta)$	**A5** BUCK-BOOST (**a**) $\frac{-\delta}{1-\delta}$	**C5** Cuk (**b**) $\frac{-\delta}{1-\delta}$	**G6** ZETA (**c**) $\frac{+\delta}{1-\delta}$	**G5** SEPIC (**d**) $\frac{+\delta}{1-\delta}$	**P5** NEW (**e**) $\frac{-\delta}{1-\delta}$
features	Discontinuous input current voltage source output	Continuous input and output current	Discontinuous input current Continuous output current	Continuous input current voltage source output	Discontinuous input current voltage source output
Magnetic coupling					
Coupling mechanism	*Magnetic storage coupling*	*Transformer coupling*	*Transformer coupling*	*Transformer coupling*	*Transformer coupling*
Average capacitor voltage	NA	$E_i + \lvert V_o \rvert$	V_o	E_i	0
Primary dc bias	δI_i	E_i	0	E_i	0
Secondary dc bias		V_o	V_o	**0**	**0**

20.7 Transformer isolated buck-boost dc-dc converters

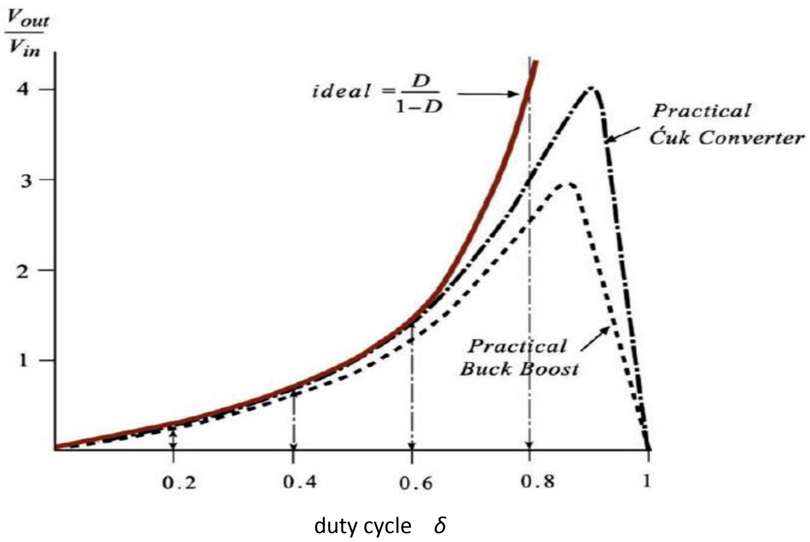


Figure 20.9. *Gain loss at high duty cycles due to circuit parasitic resistance/loss elements*

Converter gain factors above 4 to 5 for boost and buck boost converters are practically challenging, due to circuit loss elements, as illustrated in figure 20.9. Converter transformer coupling for voltage matching alleviates this problem, particularly for buck boost type dc to dc converters.
The basic buck-boost converter A5 can be readily isolated via a coupled magnetic circuit as shown in Chapter 19.8, figure 19.22 and Table 20.6. Additional features to isolation are voltage matching and better semiconductor utilization, but the limitation is energy is temporarily stored in the coupled circuit core. Thus for a given magnetic material, maximum energy transfer is limited by core volume, viz. ½*BH*×Volume. This is the case not only for the buck-boost converter but also the inductor coupled versions of the sepic and zeta converters, as in figure 20.10. All smps inductive components must be able to operate with a current (flux) bias, but when coupled windings are used, the windings must be closely coupled, meaning air gap energy storage cannot be exploited.

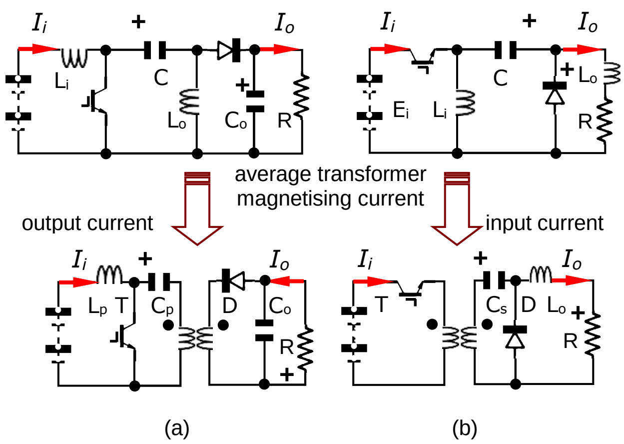


Figure 20.10. *Inductor coupled circuit magnetizing dc bias current of (a) sepic and (b) zeta converters.*

The core volume is utilized differently (better) if magnetic energy transfer is through transformer action rather than intermediate energy storage. If energy is transferred from the source to the load solely via the ripple current through a series capacitor, as with buck-boost converters C5, C6, G5 and P5, then that capacitor can be split so as to facilitate an interposed high magnetizing inductance transformer as shown in figure 20.11 and Table 20.6. A series capacitor blocks dc, so the ac circuit can be transformer coupled as shown in figure 20.11b. The table shows that the Cuk, sepic, zeta, and new buck-boost converters, all with a buck-boost magnitude transfer function, fulfil the series energy transfer capacitor requirement. The transformer acts in a current controlled mode where the voltages and currents adjusts to meet the transformer equation ($I_{in} / I_{out} = V_{out} / V_{in} = N_{out} / N_{in}$) but simultaneously through the input current, the converter current/voltage transfer function ($I_i / I_o = V_o / E_i = |\delta/1\text{-}\delta|$) is enforced since both equations must comply with energy conservation. This operation is not to be confused with the problematic so called 'verge of coupled circuit and transformer operation'. In the Cuk, sepic and zeta cases the series split capacitor pair must fulfil the important function of blocking and supporting a dc component from the magnetic coupling circuit. Table 20.6 shows the dc component each of the split capacitors must block, which is independent of capacitance. Whereas the Cuk converter experiences a dc component on both windings (E_i and V_o), the sepic (E_i) and zeta (V_o) only experience dc voltage on one winding (secondary and primary respectively). The dc component is catered for, blocked, by using large capacitance. The new buck-boost converter P5 develops no dc component on the primary or the secondary, because each is in parallel with inductance, which as for the zeta primary and sepic secondary, has zero average voltage. Large capacitance is therefore not necessary. In practice, any dc voltage bias is modified (increased or decreased) due to component voltage drops, including inductor and transformer winding resistance associated voltages.

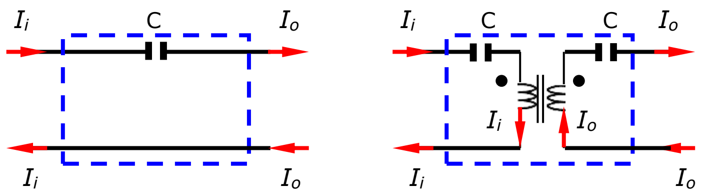


Figure 20.11. *AC equivalent circuits.*

The capacitance technically is not split, not halved or doubled for a 1:1 turns ratio transformer. The capacitor on the secondary emulates the primary capacitor, ac wise, via the transformer.

The energy transferred is independent of the transformer turns ratio and the capacitance magnitude, and is the load power $V_o I_o$ (and input power $E_i I_i$) over the switching cycle period τ, which must equate to change in Since the capacitor on the zero average-voltage-side does not need significant dc blocking capability, the capacitance is dimensioned based on any circuit instantaneous voltage restrictions (as opposed to average voltage values).
In all cases, the capacitance magnitude, in conjunction with the energy to be transferred, determines the ripple voltage magnitude superimposed on the blocked dc voltage. The dc bias on each capacitor is not the same (except for topology P5, where both are zero biased). This is of particular advantage with the Cuk converter, where the input side capacitor is bias by E_i and the output side capacitor voltage bias is V_o. The capacitor voltage stressing in the uncoupled Cuk is associated with a bias voltage of $E_i + |V_o|$.
Because of the isolated coupling, the output voltage can be either polarity, with respect to the input, so all the topologies in Table 20.6 have the same buck-boost transfer function.
With capacitor coupling, each capacitor has a dc bias (a stored electrical energy bias) and operation relies on zero average capacitor current (otherwise the capacitor will over-volt). With inductor coupling, as in figure 20.10, the inductor has a dc bias current (a stored magnetic energy bias) and operation relies on zero average inductor voltage (otherwise the core will saturate). Without coupling the inductor can utilise an optimised air gap to store energy, but if used in a coupled mode, any air gap adversely decreases the coupling factor and increases leakage.
Observe that transformer action involves a series capacitor transferring energy between the converter input and output, while inductor coupling action involves modification to a shunt inductor. Figure 20.12 shows two buck-boost topologies (from figure 20.7, topologies D1 and D2) that fulfil both coupling concepts, viz. series capacitor and shunt inductor, without any other path between the input and output.

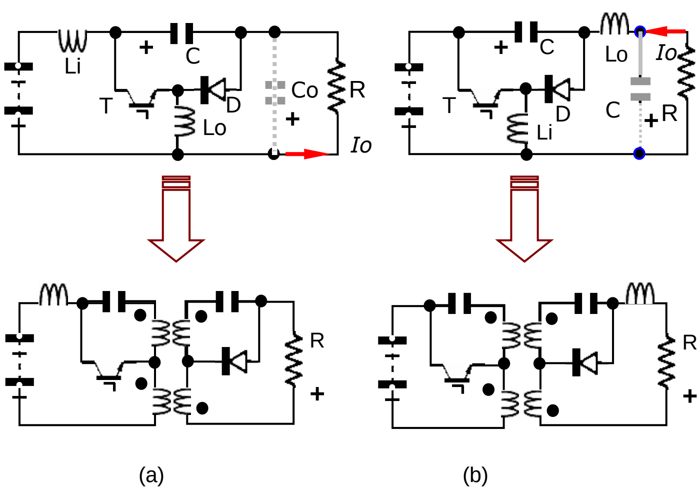


Figure 20.12. *Inductor and transformer coupling (a) D1 and (b) D2 converters*.

Like the Cuk buck-boost converter, these two buck-boost converters have continuous input and output currents. The only redeeming feature of these two complex topologies, over the same function with an isolated Cuk converter, is that the input or output current ripple is less than with the same components in a Cuk converter. The core operates in both transformer and storage coupled circuit modes.
This coupling circuit concept for isolation, can be applied to some of the tapped inductor topologies in section 20.10, specifically (like T3+ and T3-), buck-boost topologies S3± and S3-.

20.8 Capacitor ripple voltage

Output capacitor ripple voltage for the basic converters was considered in chapter 19. Two main components that specify the ripple are the output capacitor equivalent series resistance voltage and the voltage change due to charging and discharging within the switching period, where discontinuous conduction increases the ripple.
The charge change, whence voltage change in a capacitor is given by

$$\Delta v_c = \frac{\Delta Q}{C} = \frac{\int i \Delta t}{C}$$

Generally two capacitor charge state regimes occur depending on if the capacitor provides the full load current for any period, or not.

Case I
Figure 20.13a shows the typical capacitor charge regime for the buck, forward, Cuk, and zeta converters. Analysis is based on charge balance, that is, the output capacitor operates with a steady state dc voltage, with a constant load condition. Thus the hatched area (charge) above the average output current $I_o = V_o / R$ must equal the (charge) area below I_o.

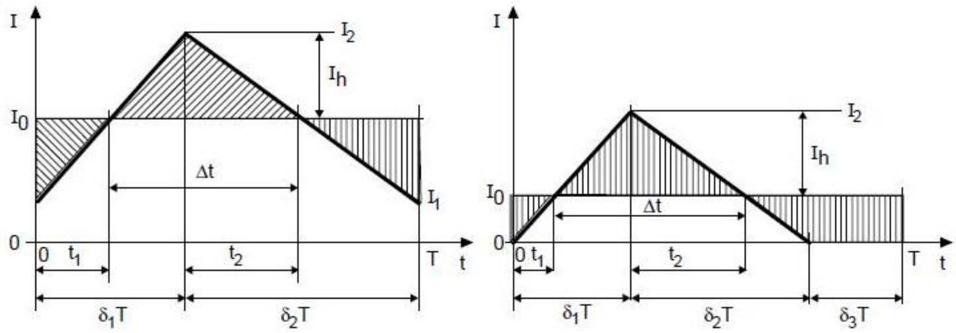


Figure 20.13. *Case I capacitor current waveforms.*

For continuous conduction

$$\Delta Q = \tfrac{1}{4}\tau \left(I_o - I_1 \right) \tag{20.1}$$

while for discontinuous conduction

$$\Delta Q = \tfrac{1}{4}\tau I_o \left(2 - \delta - \delta_2 \right) \tag{20.2}$$

where $I_o = \tfrac{1}{2}(\delta_1 + \delta_2)(I_1 + I_2)$

Case II
Figure 20.14 shows the four typical capacitor charge regime for the boost, buck-boost and sepic converters, where output current only flows from the capacitor when the diode does not conducts.

For continuous conduction $I_1 \geq I_o$ (a and b when $I_1 = I_o = V_o / R$)

$$\Delta Q = I_o \delta \tau \qquad \Delta V = \frac{1}{C}\frac{V_o}{R}\delta\tau \tag{20.3}$$

On the conduction boundary in (c), $\delta_1 + \delta_2 = 1$ and $I_1 = 0$

$$\Delta Q = \tfrac{1}{2} t_2 \left(I_o - I_1 \right) \text{ where } t_2 = \delta_2 \tau \left(1 - \tfrac{1}{2}\delta_2 \right) \tag{20.4}$$

while for discontinuous conduction figure 20.14d, $\delta_1 + \delta_2 < 1$

$$\Delta Q = \tfrac{1}{4} \frac{\tau I_o}{\left(2 - \delta_2 \right)^2} \tag{20.5}$$

where the average output current is $I_o = \tfrac{1}{2}\delta_2 \left(I_1 + I_2 \right) = V_o / R$

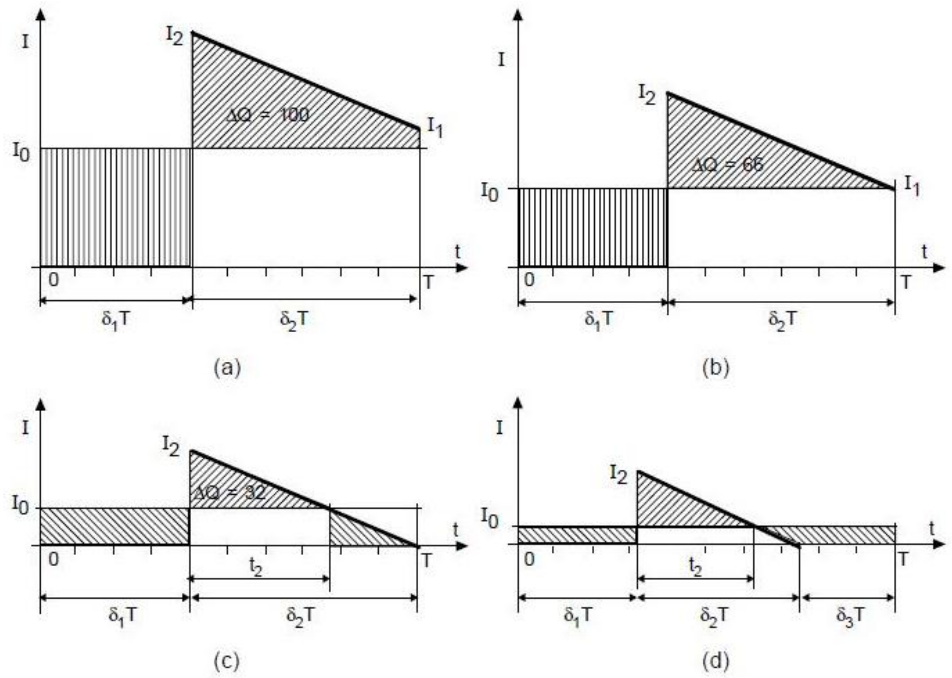


Figure 20.14. *Case II capacitor current waveforms.*

20.9 Current-Doubler Rectifier

Section 19.9 considered transformer isolated smps with full wave rectifier output stages involving a centre tapped transformer secondary, as shown in figure 20.15.

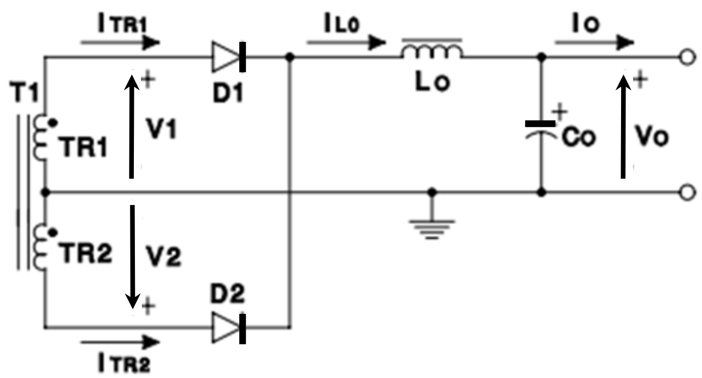


Figure 20.15. *Transformer centre-tapped secondary full-wave rectifier.*

The current-doubler rectifier in figure 20.16a is an alternative rectification method which offers a simple structure and better secondary utilization of the isolation transformers in push-pull, half-bridge and bridge power stages where usually full-wave rectification of bipolar phase shifted rectangular voltage waveforms are produced on the transformer secondary side. The current doubler rectifier is composed of one transformer secondary winding, which is not centre tapped, two rectifier diodes, two identical filter inductors and an output capacitor, as shown in figure 20.16a.

Where as with centre-tapped windings each has a 50% duty cycle at half the total secondary voltage, the current doubler winding has double the duty cycle at the total secondary voltage level. The net effect is the current doubler secondary winding carries half the current of the tapped configuration windings.

(a)

(b)

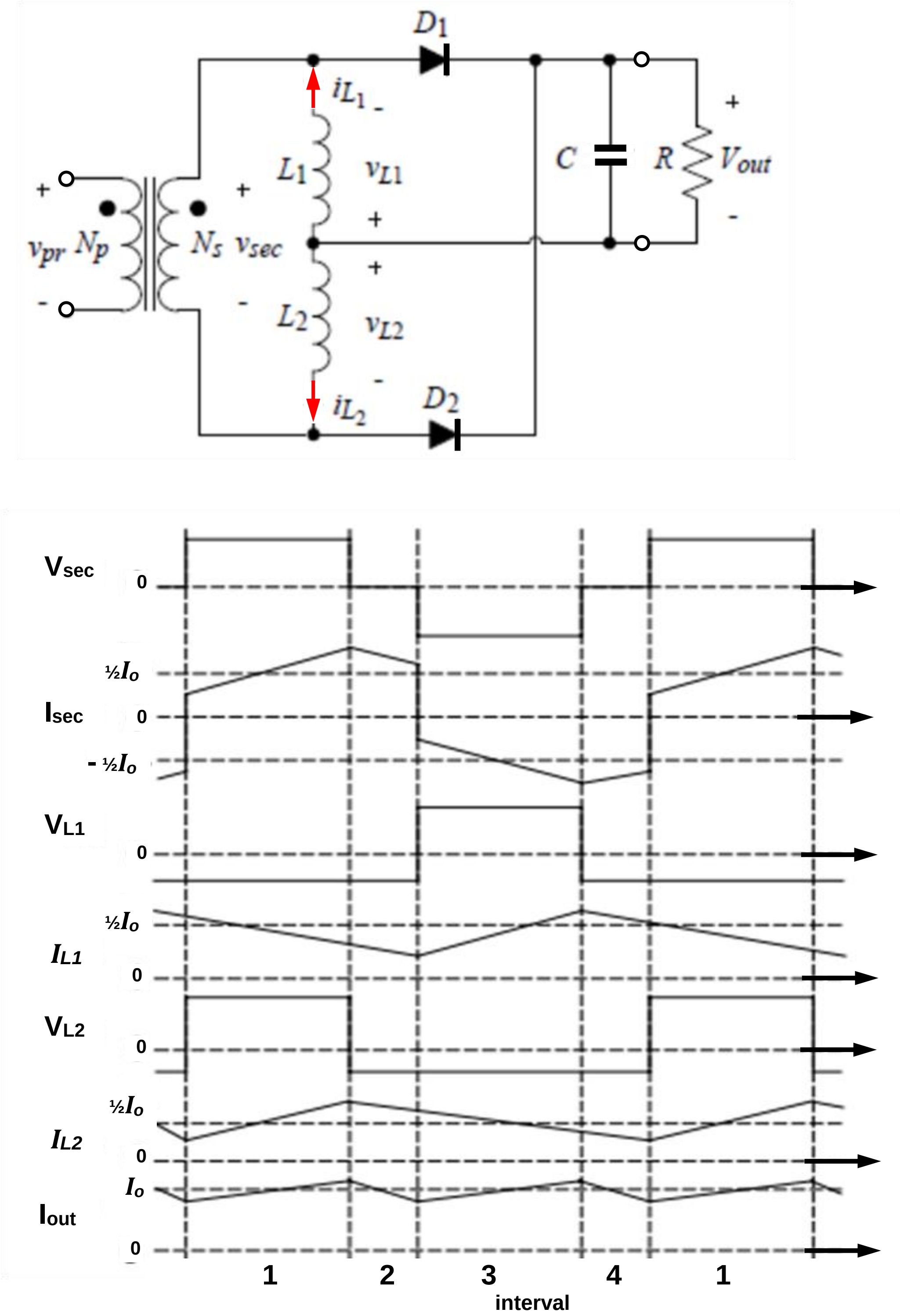


Figure 20.16. *Secondary winding current doubler rectifier: (a) circuit and (b) waveforms.*

With reference to figure 20.16b waveforms, current double operation of figure 20.16a is as follows:

Interval 1:- When the voltage across the transformer secondary winding, V_{SEC} is positive, current flows in the positive direction in both filter inductors, L_1 and L_2, as shown in figure 20.16b. During this period D1 is forward biased while D_2 is biased off by V_{SEC} and the current in L_1 charges the output capacitor C_o through D_1; a path not involving the transformer secondary winding. The current in inductor L_2 flows through the transformer winding and D_1 into the output capacitor. Hence the output current is the sum of the dc components of the two inductor currents, whilst the transformer secondary experiences only half of the load current. During this period, the voltage across L_1 is negative and equal to the output voltage, causing the current in L_1 to decrease. V_{L2} across L_2 is positive, forcing the current in L_2 to increase.

Interval 2:- During the primary circuit freewheeling period, no voltage is induced across the secondary winding, V_{SEC}=0. The voltage across L_2, V_{L2} becomes negative, and equal to the output voltage magnitude, producing a negative slope in the current I_{L2}. Because of transformer leakage, I_{L2} continuous through the secondary, rather than through D_2. The conditions for L_1 do not change.

Interval 3:- With a negative voltage across the transformer output, D_1 turns off while D_2 becomes forward biased. The secondary current rapidly changes direction, being forced to equal the current in L_1. The current in L_2 commutates to D_2, into the output capacitor, and decreases at a rate determined by the inductance and the output voltage. With V_{L1} positive across L_1, current starts building up in L_1.

Interval 4:- With V_{SEC}=0, $-V_o$ is impressed across L_1 causing its current to decrease and there is no change in the condition of L_2.

The characteristics of the current-doubler rectifier are:

- primary side operation, including duty cycle, is unchanged
- no transformer center-tap, so simpler structure – but same total number of secondary turns
- transformer secondary carries approximately half of the output current
- diode and output capacitor stresses are identical to the full-wave technique
- additional filter inductor required
- each filter inductor carries only half the dc output current, hence quarter the I^2R loss/inductor
- ripple currents cancel in output capacitor C_o
- current-mode control to ensure equal inductor currents

20.10 Tapped inductor operation

By tapping the converter inductor, different variations of the basic converters can be realised, as shown in figure 20.17. The tapping point can be referenced with respect to the diode, switch or input/output terminals, with the resultant transfer function shown in Table 20.7, where tap turns ratio is defined by $N=n_1/n_1+n_2$ when the windings are cumulatively connected and $N=n_1/n_1-n_2$ if $n_1<n_2$ when differentially connected. The complete permutation of possibilities is shown in figure 20.18.

The resultant characteristics, with respect to the basic converters, can be summarised as follows.

The switch tap versions give higher conversion magnitudes than the basic converters.
The diode tap versions give lower conversion magnitudes than the basic converters.
The rail tap versions give both higher and lower conversion magnitudes than the basic converters.

Thus it is possible to increase the gain in order to obtain higher output voltage magnitudes. As with any single ended coupled circuit, semiconductor over-voltage, due to leakage stored energy, is a problem.

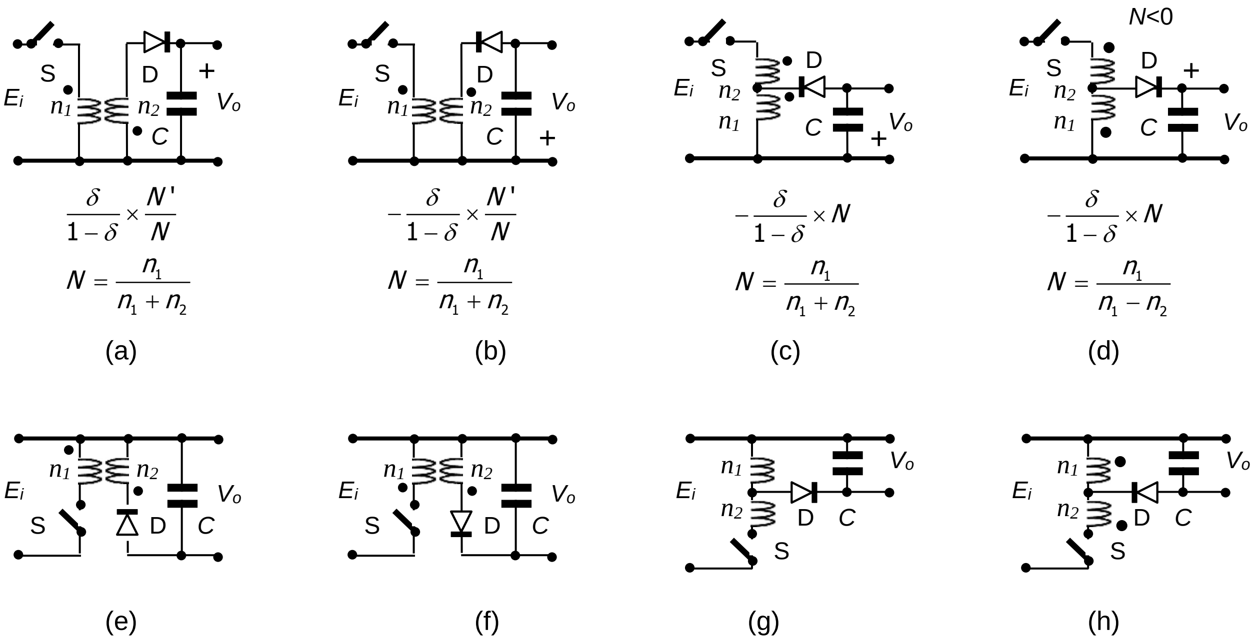


Figure 20.17. *Tapped inductor seeding buck-boost converters (a) terminal inductor tap connection converter T3+, (b) differentially connected inductor tap converter T3-, (c) diode connected, cumulative inductor tap converter S3, (d) diode connected, differentially inductor tap converter S3-, and (e), (f), (g), and (h) positive rail reference versions of (a), (b), (c), and (d).*

Different viable seeding topologies（those shown in figure 20.17）can generate other sequences of different viable converter topologies, by repeated flipping (output becomes the second output port) and inverse operations (interchange the input and output ports), eventually cycling back to the original topology as shown in Table 20.7. The two flip/inverse sequencing processes are summarized mathematically as follows:

i. The flipped transfer function is unity minus the transfer function.

ii The transfer function for inverse operation can also be defined mathematically. The inverse transfer function is derived by interchanging δ and δ', then invert the function. Interchanging the on-state and off-state duty cycles reflects the switch and diode interchange, while inverting reflects the reversing of the input and output ports of the converter (that is V_o / E_i becomes E_i / V_o). This mathematical transformation is applicable to inductor tapped circuits, provided the tap connected element is an internal canonical cell component, specifically the diode or the switch. The canonical cell derivation assumes a *Tee* cell arrangement. The inductor tap connected to an external cell dc voltage bias (not the common input to output reference, 0V) inhibits internal cell element rotation, and this restriction is accounted for by interchanging N and N', in addition to δ and δ', before inverting the function to give the inverse transfer function. The inversion process is not necessarily commutative if asymptotes exist in the transfer function.

The significance of these two sequencing processes is that topologies and transfer functions can be determined mathematically without recourse to circuit analysis.

Flipping a topology may results in a configuration requiring interchange of D and S, whence δ and δ' are also interchanged (effectively necessitating two switch/diode pairs, which guarantees CCM). Such interchanging of the switch and diode in any sequence is always avoided if a pair of bidirectional switches are employed.
The columns in figure 20.18 shows numerous inductor tapped topologies and their vertical sequence flip/inverse order, while Table 20.7 shows the corresponding voltage transfer functions (assuming ideal components) and the associated sequence order, for five sequence groups. All previous publications have assumed the tapped inductor is cumulatively winding connected which gives two major classes of topologies (viz., columns headed S_{cum} and T_{cum} in figure 20.18). Here topologies are incorporated into a general matrix of topologies and transfer functions, with two mirroring classes when the tapped inductor windings are differentially connected (viz., columns S_{diff} and T_{diff} in figure 20.18).

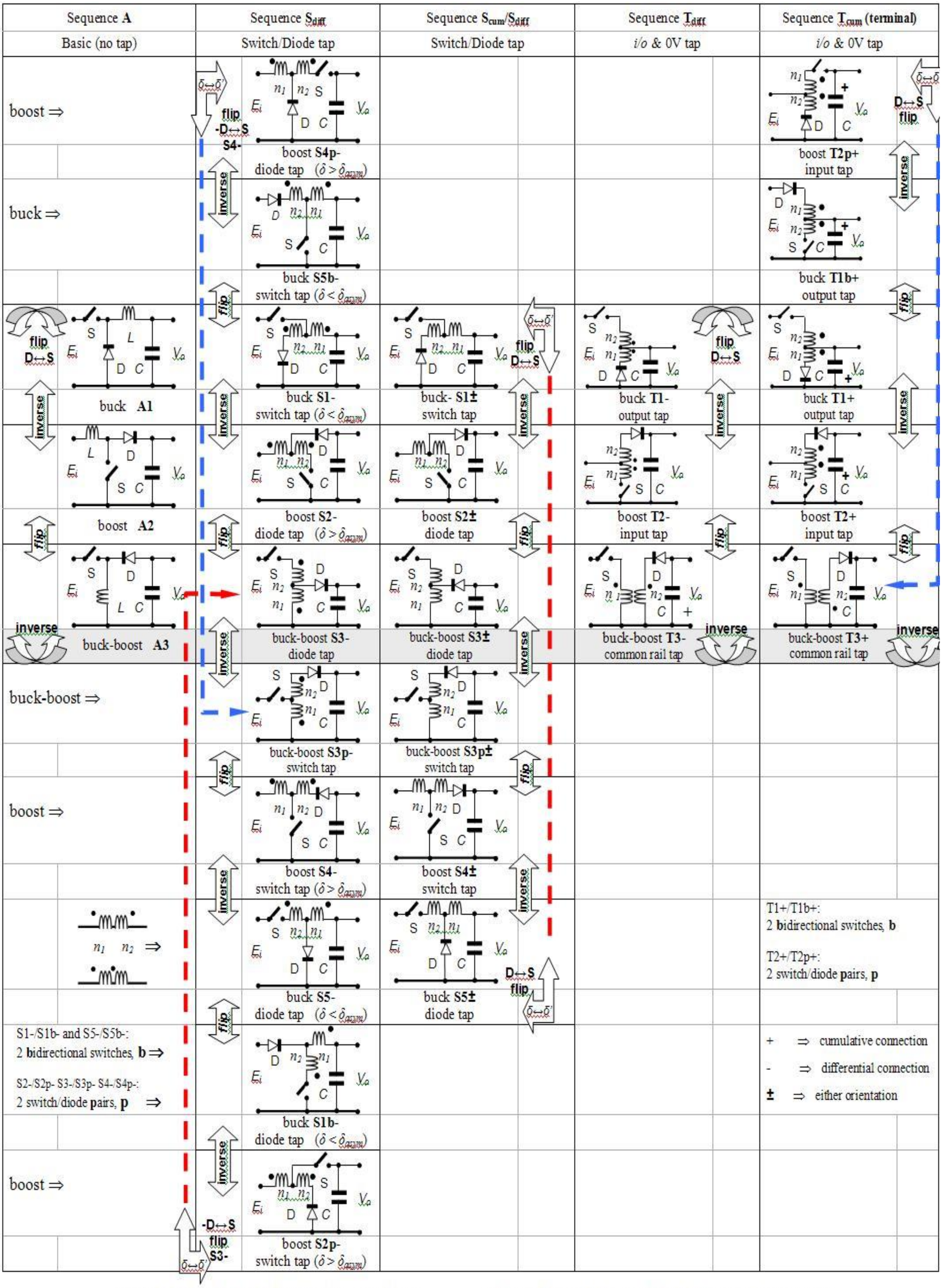
The critical load resistance, defining the CCM-DCM boundary, for each of the three T_{diff} (and five T_{cum}) topologies, in terms of the voltage transfer function, *TF*, is given by the unified equation:

$$R_{o\,crit} = |TF_v| \times \frac{N'}{N} \times \frac{2L_1}{\delta\tau\delta'}, \quad \text{where} \quad \frac{L_1}{L_2}\,\alpha\,\frac{n_1^2}{n_2^2} = \left(\frac{N}{N'}\right)^2 \qquad (6)$$

Analysis assumes a losses circuit, where the output power is equal to the input power, $E_iI_i = v_oI_o$, whence impedance transfers in the ratio of the voltage transfer function TF, squared, $R_o = R_i \times TF^2$. The critical load resistance, defining the CCM-DCM boundary, for each of the S_{diff} (and the six S_{cum}) topologies, in terms of the voltage transfer function, *TF*, is given by

$$R_{o\,crit} = |TF_v| \times \frac{1}{N} \times \frac{2L_1}{\delta\tau\delta'}, \quad \text{where} \quad \frac{L_1}{L_2}\,\alpha\,\frac{n_1^2}{n_2^2} = \left(\frac{N}{N'}\right)^2 \qquad (7)$$

Because some transfer functions have asymptotes, switch diode pairs, or even a pair of bidirectional (ideal) switches are needed if the full duty cycle range is to be exploited. Such topologies (requiring more than one diode and one switch) are shown in Table 20.8.



subscript ***p*** denote diode/switch antiparallel connect pairs
subscript **cum** and '**+**' denote cumulatively connected windings

subscript ***d*** denote bidirectional conducting and blocking switches
subscript **diff** and '-' denote differentially connected windings

Figure 20.18. *Tapped inductor converters.*

Table 20.7: Voltage transfer functions of converters, including inductor tapped converters.

Sequence **A** Basic (no tap)	Sequence **S**$_{diff}$ Switch/Diode tap G sequence TF N = -1	Sequence **S**$_{cum}$/**S**$_{diff}$ Switch/Diode tap A sequence, N = 1	Sequence **T**$_{diff}$ *i/o* & 0V tap A sequence TF, N = ½	Sequence **T**$_{cum}$ *i/o* & 0V tap G sequence TF, N=½
boost ⇒	$\frac{\delta'+N\delta}{N\delta}$ **S4b- (G2/4)** 1>δ>1/(1-N)			$\frac{N\delta'-N'\delta}{N'\delta}$ boost **T2b**$^+$ input tap **(G4/2)**
buck ⇒	$\frac{N\delta'}{\delta+N\delta'}$ **S5b- (G1/3)** 0<δ<N/(N-1)			$\frac{N\delta'}{N\delta'-N'\delta}$ buck **T1b**$^+$ output tap **(G3/1)**
δ buck **A1**	$\frac{\delta}{\delta+N\delta'}$ **S1- (G1/3)** 0<δ<N/(N-1)	$\frac{\delta}{\delta+N\delta'}$ buck **S1±** switch tap **(A1)**	$\frac{N'\delta}{N\delta'+N'\delta}$ buck **T1-** **(A1)**	$-\frac{N'\delta}{N\delta'-N'\delta}$ buck **T1**$^+$ output tap **(G1/3)**
$\frac{1}{1-\delta}$ boost **A2**	$\frac{\delta N+\delta'}{\delta'}$ **S2- (G2/4)** 1>δ>1/(1-N)	$\frac{\delta N+\delta'}{\delta'}$ boost **S2±** diode tap **(A2)**	$\frac{N\delta'+N'\delta}{\delta'N}$ boost **T2-** **(A2)**	$\frac{N\delta'-N'\delta}{\delta'N}$ boost **T2**$^+$ input tap **(G4/2)**
$-\frac{\delta}{1-\delta}$ buck-boost **A3**	$-\frac{\delta}{1-\delta}\times N$ buck-boost **S3-** **(G5/6)** 1>δ>0	$-\frac{\delta}{1-\delta}\times N$ buck-boost **S3±** diode tap **(A3)**	$-\frac{\delta}{1-\delta}\times\frac{N'}{N}$ buck-boost **T3-** **(A3)**	$\frac{\delta}{1-\delta}\times\frac{N'}{N}$ buck-boost **T3**$^+$ ground tap **(G5/6)**
differential coupling $N=\frac{n_1}{n_1-n_2}$ $n_1<n_2$ ⇒ $N<0$	$-\frac{\delta}{1-\delta}\times\frac{1}{N}$ buck-boost **S3b-** **(G6/5)** 1>δ>0	$-\frac{\delta}{1-\delta}\times\frac{1}{N}$ buck-boost **S3b±** switch tap **(A3)**	$\frac{\delta}{1-\delta}\times\frac{N}{N'}$ buck-boost T3p$^-$ ground tap	$\frac{\delta}{1-\delta}\times\frac{N}{N'}$ buck-boost **T3p**$^+$ ground tap
	$\frac{\delta+N\delta'}{\delta'N}$ **S4- (G4/2)** 1>δ>N/(N-1)	$\frac{\delta+N\delta'}{\delta'N}$ boost **S4±** switch tap **(A2)**	cumulative coupling ⇐ $N=\frac{n_1}{n_1+n_2}$ differential coupling $N=\frac{n_1}{n_1-n_2}$ ⇐ $n_1>n_2$ $N>0$	
	$\frac{\delta N}{\delta N+\delta'}$ **S5- (G3/1)** 0<δ<1/(1-N)	$\frac{\delta N}{\delta N+\delta'}$ buck **S5±** diode tap **(A1)**		
buck ⇒	$\frac{\delta'}{N\delta+\delta'}$ **S1b- (G3/1)** 0<δ<1/(1-N)			
boost ⇒	$\frac{N\delta'+\delta}{\delta}$ **S2b- (G2/4)** 1>δ>N/(N-1)			

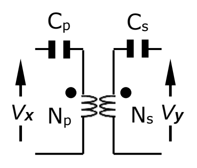
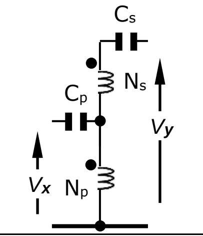
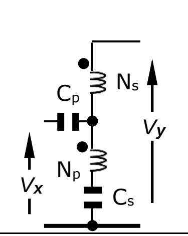
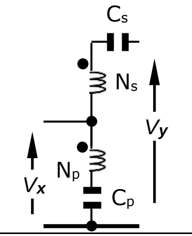
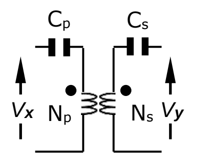
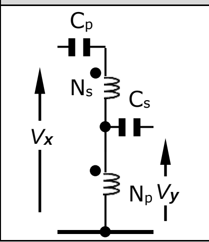
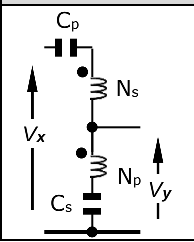
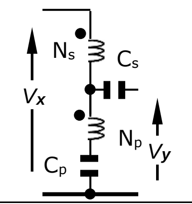
subscript ***p*** denotes diode/switch antiparallel connect pairs subscript ***d*** denotes bidirectional conducting and blocking switches

Table 20.8: Topology constraints and simplifications

sequence	number of topologies min/max	Switch/diode pairs p	Bidirectional switches b	δ_{asym} asymptote (1 switch 1 diode)	turns restriction	Degenerate sequence
A	3/3	A1, A2, A3	-	*none*	*not applicable*	*not applicable*
T_{cum}	3/6	 T2+/T2p+ T3+/T3p+	T1+/T1b+	$<N$, $>N$ *none* *none*	$0<N<1$ $N\neq1$: $n_2\neq0$ $N\neq0$: $n_1\neq0$	$N = ½$, $n_1 = n_2 \Rightarrow$ G sequence not G topology
T_{diff}	3/3	T1- T2- T3-	0	*none* *none* *none*	$0<N<1$ n_2 *and/or* n_2 $\neq0$	$N = ½$, $n_1 = n_2 \Rightarrow$ A sequence not A topology
S_{cum}	5/5	S1± S2± S3±/S3p± S4± S5±	0	*none* *none* *none* *none* *none*	*cum and diff* $N>0$	$N = 1$, $n_2 = 0 \Rightarrow$ A sequence A topology
S_{diff}	5/10	S2-/S2p- S3-/S3p- S4-/S4p-	S1-/S1b- S5-/S5b-	$\delta<N/(N-1)$, $\delta<1/(N-1)$ $\delta>1/(N-1)$, $\delta>N/(N-1)$ *none* $\delta>N/(N-1)$, $\delta>1/(N-1)$ $\delta<1/(N-1)$, $\delta<N/(N-1)$	*diff* $N<0$ $n_1\neq n_2$	$N = -1$, $n_1/n_2 = ½ \Rightarrow$ G sequence $N = 1$, $n_2 = 0 \Rightarrow$ A sequence A topology

The transformer coupled topologies (namely Cuk, sepic and zeta topologies) in Table 20.6 can also be tapped inductor connected, with capacitor and winding possibilities as shown in Table 20.9. A myriad of topologies and transfer functions are generated by performing sequential flipping/inversion operations, when the windings are cumulatively and differentially connected.

Table 20.9: Other tapped inductor configurations

converter		Independent C		Independent C		Series i/p C		Series o/p C	
Step up									
average dc voltage		V_x	V_y	V_x	V_y	V_x	V_y	V_x	V_y
C5	Cuk	E_i	$-V_o$	E_i	$-V_o$	E_i	$-V_o$	E_i	$-V_o$
G6	zeta	0	$+V_o$	0	$+V_o$	0	$+V_o$	0	$+V_o$
G5	sepic	E_i	0	E_i	0	E_i	0	E_i	0
P5	new	0	0	0	0	0	0	0	0
Capacitor dc offset bias		V_{Cp}	V_{Cs}	V_{Cp}	V_{Cs}	V_{Cp}	V_{Cs}	V_{Cp}	V_{Cs}
C5	Cuk	E_i	$-V_o$	E_i	$-V_o$	E_i+V_o	$-V_o$	E_i	$-V_o-E_i$
G6	zeta	0	V_o	0	V_o	E_i	V_o	0	V_o
G5	sepic	E_i	0	E_i	0	E_i	0	E_i	E_i
P5	new	0	0	0	0	0	0	0	0
Step down									

20.10i Reversible tapped inductor smps

The twenty-four tapped inductor converters in figure 20.18 are reversible if two switch/diode pairs are employed, as shown in the buck-boost bidirectional converters (BDCs) on the left in figure 20.19. The switch/diode combination need not be parallel connected, as in the circuits on the right. Generally, either or both diode/switch pairs can be separated, although some of the generated topologies may require the use of RBIGBTs and/or SCRs.

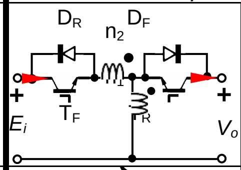
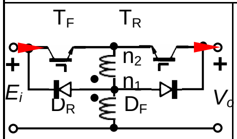
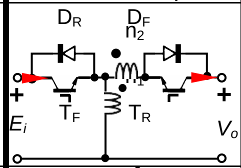
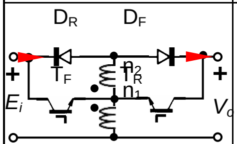
differentially-connected coupled-circuit **buck-boost** BDCs, $\mathbf{S_{diff}}$					
forward buck-boost		inverse: buck-boost switch/diode pairs		inverse: buck-boost 2 switches + 2 diodes	
S3- diode tap	$\frac{\delta}{1-\delta}\times N$		S3p- switch tap $\frac{\delta}{1-\delta}\times\frac{1}{N}$		S3- diode tap $\frac{\delta}{1-\delta}\times N$
S3- →		S3- ⇄ S3p-	← S3p-	S3- ⇄ S3-	← S3-
S3p- switch tap	$\frac{\delta}{1-\delta}\times\frac{1}{N}$		S3- diode tap $-\frac{\delta}{1-\delta}\times N$		S3p- switch tap $\frac{\delta}{1-\delta}\times\frac{1}{N}$
S3p- →		S3p- ⇄ S3-	← S3-	S3p- ⇄ S3p-	← S3p-
$n_1<n_2$ $N=n_1/n_1-n_2$ $N<0$					

Figure 20.19*. Switch/diode connected, differentially-formed tap, S2/S4 buck-boost BDCs.*

A feature of splitting the diode/switch pair is the generation of unusual forward and reverse transfer function pairs. Consider the bottom right bidirectional buck converter in figure 20.19. The input and output voltage polarity is the same, with respect to the zero reference. Also the voltage transfer function is the same in both directions, namely

$$\frac{\delta}{1-\delta}\times\frac{1}{N} \qquad (20.8)$$

20.10ii Coupled circuit leakage inductance

Magnetically coupled circuits, in single ended topologies suffer from stress energy associated with leakage inductance. Release of leakage energy usually manifests itself as an over voltage on semiconductor devices. At lower power, dissipative clamping circuits may be effective and viable. An alternative is to use an asymmetrical bridge configuration.

The diode-tap buck converter S5± in figure 20.18 is shown in figure 20.20a, where leakage energy stresses the switch S at switch turn-off. A convention clamp is ineffective because at switch turn-off, theoretically the switch experiences the supply voltage E_i plus the voltage induce across n_2 because of conditions on n1, $V_o\, n_2/n_1$. The leakage energy creates of voltage in excess of $E_i + V_o\, n_2/n_1$. Thus clamping the stressed device to the voltage rail E_i would undesirably load the coupled circuit winding n_2.

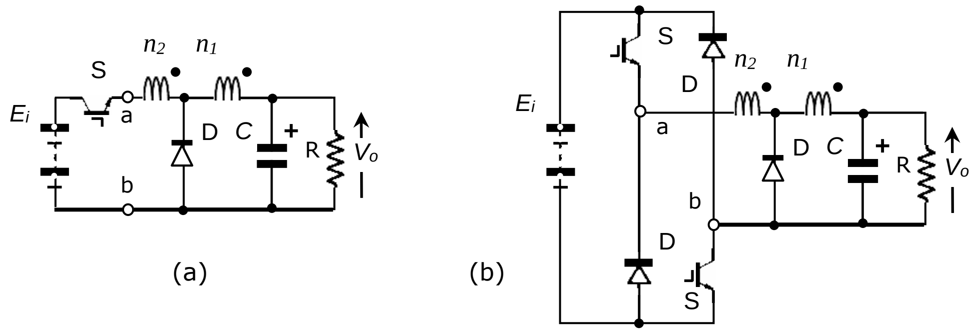


Figure 20.20. *Tapped inductor buck converter S5±: (a) basic topology and (b) asymmetrical bridge input stage to clamp leakage energy release voltage.*

In the asymmetrical bridge in figure 20.20b, the topology is energised through the two bridge switches, then leakage energy reset occurs through the two bridge diodes, back into the dc rail. The voltage releasing the leakage energy is E_i - $V_o n_2 /n_1$, while the two switch off-state voltages are constrained to within dc voltage rails. The two switches must be both on or both off, simultaneously. Phase shifting leg signals by 180° to reduce the switching losses is not possible. In creating alternating 0V loops, the winding n_2 induced voltage $V_o n_2 /n_1$ would be short circuited. The penalties incurred are two additional diodes, one switch and its gate drive, plus the input and output voltage no longer share a common dc reference. The asymmetrical bridge arrangement caters for converter reverse operation, with the output (former input) being sourced through the two bridge diodes.

20.11 HV referenced dc to dc converter

Derivation of low voltage dc supplies from sources greater than a hundred volts can be inefficient, particularly at over 1kV. The extremely low duty cycle requirement implies the need of transformer matching. At 10kV, as with submarine cables, a Zener diode plus resistor approach offers low efficiency, below 1%, so such an approach is limited to a start-up function. The availability of a normally-on SiC 1200V 45mΩ JFET means series smps circuits, based on current source topologies can be employed as shown in figure 20.21. In a default mode current is conducted by the normally on JFET. By turning off JFET T, current is diverted to the bulk storage capacitor C, which is maintained charged in a burst mode. Start-up energy is sourced by the Zener diode-resistor across the high voltage supply, which is deactivated once C has charged. The deactivation function is achieved with a normally-closed contact of a high voltage relay (see Chapter 33, Table 33.14 for a 70kV, 10A dc relay).

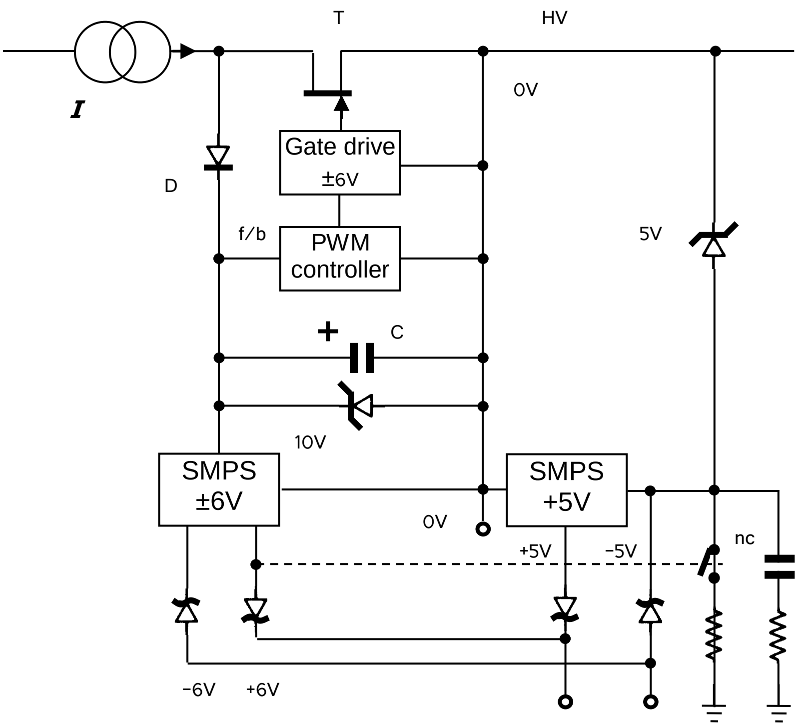


Figure 20.21. *HV referenced low-voltage dc to dc converter.*

20.12 Current sourced dc to dc converters

Three multiple switch current sourced converters are shown in figure 20.22. Such multiple switch current sourced dc to dc voltage converters are characterised by switch overlap operation, so as to always provide a current path for the current source.

Table 20.10 Current source dc to dc converter characteristics

Current source converter boost Text figure number	TF $\frac{V_{out}}{E_{in}} = \frac{I_{in}}{I_{out}} =$	i/p & o/p capacitance $\Delta v_{Cin} C_{in} = Q_{Cin}$ $\Delta v_{Cin} C_{out} = i_{Cout} \Delta t$	inductance $\Delta i_{L1} L_i = v_{Li} \Delta t$ Δ is p-p	semiconductor utilisation factor $u_f = \frac{P_{rated}}{\sum_{\forall Si} V_{max} I_{rms}}$
Half bridge $L_1 = L_2$ 1:*N* Fig 20.22b	$\frac{1}{1-\delta} N$ $\delta > ½$	$\frac{\frac{1}{16}\Delta i_{L1}(2\delta - 1)\tau}{\delta}$ $I_{out}(\delta - ½)\tau$	$E_{in}\delta\tau$	$\frac{\delta'}{2\sqrt{\delta'} + \sqrt{1+2\delta'}}$
Full bridge 1:*N* Fig 20.22c	$\frac{1}{1-2\delta} N$ $\delta > ½$	$\frac{1}{16}\Delta i_{L1}\tau$ $I_{out}\delta\tau$	$E_{in}\delta\tau$	$\frac{2\delta' - 1}{2\sqrt{2}\left[\sqrt{\delta'} + \sqrt{2\delta' - 1}\right]}$
Push pull 1:1:N Fig 20.22a	$\frac{1}{1-2\delta} N$ $\delta > ½$	$\frac{1}{16}\Delta i_{L1}\tau$ $I_{out}\delta\tau$	$E_{in}\delta\tau$	$\frac{2\delta' - 1}{2\sqrt{2}\left[\sqrt{\delta'} + \sqrt{2\delta' - 1}\right]}$

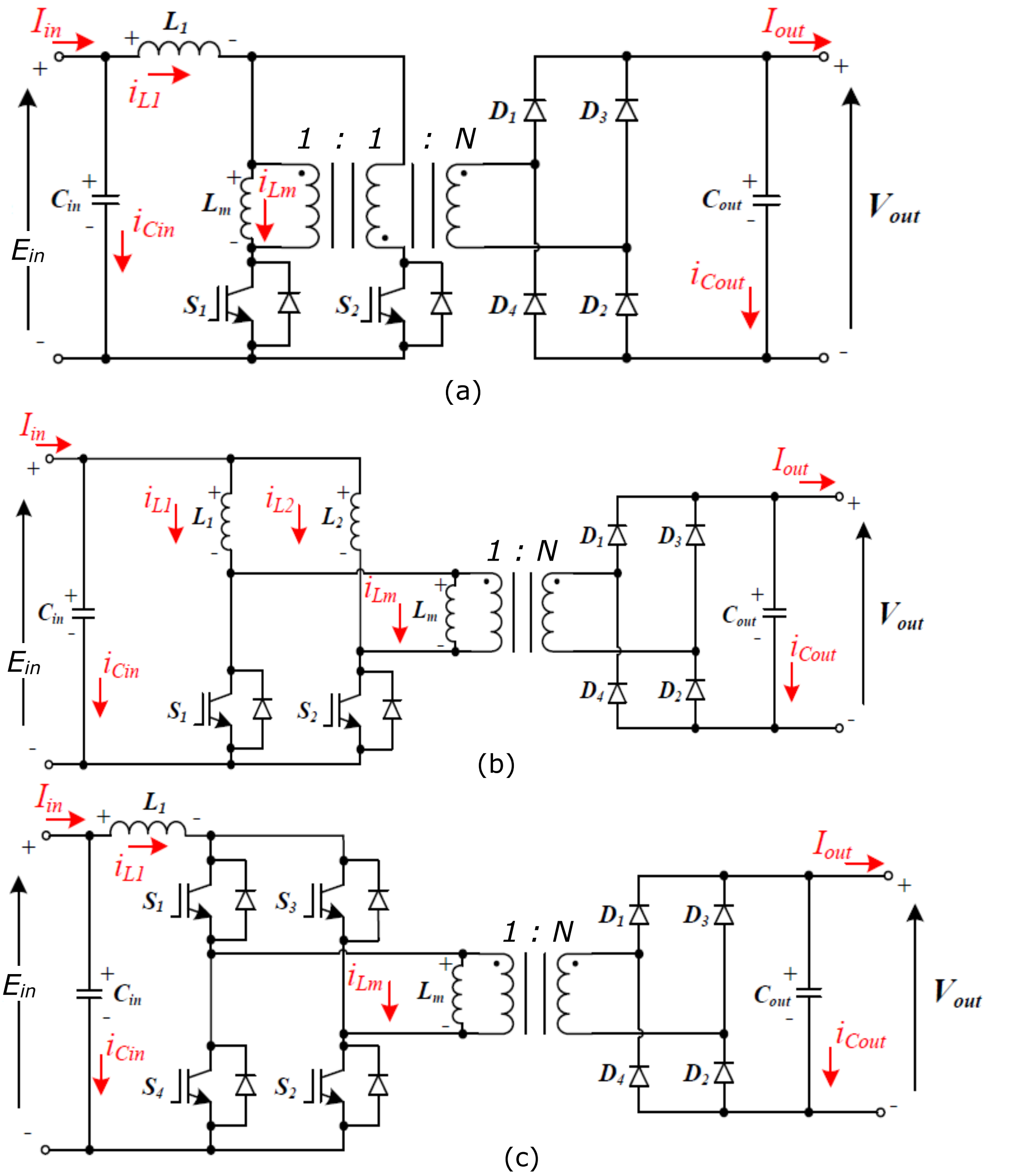


Figure 20.22. *Isolated multi switch, current sourced dc to dc converters: (a) push-pull, (b) half-bridge, and (c) full-bridge.*

20.13 Analysis of non-continuous inductor current operation

Operation with constant input voltage, E_i

In applications were the input voltage E_i is fixed, as with rectifier ac voltage input circuits and battery supplies, the output voltage v_o can be controlled by varying the duty cycle.
In the continuous inductor conduction region, the transfer function for the three basic converters is determined solely in terms of the on-state duty cycle, δ. Operation in the discontinuous inductor current region, for a constant input voltage, can be characterised for each converter in terms of duty cycle and the normalised output or input current, as shown in figure 20.27. Region and boundary equations, for a constant input voltage E_i, are summarised in tables 20.11 and 20.12.

Operation with constant output voltage, v_o

In applications were the output voltage v_o is fixed, as required with regulated dc power supplies, the effects of varying input voltage E_i can be controlled and compensated by varying the duty cycle.
In the inductor continuous current conduction region, the transfer function is determined solely in terms of the on-state duty cycle, δ. Operation in the discontinuous region, for a constant output voltage, can be characterised in terms of duty cycle and the normalised output or input current, as shown in figure 20.28. Region and boundary equations, for a constant output voltage v_o, are summarised in tables 20.13 and 20.14.

Because of the invariance of power, the output current $\overline{I}_o$ characteristics for each converter with a constant input voltage E_i , shown in figure 20.24, are the same as those for the input current $\overline{I}_i$ when the output voltage v_o is maintained constant, as shown in figure 20.28. [That is, the right hand side of each plot in figures 20.27 and 20.28 (or figures 20.23 and 20.24) are the same.]

Generalised characteristics, with operating condition k (=$R\tau/L$), for the three basic converters, are summarised in Table 20.15. The associated monographs in figures 20.30, 20.31, and 20.32, with a specific load condition, k, for each converter, yield the inductor current waveforms for any on-state duty cycle δ. The three graphs illustrate operational boundaries between continuous inductor current at high δ and discontinuous inductor current at lower δ.

The graphs for the boost converter in figure 20.31 highlight a little appreciated feature that, if $k > 13½$, then discontinuous inductor current having appeared, disappears at lower and higher duty cycles. Specifically, continuous inductor current occurs for low duty cycles, where the same theoretical equation is interpreted to the contrary. That is, from Table 19.2, the roots of

$$\delta\left(1-\delta\right)^2 \le \frac{2}{k} \qquad (20.9)$$

are not interpreted correctly. The correct interpretation of δ and k (= $R\tau/L$) gives:

- for $k < 13½$, discontinuous inductor current never occurs, independent of δ (equation (20.9) has two imaginary roots)
- for $k = 13½$, discontinuous inductor current occurs at only $\delta = ⅓$ (equation (20.9) has three roots, two of which are coincident at $\delta = ⅓$)
- for $k > 13½$, discontinuous inductor current occurs for δ around ⅓ as given by the two (of the three) real roots of equation (20.9) associated with the local minimum turning point of the cubic equation (20.9).

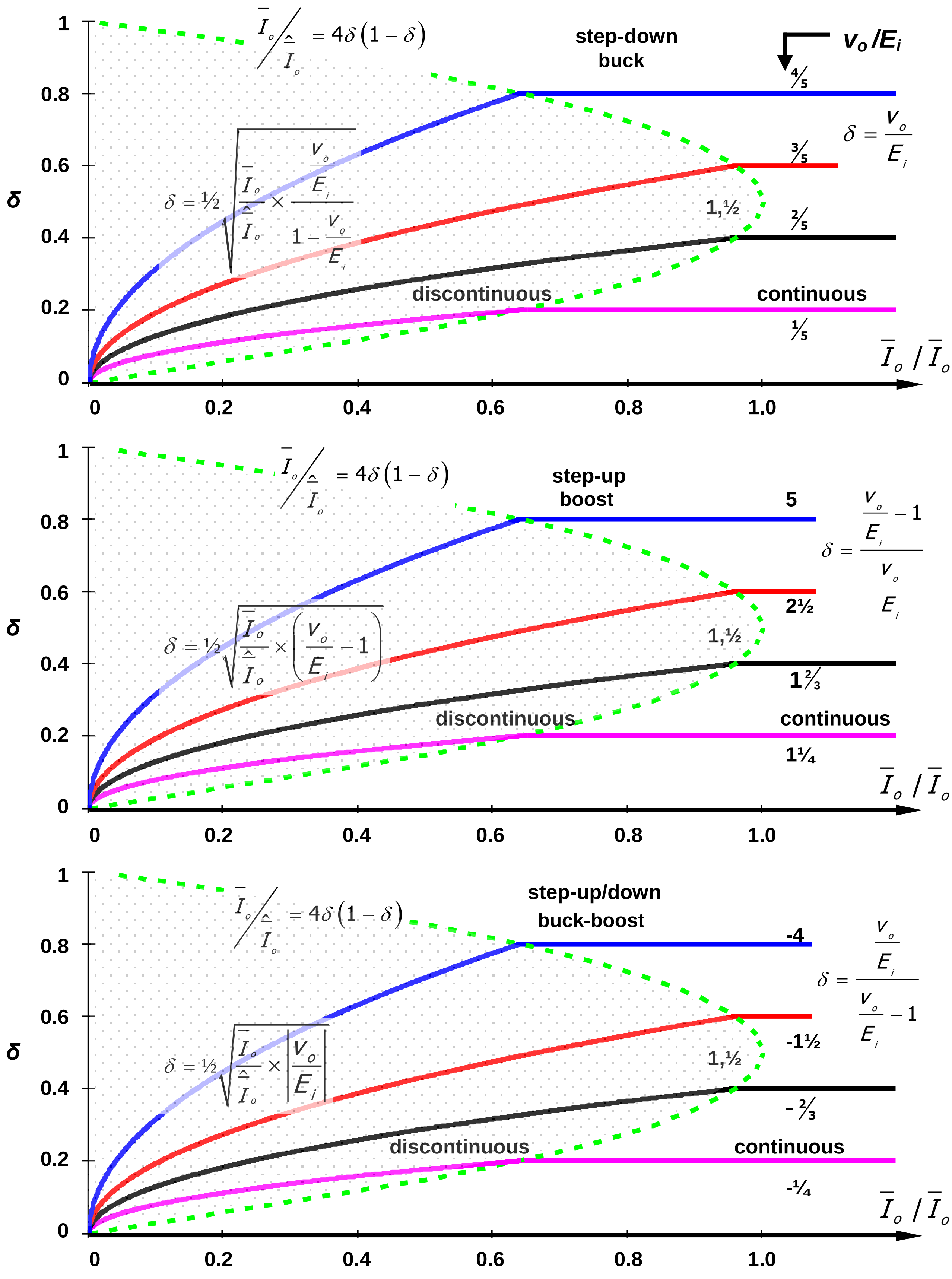


Figure 20.23. *Characteristics for three dc-dc converters with respect to* $\bar{I}_o$*, when the input voltage* E_i *is held constant. See Table 20.11.*

Table 20.11: Transfer functions with constant input voltage, E_i, with respect to $\bar{I}_o$

E_i constant	converter		
	step-down (buck)	step-up (boost)	step-up/down (buck-boost)
reference equation	**(19.4)**	**(19.50)**	**(19.80)**
continuous inductor current conduction (and change of variable)	$\dfrac{V_o}{E_i} = \delta$	$\dfrac{V_o}{E_i} = \dfrac{1}{1-\delta}$	$\dfrac{V_o}{E_i} = \dfrac{-\delta}{1-\delta}$
	$\delta = \dfrac{V_o}{E_i}$	$\delta = \dfrac{\dfrac{V_o}{E_i} - 1}{\dfrac{V_o}{E_i}}$	$\delta = \dfrac{\dfrac{V_o}{E_i}}{\dfrac{V_o}{E_i} - 1}$
reference equation	**(19.21)**	**(19.65)**	**(19.96)**
discontinuous inductor current conduction	$\dfrac{V_o}{E_i} = \dfrac{1}{1 + \dfrac{2L\bar{I}_o}{\delta^2 \tau E_i}}$	$\dfrac{V_o}{E_i} = 1 + \dfrac{\delta^2 E_i \tau}{2L\bar{I}_o}$	$\dfrac{V_o}{E_i} = -\dfrac{\delta^2 E_i \tau}{2L\bar{I}_o}$
normalised $\dfrac{V_o}{E_i} =$ where $\hat{\bar{I}}_o = \dfrac{E_i \tau}{8L}$	$\dfrac{V_o}{E_i} = \dfrac{1}{1 + \dfrac{1}{4\delta^2} \times \dfrac{\bar{I}_o}{\hat{\bar{I}}_o}}$	$\dfrac{V_o}{E_i} = 1 + 4\delta^2 / \dfrac{\bar{I}_o}{\hat{\bar{I}}_o}$	$\dfrac{V_o}{E_i} = -4\delta^2 / \dfrac{\bar{I}_o}{\hat{\bar{I}}_o}$
$\bar{I}_o = \hat{\bar{I}}_o = 1$pu @	$\delta = ½;\ \dfrac{V_o}{E_i} = ½$	$\delta = ½;\ \dfrac{V_o}{E_i} = 2$	$\delta = ½;\ \dfrac{V_o}{E_i} = -1$
change of variable $\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} =$	$\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} = 4\delta^2 \times \dfrac{\left(1 - \dfrac{V_o}{E_i}\right)}{\dfrac{V_o}{E_i}}$	$\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} = 4\delta^2 \times \dfrac{1}{\dfrac{V_o}{E_i} - 1}$	$\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} = -4\delta^2 \times \dfrac{1}{\dfrac{V_o}{E_i}}$
change of variable $\delta =$ all with a boundary $\delta = ½ + ½\sqrt{1 - \dfrac{\bar{I}_o}{\hat{\bar{I}}_o}}$	$\delta = ½\sqrt{\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} \times \dfrac{\dfrac{V_o}{E_i}}{1 - \dfrac{V_o}{E_i}}}$	$\delta = ½\sqrt{\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} \times \left(\dfrac{V_o}{E_i} - 1\right)}$	$\delta = ½\sqrt{\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} \times \left\lvert\dfrac{V_o}{E_i}\right\rvert}$
conduction boundary $\delta = ½ + ½\sqrt{1 - \dfrac{\bar{I}_o}{\hat{\bar{I}}_o}}$	$\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} = 4 \times \dfrac{V_o}{E_i}\left(1 - \dfrac{V_o}{E_i}\right)$ $= 4\delta(1-\delta)$	$\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} = 4 \times \dfrac{\left(\dfrac{V_o}{E_i} - 1\right)}{\left(\dfrac{V_o}{E_i}\right)^2}$ $= 4\delta(1-\delta)$	$\dfrac{\bar{I}_o}{\hat{\bar{I}}_o} = -4 \times \dfrac{\dfrac{V_o}{E_i}}{\left(1 - \dfrac{V_o}{E_i}\right)^2}$ $= 4\delta(1-\delta)$

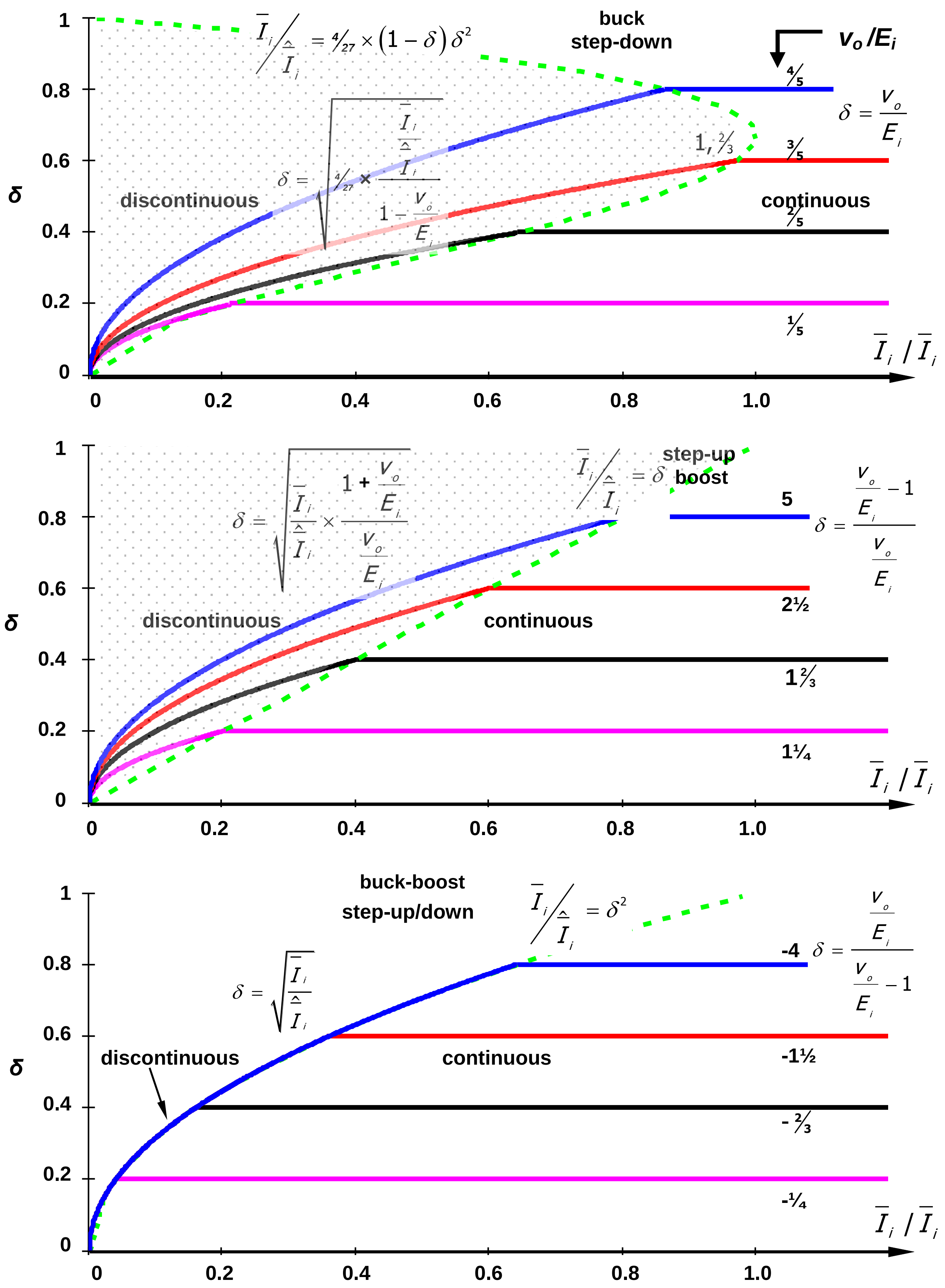


Figure 20.24. *Characteristics for three dc-dc converters with respect to $\bar{I}_i$, when the input voltage E_i is held constant. See Table 20.12.*

Table 20.12: Transfer functions with constant input voltage, E_i, with respect to $\bar{I}_i$

E_i constant	converter step-down (buck)	step-up (boost)	step-up/down (buck-boost)
reference equation	**(19.4)**	**(19.50)**	**(19.80)**
continuous inductor current conduction (and change of variable)	$\frac{V_o}{E_i} = \delta$	$\frac{V_o}{E_i} = \frac{1}{1-\delta}$	$\frac{V_o}{E_i} = \frac{-\delta}{1-\delta}$
	$\delta = \frac{V_o}{E_i}$	$\delta = \frac{\frac{V_o}{E_i} - 1}{\frac{V_o}{E_i}}$	$\delta = \frac{\frac{V_o}{E_i}}{\frac{V_o}{E_i} - 1}$
reference equation	**(19.20)**	**(19.66)**	**(19.96)**
discontinuous inductor current conduction	$\frac{V_o}{E_i} = 1 - \frac{2L\bar{I}_i}{\delta^2 \tau E_i}$	$\frac{V_o}{E_i} = \frac{1}{1 - \frac{E_i \tau \delta^2}{2L\bar{I}_i}}$	$\frac{V_o}{E_i} = \frac{V_o \tau \delta^2}{2L\bar{I}_i}$
normalised $\frac{V_o}{E_i} =$	$\frac{V_o}{E_i} = 1 - \frac{4}{27\delta^2} \times \frac{\bar{I}_i}{\hat{\bar{I}}_i}$ where $\hat{\bar{I}}_i = \frac{4}{27} \times \frac{E_i \tau}{2L}$	$\frac{V_o}{E_i} = \frac{1}{1 - \delta^2 / \left(\frac{\bar{I}_i}{\hat{\bar{I}}_i}\right)}$ where $\hat{\bar{I}}_i = \frac{E_i \tau}{2L}$	$1 = \delta^2 / \frac{\bar{I}_i}{\hat{\bar{I}}_i}$ where $\hat{\bar{I}}_i = \frac{E_i \tau}{2L}$
$\bar{I}_i = \hat{\bar{I}}_i = 1$pu @	$\delta = ⅔$; $\frac{V_o}{E_i} = ⅔$	$\delta = 1$; $\frac{V_o}{E_i} \rightarrow \infty$	$\delta = 1$; $\frac{V_o}{E_i} \rightarrow -\infty$
change of variable $\frac{\bar{I}_i}{\hat{\bar{I}}_i} =$	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = {}^{27}\!/\!_4\, \delta^2 \left(1 - \frac{V_o}{E_i}\right)$	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = \delta^2 \times \frac{\frac{V_o}{E_i}}{\left(\frac{V_o}{E_i} - 1\right)}$	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = \delta^2$
change of variable $\delta =$	$\delta = \sqrt{{}^4\!/\!_{27} \times \frac{\bar{I}_i}{\hat{\bar{I}}_i} \times \frac{1}{1 - \frac{V_o}{E_i}}}$	$\delta = \sqrt{\frac{\bar{I}_i}{\hat{\bar{I}}_i} \times \frac{\frac{V_o}{E_i} - 1}{\frac{V_o}{E_i}}}$	$\delta = \sqrt{\frac{\bar{I}_i}{\hat{\bar{I}}_i}}$
conduction boundary	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = {}^{27}\!/\!_4 \times \left(1 - \frac{V_o}{E_i}\right)\left(\frac{V_o}{E_i}\right)^2$ $= {}^{27}\!/\!_4\, \delta^2 (1-\delta)$	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = \frac{\left(\frac{V_o}{E_i} - 1\right)}{\frac{V_o}{E_i}}$ $= \delta$	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = \left(\frac{\frac{V_o}{E_i}}{\frac{V_o}{E_i} - 1}\right)^2$ $= \delta^2$
conduction boundary	$\frac{\bar{I}_i}{\hat{\bar{I}}_i} = {}^{27}\!/\!_4\, \delta^2 (1-\delta)$	$\delta = \frac{\bar{I}_i}{\hat{\bar{I}}_i}$	$\delta = \sqrt{\frac{\bar{I}_i}{\hat{\bar{I}}_i}}$

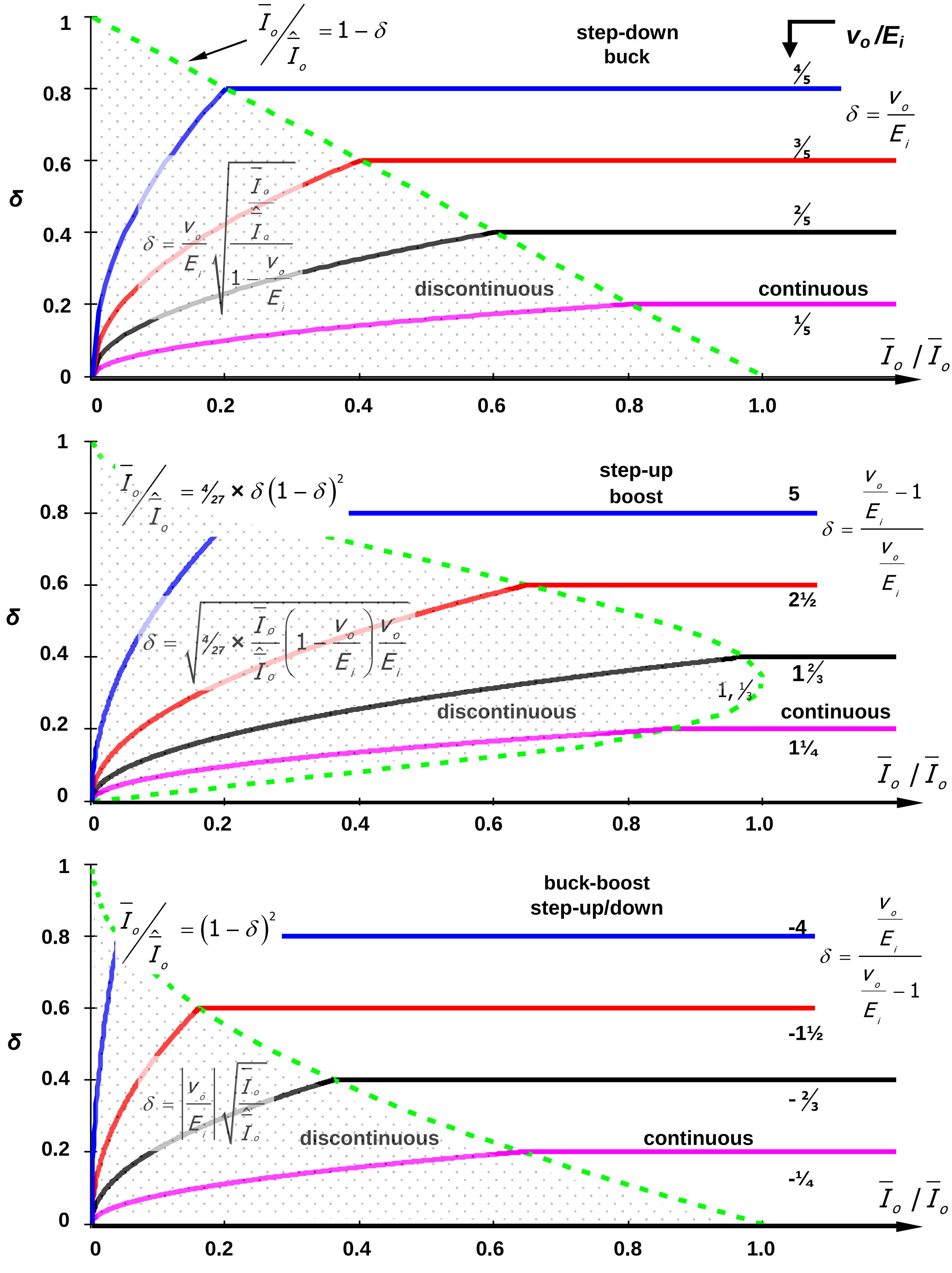


Figure 20.25. *Characteristics for three dc-dc converters with respect to $\bar{I}_o$, when the output voltage v_o is held constant. See Table 20.13.*

Table 20.13: Transfer functions with constant output voltage, v_o, with respect to $\bar{I}_o$

v_o constant	converter		
	step-down (buck)	step-up (boost)	step-up/down (buck-boost)
reference equation	**(19.4)**	**(19.50)**	**(19.80)**
continuous inductor current conduction (and change of variable)	$\frac{v_o}{E_i} = \delta$	$\frac{v_o}{E_i} = \frac{1}{1-\delta}$	$\frac{v_o}{E_i} = \frac{-\delta}{1-\delta}$
	$\delta = \frac{v_o}{E_i}$	$\delta = \frac{\frac{v_o}{E_i}-1}{\frac{v_o}{E_i}}$	$\delta = \frac{\frac{v_o}{E_i}}{\frac{v_o}{E_i}-1}$
reference equation	**(19.20)**	**(19.66)**	**(19.96)**
discontinuous inductor current conduction	$\frac{v_o}{E_i} = 1 - \frac{2L\bar{I}_i}{\delta^2 \tau E_i}$	$\frac{v_o}{E_i} = \frac{1}{1 - \frac{E_i \tau \delta^2}{2L\bar{I}_i}}$	$\frac{v_o}{E_i} = \frac{v_o \tau \delta^2}{2L\bar{I}_i}$
normalised $\frac{v_o}{E_i} =$	$\frac{v_o}{E_i} = 1 - \frac{1}{4\delta^2} \times \frac{\bar{I}_o}{\hat{\bar{I}}_o} \times \left(\frac{v_o}{E_i}\right)^2$ where $\hat{\bar{I}}_o = \frac{v_o \tau}{2L}$	$\frac{v_o}{E_i} = \frac{1}{1 - {}^{27}\!/_{4}\,\delta^2 \Big/ \left(\frac{\bar{I}_o}{\hat{\bar{I}}_o} \times \left(\frac{v_o}{E_i}\right)^2\right)}$ where $\hat{\bar{I}}_o = \frac{4}{27} \times \frac{v_o \tau}{2L}$	$\frac{v_o}{E_i} = \delta^2 \Big/ \left(\frac{\bar{I}_o}{\hat{\bar{I}}_o} \times \frac{v_o}{E_i}\right)$ where $\hat{\bar{I}}_o = \left\vert\frac{v_o \tau}{2L}\right\vert$
$\bar{I}_o = \hat{\bar{I}}_o$ = 1pu @	$\delta = 0$; $\frac{v_o}{E_i} = 0$	$\delta = ⅓$; $\frac{v_o}{E_i} = 1½$	$\delta = 0$; $\frac{v_o}{E_i} = 0$
change of variable $\frac{\bar{I}_o}{\hat{\bar{I}}_o} =$	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = \delta^2 \times \frac{\left(1 - \frac{v_o}{E_i}\right)}{\left(\frac{v_o}{E_i}\right)^2}$	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = {}^{27}\!/_{4}\,\delta^2 \times \frac{1}{\left(\frac{v_o}{E_i}-1\right)\frac{v_o}{E_i}}$	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = \delta^2 \times \frac{1}{\left(\frac{v_o}{E_i}\right)^2}$
change of variable $\delta =$	$\delta = \frac{v_o}{E_i}\sqrt{\frac{\bar{I}_o}{\hat{\bar{I}}_o} \times \frac{1}{1 - \frac{v_o}{E_i}}}$	$\delta = \sqrt{{}^{4}\!/_{27} \times \frac{\bar{I}_o}{\hat{\bar{I}}_o} \times \left(\frac{v_o}{E_i}-1\right)\frac{v_o}{E_i}}$	$\delta = \left\vert\frac{v_o}{E_i}\right\vert\sqrt{\frac{\bar{I}_o}{\hat{\bar{I}}_o}}$
conduction boundary	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = 1 - \frac{v_o}{E_i}$ $= 1 - \delta$	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = {}^{27}\!/_{4} \times \frac{\left(\frac{v_o}{E_i}-1\right)}{\left(\frac{v_o}{E_i}\right)^3}$ $= {}^{27}\!/_{4}\,\delta\left(1-\delta\right)^2$	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = \frac{1}{\left(1 - \frac{v_o}{E_i}\right)^2}$ $= \left(1-\delta\right)^2$
conduction boundary	$\delta = 1 - \frac{\bar{I}_o}{\hat{\bar{I}}_o}$	$\frac{\bar{I}_o}{\hat{\bar{I}}_o} = {}^{27}\!/_{4}\,\delta\left(1-\delta\right)^2$	$\delta = 1 - \sqrt{\frac{\bar{I}_o}{\hat{\bar{I}}_o}}$

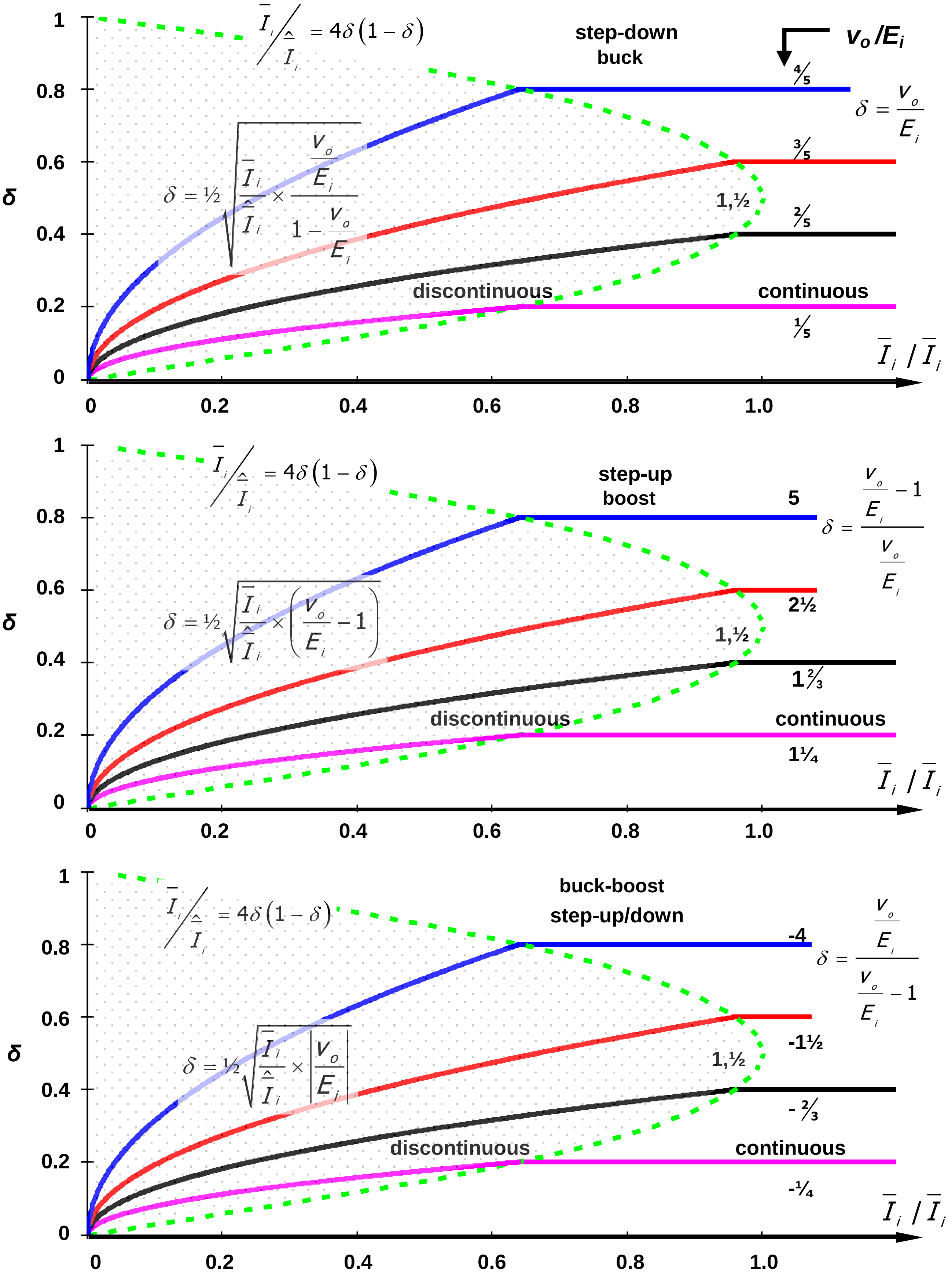


Figure 20.26. *Characteristics for three dc-dc converters with respect to* $\bar{I}_i$*,* *when the output voltage* v_o *is held constant. See Table 20.14.*

Table 20.14: Transfer functions with constant input voltage, v_o, with respect to $\bar{I}_i$

v_o constant	converter		
	step-down (buck)	step-up (boost)	step-up/down (buck-boost)
reference equation	**(19.4)**	**(19.50)**	**(19.80)**
continuous inductor current conduction (and change of variable)	$\frac{V_o}{E_i}=\delta$	$\frac{V_o}{E_i}=\frac{1}{1-\delta}$	$\frac{V_o}{E_i}=\frac{-\delta}{1-\delta}$
	$\delta=\frac{V_o}{E_i}$	$\delta=\frac{\frac{V_o}{E_i}-1}{\frac{V_o}{E_i}}$	$\delta=\frac{\frac{V_o}{E_i}}{\frac{V_o}{E_i}-1}$
reference equation	**(19.21)**	**(19.65)**	**(19.96)**
discontinuous inductor current conduction	$\frac{V_o}{E_i}=\frac{1}{1+\frac{2L\bar{I}_i}{\delta^2\tau V_o}}$	$\frac{V_o}{E_i}=1+\frac{\delta^2 V_o\tau}{2L\bar{I}_i}$	$\frac{V_o}{E_i}=-\frac{\delta^2 V_o\tau}{2L\bar{I}_i}$
normalised $\frac{V_o}{E_i}=$ where $\hat{I}_i=\left\lvert\frac{V_o\tau}{8L}\right\rvert$	$\frac{V_o}{E_i}=\frac{1}{1+\frac{1}{4\delta^2}\times\frac{\bar{I}_i}{\hat{I}_i}}$	$\frac{V_o}{E_i}=1+4\delta^2/\frac{\bar{I}_i}{\hat{I}_i}$	$\frac{V_o}{E_i}=-4\delta^2/\frac{\bar{I}_i}{\hat{I}_i}$
$\bar{I}_i=\hat{I}_i=1$pu @	$\delta=½$; $\frac{V_o}{E_i}=½$	$\delta=½$; $\frac{V_o}{E_i}=2$	$\delta=½$; $\frac{V_o}{E_i}=-1$
change of variable $\frac{\bar{I}_i}{\hat{I}_i}=$	$\frac{\bar{I}_i}{\hat{I}_i}=4\delta^2\times\frac{\left(1-\frac{V_o}{E_i}\right)}{\frac{V_o}{E_i}}$	$\frac{\bar{I}_i}{\hat{I}_i}=4\delta^2\times\frac{1}{\frac{V_o}{E_i}-1}$	$\frac{\bar{I}_i}{\hat{I}_i}=-4\delta^2\times\frac{1}{\frac{V_o}{E_i}}$
change of variable $\delta=$ all with a boundary $\delta=½+½\sqrt{1-\frac{\bar{I}_o}{\hat{I}_o}}$	$\delta=½\sqrt{\frac{\bar{I}_i}{\hat{I}_i}\times\frac{\frac{V_o}{E_i}}{1-\frac{V_o}{E_i}}}$	$\delta=½\sqrt{\frac{\bar{I}_i}{\hat{I}_i}\times\left(\frac{V_o}{E_i}-1\right)}$	$\delta=½\sqrt{\frac{\bar{I}_i}{\hat{I}_i}\times\left\lvert\frac{V_o}{E_i}\right\rvert}$
conduction boundary $\delta=½+½\sqrt{1-\frac{\bar{I}_o}{\hat{I}_o}}$	$\frac{\bar{I}_i}{\hat{I}_i}=4\frac{V_o}{E_i}\left(1-\frac{V_o}{E_i}\right)$ $=4\delta(1-\delta)$	$\frac{\bar{I}_i}{\hat{I}_i}=4\times\frac{\left(\frac{V_o}{E_i}-1\right)}{\left(\frac{V_o}{E_i}\right)^2}$ $=4\delta(1-\delta)$	$\frac{\bar{I}_i}{\hat{I}_i}=-4\times\frac{\frac{V_o}{E_i}}{\left(1-\frac{V_o}{E_i}\right)^2}$ $=4\delta(1-\delta)$

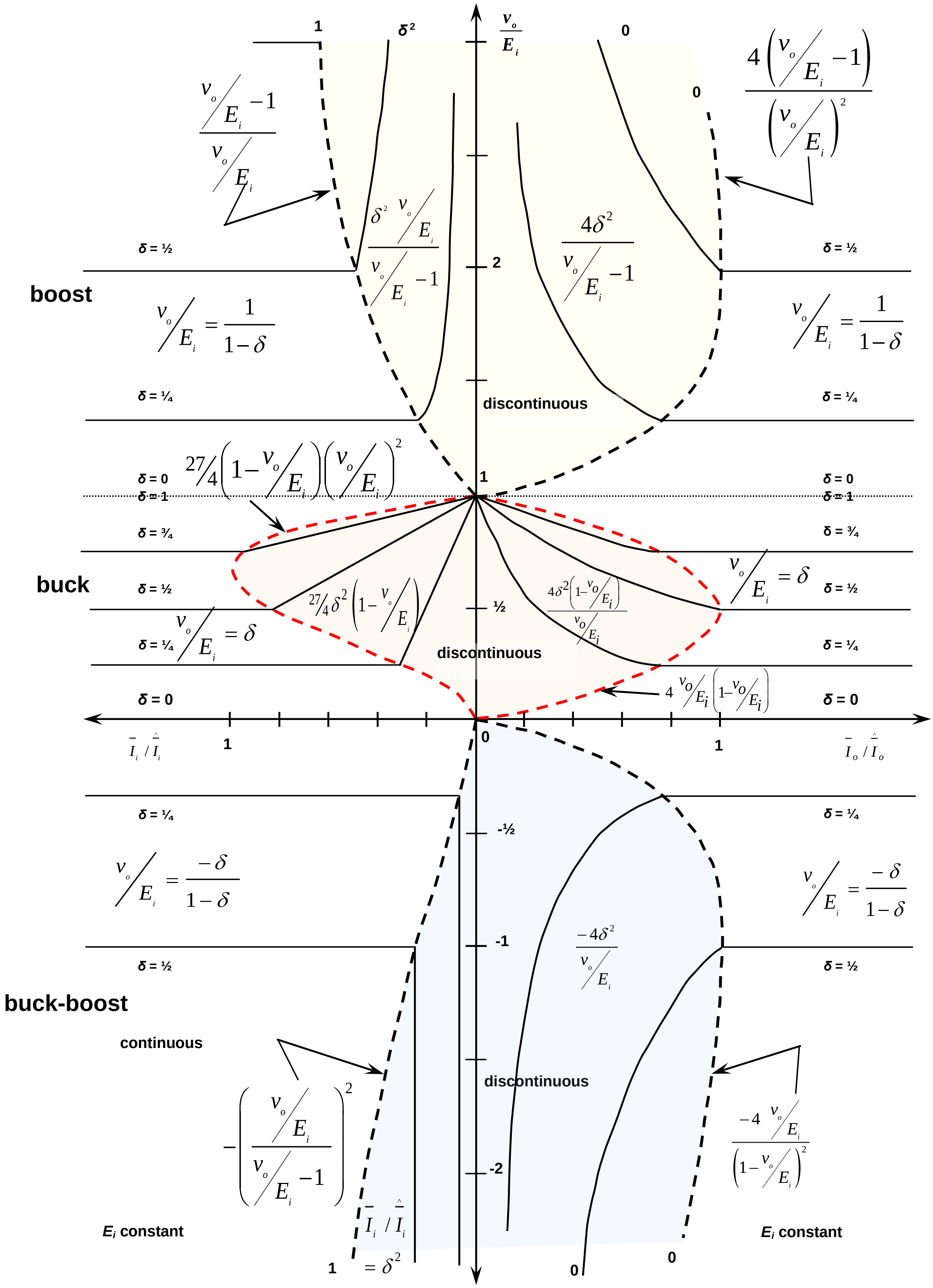


Figure 20.27. *Characteristics for three dc-dc converters, when the input voltage E_i is held constant.*

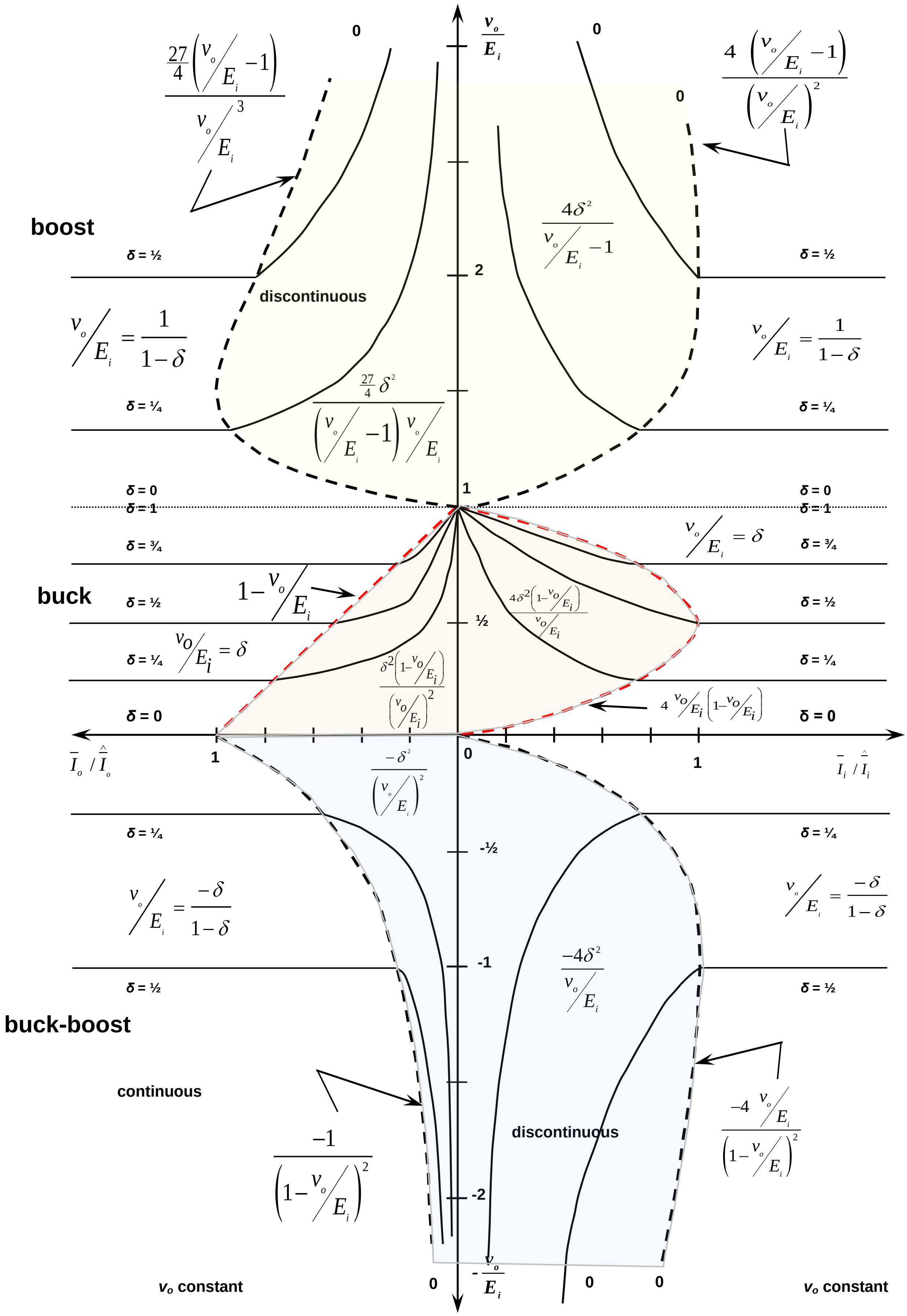


Figure 20.28. *Characteristics for three dc-dc converters, when the output voltage v_o is held constant.*

Table 20.15: Converter parameters for discontinuous and continuous inductor conduction regions and boundaries

$k=\frac{R\tau}{L};\ 0\le\delta=\frac{t_T}{\tau}\le 1$	Converter: Forward (buck) step-down		Flyback (boost) step-up		Flyback (buck-boost) step-up/down	
	discontinuous	*continuous*	*discontinuous*	*Continuous*	*discontinuous*	*continuous*
$\delta_{critical}(k)=$ $k\ge\frac{2}{\delta(1-\delta)}\times\frac{v_o}{E_i}$	$\delta\le 1-\frac{2}{k}$ $k\ge\frac{2}{1-\delta}$	$\delta\ge 1-\frac{2}{k}$ $k\le\frac{2}{1-\delta}$	$k>{}^{27}\!/_{2}$ $\delta(1-\delta)^2\le{}^{2}\!/_{k}$	$k\le{}^{27}\!/_{2}$	$\delta\le 1-\sqrt{\frac{2}{k}}$ $k\ge\frac{2}{(1-\delta)^2}$	$\delta\ge 1-\sqrt{\frac{2}{k}}$ $k\le\frac{2}{(1-\delta)^2}$
$\frac{v_o}{E_i}(k,\delta)=\frac{\bar I_i}{\bar I_o}=\bar I_o\times\frac{R}{E_i}$	$\tfrac14 k\delta^2\left[-1+\sqrt{1+\frac{8}{k\delta^2}}\right]$	δ	$\tfrac12\left[1+\sqrt{1+2k\delta^2}\right]$	$\frac{1}{1-\delta}$	$-\delta\sqrt{\tfrac12 k}$	$\frac{-\delta}{1-\delta}$
$\delta_D=\frac{t_D}{\tau}$	$\delta\times\frac{1-\frac{v_o}{E_i}}{\frac{v_o}{E_i}}$	$1-\delta$	$\delta\times\frac{1}{\frac{v_o}{E_i}-1}$	$1-\delta$	$\delta\times\frac{1}{\left\lvert\frac{v_o}{E_i}\right\rvert}$	$1-\delta$
$\delta_x=\frac{t_x}{\tau}$ $=1-(\delta+\delta_D)$	$1-\delta\times\frac{1}{\frac{v_o}{E_i}}$	0	$1\text{-}\delta\times\frac{\frac{v_o}{E_i}}{\frac{v_o}{E_i}-1}$	0	$1-\delta\times\frac{1+\left\lvert\frac{v_o}{E_i}\right\rvert}{\left\lvert\frac{v_o}{E_i}\right\rvert}$	0
$\hat I_L\times\frac{R}{E_i},\quad \check I_L\times\frac{R}{E_i}$	$k\delta\left[1-\frac{v_o}{E_i}\right],0$	$\frac{v_o}{E_i}\pm\tfrac12 k\delta(1-\delta)$	$k\delta,0$	$\frac{1}{1-\delta}\times\frac{v_o}{E_i}\pm\tfrac12 k\delta$	$k\delta,0$	$\frac{1}{1-\delta}\times\left\lvert\frac{v_o}{E_i}\right\rvert\pm\tfrac12 k\delta$
$\bar I_L\times\frac{R}{E_i}$ $\bar I_L=\tfrac12\left(\hat i_L+\check i_L\right)(\delta+\delta_D)$	$\tfrac12 k\delta^2\times\frac{1-\frac{v_o}{E_i}}{\frac{v_o}{E_i}}$	$\frac{v_o}{E_i}$	$\tfrac12 k\delta^2\times\frac{\frac{v_o}{E_i}}{\frac{v_o}{E_i}-1}$	$\frac{1}{1-\delta}\times\frac{v_o}{E_i}$	$\tfrac12 k\delta^2\times\frac{1+\left\lvert\frac{v_o}{E_i}\right\rvert}{\left\lvert\frac{v_o}{E_i}\right\rvert}$	$\frac{1}{1-\delta}\times\left\lvert\frac{v_o}{E_i}\right\rvert$
$I_c=0$ $i_L(\delta\le t\le\delta+\delta_D)\le I_o$	${}^{t}\!/_{\tau}\big\rvert_{t<t_T}={}^{\delta}\!/\,k\delta_D$	${}^{t}\!/_{\tau}\big\rvert_{t<t_T}=\tfrac12\delta\quad\forall\delta$	${}^{t}\!/_{\tau}\big\rvert_{t\le t_T}=\delta$	${}^{t}\!/_{\tau}\big\rvert_{t\le t_T}=\delta$	${}^{t}\!/_{\tau}\big\rvert_{t\le t_T}=\delta$	${}^{t}\!/_{\tau}\big\rvert_{t\le t_T}=\delta$
	${}^{t}\!/_{\tau}\big\rvert_{t_T<t\le\tau}=\delta_D-{}^{1}\!/_{k}$	$\forall\delta$ ${}^{t}\!/_{\tau}\big\rvert_{t_T<t\le\tau}=\tfrac12(1-\delta)$	$\forall\delta$ ${}^{t}\!/_{\tau}=\delta_D\left(1-{}^{{}^{v_o}\!/_{E_i}}\!/_{k\delta}\right)$ $t_T<t\le\tau$	$\delta\le 1-\sqrt{\frac{2}{k}}\qquad k\ge\frac{2}{1-\delta}\times\frac{v_o}{E_i}$ if $k\le 2$ then $\forall\delta$ ${}^{t}\!/_{\tau}\big\rvert_{t_T<t\le\tau}=\tfrac12(1-\delta)+\frac{v_o}{E_i}\Big/k$ $t_T<t\le\tau$	$\forall\delta$ ${}^{t}\!/_{\tau}=\delta_D\left(1-\frac{\left\lvert{}^{v_o}\!/_{E_i}\right\rvert}{k\delta}\right)$ $t_T<t\le\tau$	$\delta\le 1+\frac{1}{k}-\sqrt{\left(1+\frac{1}{k}\right)^2-1}$ $k\ge\frac{2}{1-\delta}\times\left\lvert\frac{v_o}{E_i}\right\rvert$ ${}^{t}\!/_{\tau}\big\rvert_{t_T<t\le\tau}=\tfrac12(1-\delta)+\left\lvert\frac{v_o}{E_i}\right\rvert\Big/k$ $t_T<t\le\tau$

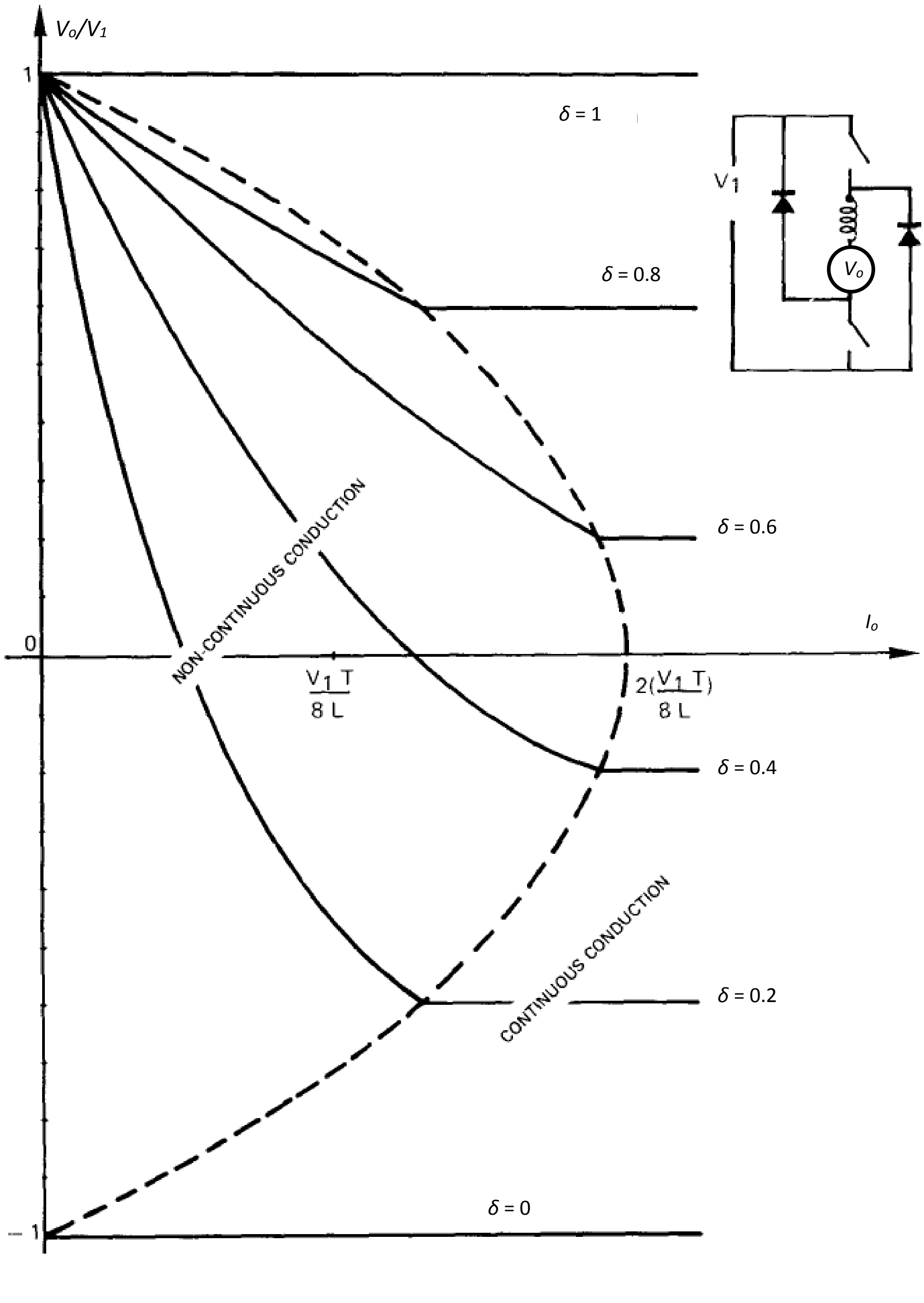


Figure 20.29. *Voltage transfer characteristics for the dc-dc converter output reversible, in terms of the output current I_o.*

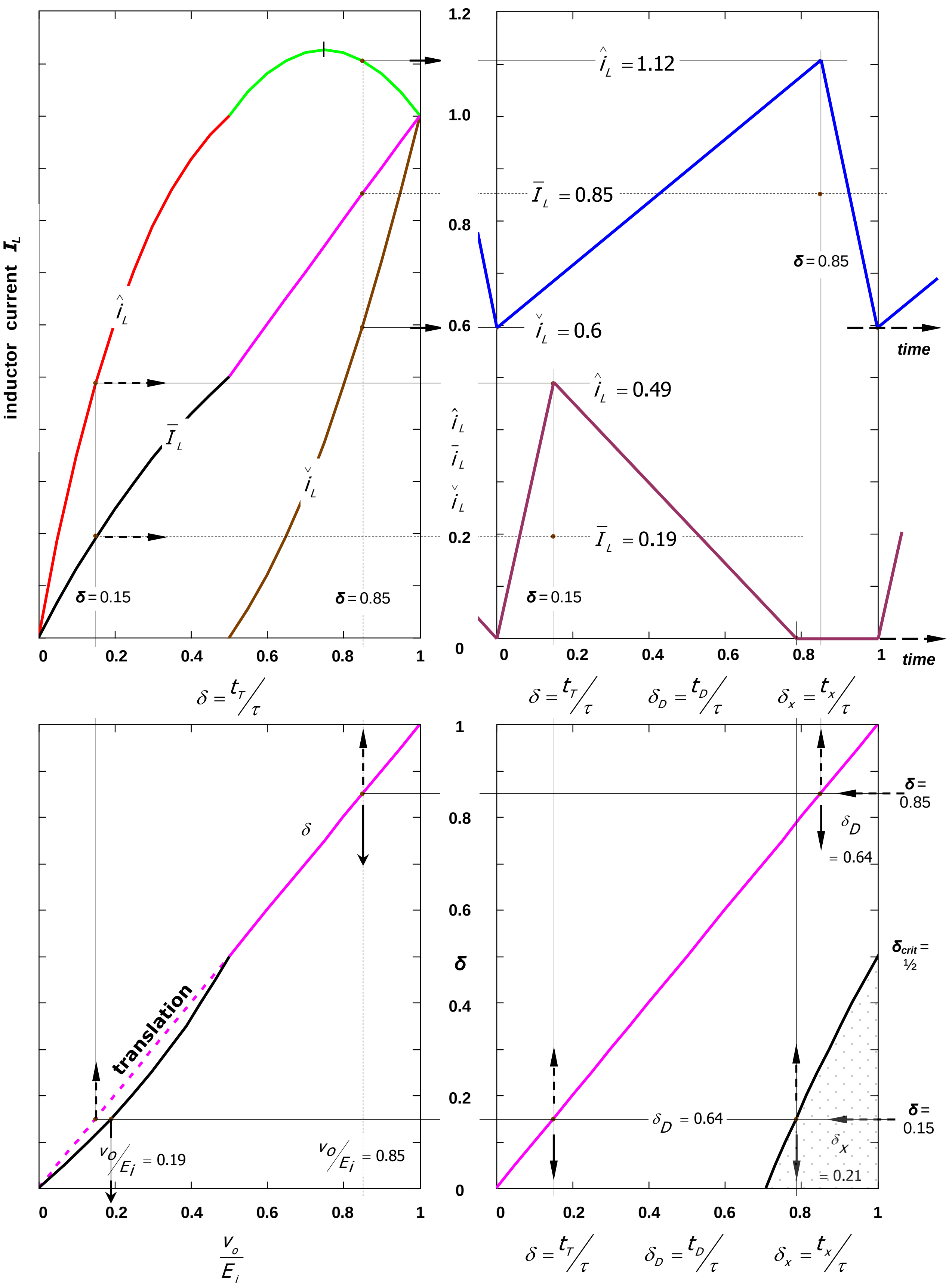


Figure 20.30. *Step-down (buck) converter normalised performance monogram for k = 4, giving discontinuous inductor conduction for $\delta_{crit} \le ½$. Inductor time domain current waveforms for δ_{cont} = 0.85 (continuous inductor current) and δ_{dis} = 0.15 (discontinuous inductor current).*

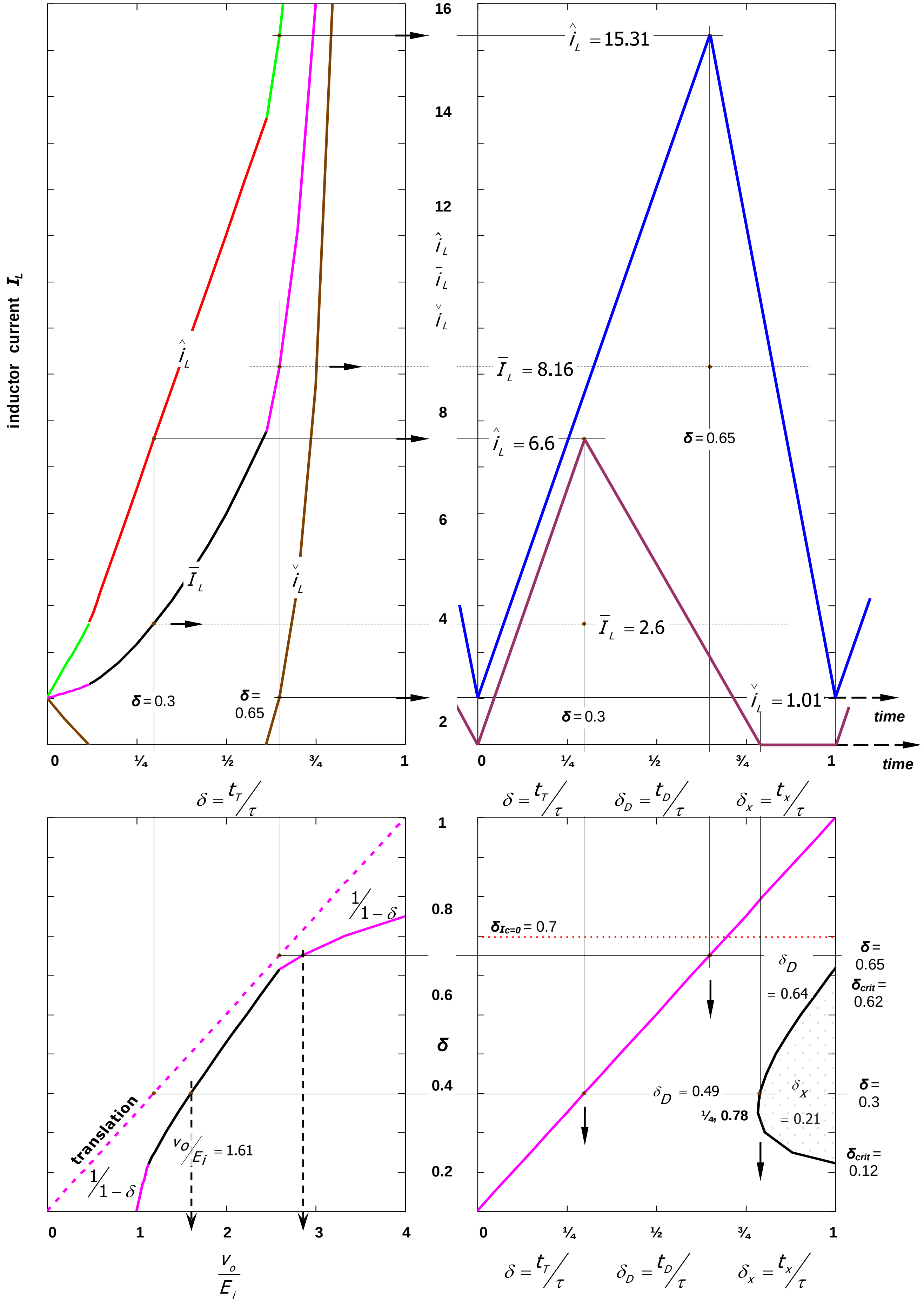


Figure 20.31. *Step-up (boost) converter normalised performance monogram for k = 22, giving discontinuous inductor current for 0.12 ≤ δ_{crit} ≤ 0.62. Inductor time domain current waveforms for δ_{cont} = 0.65 (continuous inductor current) and δ_{dis} = 0.3 (discontinuous inductor current). Capacitor discharge in switch-off period when δ ≤ 0.7.*

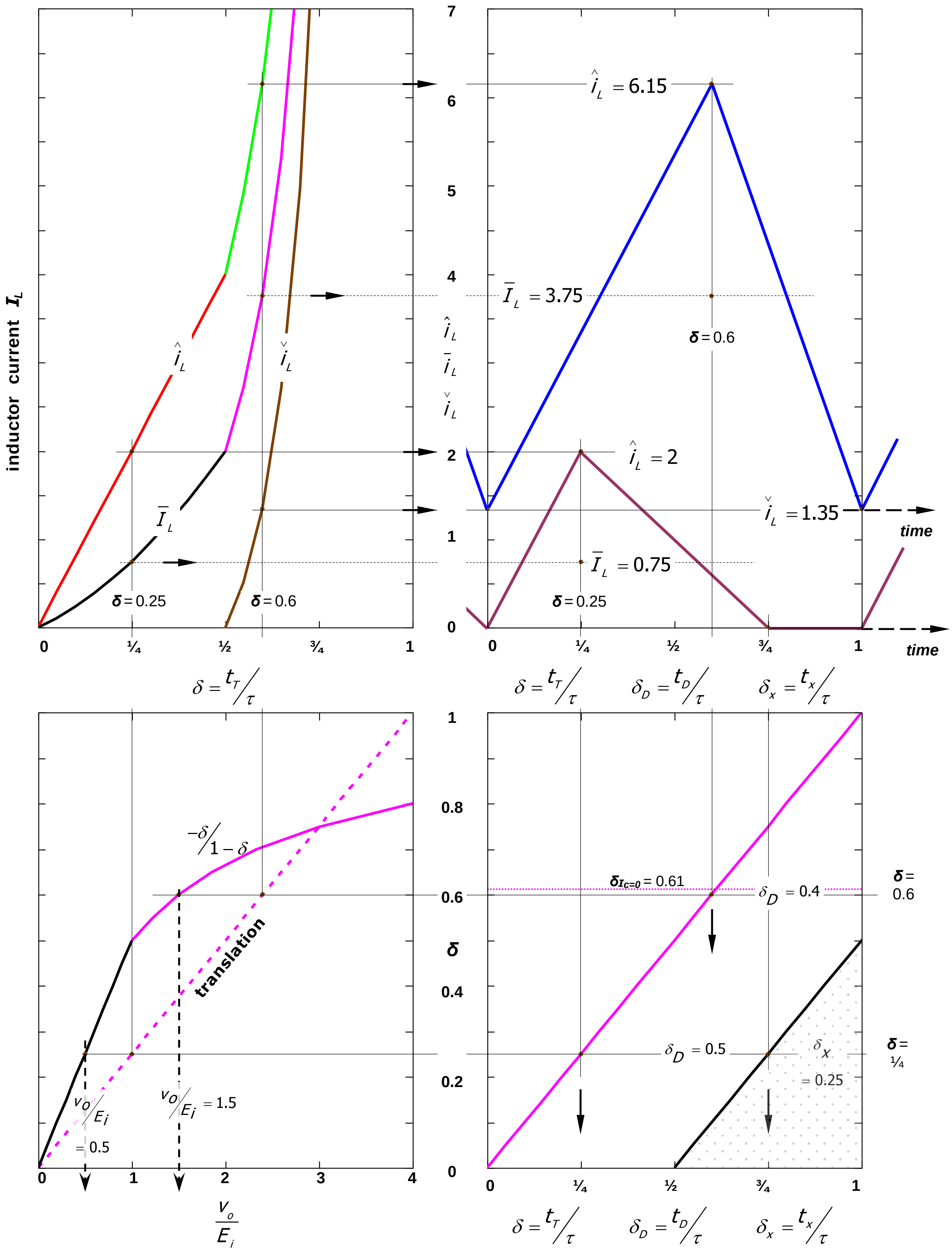


Figure 20.32. *Step-up/down (buck-boost) converter normalised performance monogram for k = 8, giving discontinuous inductor current for $\delta_{crit} \le ½$. Inductor time domain current waveforms for δ_{cont} = 0.6 (continuous inductor current) and δ_{dis} = ¼ (discontinuous inductor current). Capacitor discharge in switch-off period when δ ≤ 0.61.*

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